Accurate Velocity Estimation for Steep Fault Zones Using Full-Waveform Inversion With Edge-Guided Regularization

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ABSTRACT

Reverse-time migration has the potential to directly image steeply-dipping fault zones. However, it requires an accurate velocity model. Full-waveform inversion is a promising tool for velocity estimation. Because of the ill-posedness of full-waveform inversion, it is a great challenge to accurately obtain velocity estimation, particularly in the deep regions and fault zones. To improve velocity estimation, we develop a full-waveform inversion method with an edge-guided modified total-variation regularization scheme to improve the inversion accuracy and robustness, particularly for steeply-dipping fault zones with widths of much smaller than the seismic wavelength. The new regularization scheme incorporates the edge information into waveform inversion. The edge information of subsurface interfaces is obtained iteratively using migration imaging during waveform inversion. Seismic migration is a robust tool for subsurface imaging. Our new full-waveform inversion takes advantage of the robustness of migration imaging to improve velocity estimation. We validate the improved capability of our new full-waveform inversion method using synthetic seismic data for a complex model containing several steeply-dipping fault zones. Our inversion results are much better than those produced without using edge-guided regularization. Full-waveform inversion with an edge-guided modified total-variation regularization scheme has the potential to estimate velocities within steep fault zones, which would significantly improve direct imaging of these faults.

1. Introduction

It is crucial to directly image steeply-dipping fault zones because they may provide paths for hydrothermal flow or confine the boundaries of geothermal reservoirs. Huang et al. (2011) demonstrated that reverse-time migration has the potential to image steep fault zones, but the method requires an accurate velocity model. Full-waveform inversion (FWI) is a quantitative method for estimating subsurface geophysical properties. It is a great challenge to use FWI for practical applications (Sirgue and Pratt, 2004; Pratt et al., 1998; Tarantola, 1984; Mora, 1987). Many different methods have been developed to alleviate the ill-posedness problem of waveform inversion caused by the limited coverage, including regularization techniques (Hu et al., 2009; Burstedde and Ghattas, 2009; Ramirez and Lewis, 2010; Guitton, 2011), preconditioning approaches (Guitton and Ayeni, 2010; Tang and Lee, 2010), dimensionality reduction methods (Moghaddam and Herrmann, 2010; Habashy et al., 2011; Abubakar et al., 2011), and using *a priori* information (Ma et al., 2010; Ma and Hale, 2011).

Edges are important features for image reconstructions. It has been proved in medical imaging that the reconstruction accuracy can be significantly improved using the edge information. Dewaraja et al. (2010) used the boundary information to improve the reconstruction quality for SPECT imaging. Guo and Yin (2012) encoded the edge information in MRI reconstructions, which also yields better imaging results. Baritaux and Unser (2010) applied the edge information to the Fluorescence Diffuse Optical Tomography, and obtained enhanced reconstructions compared to results yielded without using the edge information.

We develop a full-waveform inversion method with edgeguided regularization to improve the accuracy of velocity estimation. The edge-guided regularization can be combined with any regularization techniques. In this paper, we study the improved capability of full-waveform inversion by combining the edge-guided regularization with a modified total-variation regularization.

We use synthetic data for a complex velocity model to validate the improved capability of our new full-waveform inversion for estimating velocities within steeply-dipping fault zones. The model is built using geologic features found at the Soda Lake geothermal field, and contains several steeply-dipping fault zones with widths of much smaller than the wavelength. Our results demonstrate that our new FWI method produces much more accurate estimation of velocities in the deep regions of the model and within steeply-dipping fault zones compared to those obtained without using edge-guided regularization.

2. Full-Waveform Inversion

The acoustic-wave equation in the time-domain is given by

$$\left\lfloor \frac{1}{K(\mathbf{r})} \frac{\partial^2}{\partial t^2} - \nabla \cdot \left(\frac{1}{\rho(\mathbf{r})} \nabla \right) \right\rfloor p(\mathbf{r}, t) = s(t) \delta(\mathbf{r} - \mathbf{r}_0), \tag{1}$$

where $\rho(\mathbf{r})$ is the density, $K(\mathbf{r})$ is the bulk modulus, s(t) is the source term, \mathbf{r}_0 is the source location, and $p(\mathbf{r},t)$ is the pressure field. The forward modeling using equation (1) can be written as

$$\mathbf{p} = f(\mathbf{K}, \boldsymbol{\rho}, \mathbf{s}),\tag{2}$$

where the function of f is a given nonlinear operator. Numerical techniques such as finite-difference or spectral-element methods can be used to solve equation (2). Let **m** be the model parameters, equation (2) becomes

$$\mathbf{p} = f(\mathbf{m}). \tag{3}$$

The inverse problem of equation (3) is usually posed as a minimization problem such that

$$E(\mathbf{m}) = \min_{\mathbf{m}} \left\{ \|\mathbf{p} - f(\mathbf{m})\|_{2}^{2} \right\},$$
(4)

where $E(\mathbf{m})$ is the misfit function, $\|\cdot\|_2$ stands for the L₂ norm, and **p** represents recorded waveforms. The minimization operation in equation (4) is to find a model **m** that yields the minimum difference between observed and synthetic waveforms.

3. Full-Waveform Inversion With Edge-Guided Regularization

The edge-guided regularization can be incorporated into any regularization schemes. We first provide a general form of the edge-guided regularization, and then we combine the edge-guided regularization with the modified total-variation (TV) regularization for full-waveform inversion.

A form of regularization is often written as

$$E(\mathbf{m}) = \min_{\mathbf{m}} \left\{ \left\| \mathbf{p} - f(\mathbf{m}) \right\|_{2}^{2} + \lambda \mathbf{R}(\mathbf{m}) \right\}$$
(5)

where $R(\mathbf{m})$ is the regularization term, whose form depends on the type of the regularization used. The Tikhonov regularization and the TV regularization are the most commonly used.

To incorporate the edge information, we reformulate the regularization term $R(\mathbf{m})$ as

$$\mathbf{R}(\mathbf{m}) = \mathbf{R}(w_{i,j}\,\mathbf{m}),\tag{6}$$

where the weighting parameter w controls the amount of regularization among adjacent spatial grid points. We set the weighting value as the following:

$$w_{i,j} = \begin{cases} 0 & \text{if point } (i, j) \text{ is on the edge} \\ 1 & \text{if point } (i, j) \text{ is not on the edge} \end{cases}$$
(7)

The motivation of assigning a zero weight to the points on the edges is to free them from being penalized by the regularization.

The weighting parameter w therefore relies on the detection of the edge locations.

4. Full-Waveform Inversion with Edge-Guided Modified Total-Variation Regularization

The cost function with a modified TV regularization is given by (Huang et al., 2008)

$$E(\mathbf{m}, \mathbf{u}) = \min_{\mathbf{m}, \mathbf{u}} \{ \|\mathbf{p} - f(\mathbf{m})\|_{2}^{2} + \lambda_{1} \|\mathbf{m} - \mathbf{u}\|_{2}^{2} + \lambda_{2} \|\mathbf{u}\|_{\mathrm{TV}} \}$$
(8)

where λ_1 and λ_2 are both positive regularization parameters, **u** is an auxiliary vector with a dimension equal to that of **m**, and the TV term $\|\mathbf{u}\|_{TV}$ for a 2D model is defined as the L_1 given by

$$\|\mathbf{u}\|_{\mathrm{TV}} = \|\nabla \mathbf{u}\|_{\mathrm{I}} = \sum_{i,j} \sqrt{|(\nabla_{x}\mathbf{u})_{i,j}|^{2} + |(\nabla_{z}\mathbf{u})_{i,j}|^{2}}, \qquad (9)$$

with $(\nabla_x \mathbf{u})_{i,j} = \mathbf{u}_{i+1,j} - \mathbf{u}_{i,j}$ and $(\nabla_z \mathbf{u})_{i,j} = \mathbf{u}_{i,j+1} - \mathbf{u}_{i,j}$.

To incorporate the edge information, we reformulate the TV term given by equation (9) as

$$\|\mathbf{u}\|_{\text{ETV}} = \|w\nabla\mathbf{u}\|_{1} = \sum_{i,j} \sqrt{w_{i,j}(|(\nabla_{x}\mathbf{u})_{i,j}|^{2} + |(\nabla_{z}\mathbf{u})_{i,j}|^{2})}, \quad (10)$$

where w is given by equation (7). We then obtain the edge-guided modified TV regularization scheme given by

$$E(\mathbf{m}, \mathbf{u}) = \min_{\mathbf{m}, \mathbf{u}} \{ \left\| \mathbf{p} - f(\mathbf{m}) \right\|_{2}^{2} + \lambda_{1} \left\| \mathbf{m} - \mathbf{u} \right\|_{2}^{2} + \lambda_{2} \left\| w \nabla \mathbf{u} \right\|_{1} \}.$$
(11)

We rewrite the edge-guided modified TV regularization in equation (11) as

$$E(\mathbf{m}, \mathbf{u}) = \min_{\mathbf{u}} \{ \min_{\mathbf{m}} \{ \left\| \mathbf{p} - f(\mathbf{m}) \right\|_{2}^{2} + \lambda_{1} \left\| \mathbf{m} - \mathbf{u} \right\|_{2}^{2} \} + \lambda_{2} \left\| \mathbf{u} \right\|_{\text{ETV}} \}.$$
(12)

We employ an alternating-minimization algorithm to solve the double-variable minimization problem in equation (12). Beginning with a starting model $\mathbf{u}^{(0)}$, solving equation (12) leads to the solutions of two minimization problems:

$$\begin{cases} \mathbf{m}^{(k)} = \underset{\mathbf{m}}{\operatorname{argmin}} \|\mathbf{p} - f(\mathbf{m})\|_{2}^{2} + \lambda_{1} \|\mathbf{m} - \mathbf{u}^{(k-1)}\|_{2}^{2} \\ \mathbf{u}^{(k)} = \underset{\mathbf{u}}{\operatorname{argmin}} \|\mathbf{m}^{(k)} - \mathbf{u}\|_{2}^{2} + \lambda_{2} \|\mathbf{u}\|_{\mathrm{ETV}} \end{cases}$$
(13)

for i = 1, 2, ...

5. Edge Detection

During each iteration step of full-waveform inversion, we compute forward propagation wavefields from sources and backward propagation wavefields from receivers. Therefore, we exploit these wavefields to obtain the edges of heterogeneities or interfaces of subsurface structures using reverse-time migration. Consequently, we gain the edge information during full-waveform inversion with very little additional computation costs. After the edges are determined, we apply the weighting coefficients according to equation (7).

6 Numerical Results

We use synthetic surface seismic data for the model in Fig. 1a to demonstrate the improvement of our new full-waveform

inversion method with the edge-guided modified TV regularization scheme for velocity estimation. The model is constructed using geologic features found at the Soda Lake geothermal field. It contains several steeply-dipping fault zones. There are three



(b) Initial velocity model

Figure 1. Velocity models used for FWI: (a) A velocity model from the Soda Lake geothermal field for generating synthetic seismic data; (b) A smoothed velocity model used as the starting model for FWI.



Figure 2. FWI reconstructions of the velocity model in Fig. 1a using (a) no regularization and (b) edge-guided modified TV regularization. FWI without using regularization yields a poor velocity reconstruction. FWI with edge-guided modified TV regularization produces significantly improved velocity model, particularly in the deep regions of the model and the steeply-dipping fault zones.



(b) FWI reconstruction with edge-guided modified TV regularization





Figure 4. Comparison of horizontal profiles of FWI reconstructions in Fig. 2 along the upper dashed line as shown in Fig. 1a. In each panel, the red line shows the true velocity value, the green line is the initial guess, and the blue line is the FWI reconstruction result. FWI with edge-guided modified TV regularization yields accurate velocity values. The true velocity value within the fault zones is 2125 m/s, while the velocity values of FWI with edge-guided modified TV regularization are approximately 2185 m/s, as indicated by the dashed line in (b). In contrast, the reconstructed velocity values in (a) are far away from the true value.

basalt regions in Fig. 1a with a high velocity value. Two hundred common-shot gathers of synthetic seismic data with 1000 receivers at the top surface of the model are used to invert for velocity values of the model. The shot interval is 20 m and the receiver interval is 5 m. A Ricker wavelet with a center frequency of 25 Hz is used as the source function. We plot three profiles (one vertical and two horizontal) to visualize the differences of velocity reconstructions from their true values. The locations of these three profiles are depicted with the dashed lines in Fig. 1a.

We smooth the original velocity model in Fig. 1a by averaging the slowness within two wavelengths, resulting in a model in Fig. 1b. We use this smoothed model as the starting model for FWI reconstructions.



Figure 5. Comparison of horizontal profiles of FWI reconstructions in Fig. 2 along the lower dashed line as shown in Fig. 1a. In each panel, the red line shows the true velocity value, the green line is the initial guess, and the blue line is the FWI reconstruction result. FWI with edge-guided modified TV regularization can distinguish two fault zones that are close to one another as shown in the area within the ellipse in (b), but the other method fails to separate these two fault zones as depicted in (a). In addition, as shown in the circled area in (b), FWI with edge-guided modified TV regularization accurately reconstructs the sharp interfaces between the sediment and the basalt.

Figure 2 shows FWI reconstructions with no regularization (Fig. 2a) and edge-guided modified TV regularization (Fig. 2b). The FWI result obtained with the edge-guided modified TV regularization in Fig. 2b shows accurate velocity reconstruction, particularly in the deep regions of the model that cannot be reconstructed without using any regularization. In addition, our new method greatly improves reconstruction of the steeply-dipping fault zones, as depicted in fault zone "B" in Fig. 2b.

For comparison of the velocity values estimated using the two FWI methods, we plot three profiles along three dashed lines in Fig. 1a. The vertical profile is along the center of the largest basalt region. The upper horizontal profile cuts through a sediment layer and all four fault zones, and the lower horizontal profile shows reconstructions of the large velocity contrast between the sediment and the basalt.

Figure 3 shows a comparison of the two vertical profiles of FWI reconstructions in Fig. 2. FWI without regularization produces an oscillated profile. By contrast, FWI with edge-guided modified TV regularization accurately reconstructs both velocity values and the interfaces.

Figure 4 displays the horizontal profiles of the two FWI reconstructions along the upper dashed line in Fig. 1a. Both methods reconstruct the locations of all four fault zones. FWI with edge-guided modified TV regularization significantly improves the reconstructed velocity values of the fault zones compared to FWI without regularization. The true value of the fault zones is 2125 m/s. The estimated velocity values of FWI with edge-guided modified TV regularization are approximately 2185 m/s, but the approximated reconstructed velocity value of FWI without regularization is 2450 m/s.

Figure 5 is a comparison of the horizontal profiles of the two FWI reconstructions along the lower dashed line in Fig. 1a. FWI with edge-guided modified TV regularization significantly reduces the artifacts and preserves the edges/interfaces much better than the other method, as shown in the circled areas. FWI with edgeguided modified TV regularization can distinguish two fault zones that are close to each other as depicted in the ellipses in the figure, but FWI without regularization cannot separate these fault zones.

7 Conclusions

We have developed a novel full-waveform inversion method with edge-guided modified total-variation regularization. The method employs the edge information in combination with a modified total-variation regularization scheme. We employ an alternating-minimization method to solve the optimization problem. We have validated the capability of our new full-waveform inversion method for accurate velocity reconstructions of steeplydipping fault zones and the deep regions of the model. Our full-waveform inversion results of synthetic seismic data for a Soda Lake geothermal velocity model demonstrate that our new method can accurately reconstruct not only velocity values and but also shapes of interfaces. Therefore, our novel full-waveform inversion method with edge-guided modified total-variation regularization is a powerful tool for accurate velocity estimation, which could lead to significantly improved reverse-time migration for direct imaging of steeply-dipping fault zones.

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