# TOUGH2 Flow Simulator Used to Simulate an Electric Field of a Reservoir With a Conductive Tracer for Fracture Characterization

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## ABSTRACT

This paper discusses a method for characterizing fracture paths in geothermal reservoirs using conductive fluid injection and electrical resistivity measurements. Electrical resistivity distribution of a reservoir can be determined by measuring potential differences between various points, either on the surface or inside wells, while passing an electric current through the ground. The apparent resistivity between these points decreases as conductive fluid fills up fracture paths from the injector to the producer. Therefore, the time history of the electric potential (which corresponds to the apparent resistivity) is dependent on the flow paths of the conductive fluid, i.e. the fracture network, and as a result can be used to estimate fracture characteristics.

In this study, the flow simulator TOUGH2 was first used to simulate the flow of a conductive tracer through a reservoir, and then applied to solve the electric fields by utilizing the analogy between Ohm's law that describes electrical flow and Darcy's law that describes fluid flow. A discrete fracture network was modeled and the relationship between the electric potential difference and the fracture network was studied. The fracture network was also modeled as an electric circuit and the voltage drop between an injector and a producer was calculated to verify the electric potential solved using TOUGH2. Another fracture network with one injector and three producers was analyzed as well. The apparent resistivity was mapped by kriging to illustrate the changes in resistivity with time when injecting a constant concentration solution of NaCl and into reinjection wells, resulting in increasing NaCl concentration due to steam separation.

The results from this study showed promising possibilities for characterizing fractures using electric measurements with a conductive fluid injection.

# Introduction

Fracture characterization in Enhanced Geothermal Systems (EGS) is crucial to ensure adequate supply of geothermal fluids and efficient thermal operation of the wells. The flow path characteristics control mass and heat transport in the system and inappropriate placing of injection or production wells can lead to premature thermal breakthrough. Such premature thermal breakthroughs have occurred in numerous geothermal reservoirs, as described by Horne (1982), and observed in The Geysers (Beal et al., 1994).

The goal of this study was to find ways to use direct current resistivity measurements to characterize fractures in geothermal reservoirs. Pritchett (2004) concluded based on a theoretical study that hidden geothermal resources can be explored by electrical resistivity surveys because geothermal reservoirs are usually characterized by substantially reduced electrical resistivity relative to their surroundings. The rock is normally a good insulator so the electrical current moving through the reservoir passes mainly through fluid-filled fractures and pore spaces. In these surveys, a direct current is sent into the ground through electrodes and the voltage differences between them are recorded. The input current and measured voltage difference give information about the subsurface resistivity, which can then be used to infer fracture locations. Other geophysical surveys commonly used to find hidden geothermal resources are self-potential and magnetotelluric surveys. Garg et al. (2007) described how self-potential, magnetotelluric and direct current surveys were all used to explore the Beowawe geothermal field in the Basin and Range Province of western USA. However, these surveys are usually performed on the surface with very low resolution when exploring deeper portions of the reservoirs, making it impossible to characterize fractures that are small-scaled compared to the size of the reservoir. Therefore, the possibility of placing the electrodes inside geothermal wells has been considered in this study, in order to measure the resistivity more accurately in the deeper parts of the reservoir. Due to the limited number of wells

(i.e. measurement points), the study includes investigating ways to enhance the process of characterizing fractures from sparse resistivity data.

In order to increase the contrast in resistivity between the rock and fracture zones, a conductive tracer is injected into the reservoir and the time-dependent potential difference is measured as the tracer distributes through the fracture network. Slater et al. (2000), and Singha and Gorelick (2005) have shown a way of using tracer injection with Electrical Resistivity Tomography (ERT) to observe tracer migration in experimental tanks with cross-borehole electrical imaging. In previous work, usually many electrodes were used to obtain the resistivity distribution for the whole tank at each time step. The resistivity distribution was then compared to the background distribution (without any tracer) to see resistivity changes in each block visually. These resistivity changes helped locate the saline tracer and thereby the fractures. Using this method for a whole reservoir would require a gigantic parameter space, and the inverse problem would likely not be solvable, except at very low resolution. However, in the approach considered in this study, the electrodes would be placed inside two or more geothermal wells and the potential differences between them studied. The potential difference between the wells which corresponds to changes in apparent resistivity would be measured and plotted as a function of time while the conductive tracer flows through the fracture network. The goal is to find ways to use that response, i.e. potential difference vs. time, with the tracer return curves in an inverse modeling process to characterize fracture patterns.

## The Electric Field Solved Using TOUGH2

TOUGH2 is a flow simulator that simulates fluid flow under pressure, viscous, and gravity forces according to Darcy's law. This paper describes how the flow simulator can also be used to solve Ohm's law describing the flow of an electric current due to the analogy between Darcy's law and Ohm's law, formulated by Muskat (1932). The potential distribution in steady-state flow through a porous medium is exactly the same as the potential distribution in an electrically conducting medium. Therefore, the efficiency can be increased by using TOUGH2 for both the fluid flow simulations as well as to simulate the electric current. That way, the same grid can be used when calculating the distribution of a conductive tracer in the reservoir as well as when solving the electric difference between the wells at each time step.

#### Water Flow Analogy of Electrical Flow

Ohm's law defines the relationship between electric potential, current and conductance, and can be written as,

$$J = -\sigma \nabla \phi \tag{1}$$

where J is current density  $[A/m^2]$ ,  $\sigma$  is the conductivity of the medium  $[\Omega m]$  and  $\phi$  is the electric potential [V]. Ohm's law describes the electric flow through a conductive medium instead of describing fluid flow through porous medium, but is otherwise an empirical relationship very similar to Darcy's law,

$$q = -\frac{\kappa}{\mu} \nabla p \tag{2}$$

Table 1. Analogy between electric current flow and water flow.

	Darcy's Law:	Ohm's Law:
Flux of:	Water q [m/s]	Charge J [A/m <sup>3</sup> ]
Potential:	Pressure p [Pa]	Voltage $\phi$ [V]
Medium property:	Hydraulic conductivity $\frac{k}{\mu}$ [m <sup>2</sup> /Pa·s]	Electrical conductivity $\sigma [1/\Omega m]$

where q is the flow rate [m/s], k is permeability  $[m^2]$ ,  $\mu$  is viscosity of the fluid [kg/ms] and p is pressure [Pa]. Table 1 presents the relations between the variables of Darcy's law and Ohm's law.

The similarities between these two equations show that it is possible to use a flow simulator like TOUGH2 to solve an electric field due to flow of an electric current. Then, the pressure results from TOUGH2 correspond to the electric voltage, the current density to the flow of water and the electrical conductivity corresponds to the hydraulic conductivity, i.e.

$$\sigma = \frac{k}{\mu} \tag{3}$$

Consequently, the permeability written in the TOUGH2 input file is defined as the conductivity of the field under study, multiplied by the appropriate viscosity which corresponds to the pressure (i.e. electric potential) conditions existing in the TOUGH2 simulation. However, it must be recognized that viscosity depends on pressure while conductivity of a reservoir does not depend on the electric voltage used. Also, some of the electric parameters need to be scaled when using TOUGH2 in this way.

#### Pressure Dependence of Viscosity

Magnusdottir and Horne (2012a) described how the EOS9 module in TOUGH2 was used successfully to solve an electric field due to the flow of an electric current by defining liquid viscosity, density and compressibility constant. As a result, EOS9 allows for a simulation of an electric field without the resistivity becoming dependent on the electric potential. However, problems occurred when using the EOS9 module with the Discrete Fracture Method (DFN) by Karimi-Fard et al. (2003) so the effects of pressure dependence on the simulated electric potential were studied by comparing EOS1 (which assumes pressure dependence) and EOS9, see the reference by Magnusdottir and Horne (2012b). The electric field was calculated for a simple inhomogeneous grid using both EOS1 and EOS9 and it was concluded that the difference in results due to pressure dependence was negligible, as long as the permeability is defined as the conductivity multiplied by the appropriate viscosity that corresponds to the pressure and temperature conditions in the simulation. The results of the electric field calculations using EOS1 were further verified as shown in a later section, by modeling a fracture network as an electric circuit and solving the potential drop analytically.

#### **Discrete Fracture Networks**

A Discrete Fracture Network (DFN) approach introduced by Karimi-Fard et al. (2003) was used to create realistic fracture networks by treating the fractures discretely instead of defining them by high permeability values in course-scale grid blocks. The method is based on an unstructured control volume finitedifference formulation where the element connections are assigned using a connectivity list. A MATLAB code written by Juliusson (2009) was used to generate a two-dimensional stochastic fracture network, run flow simulations on the network with TOUGH2, and plot the tracer flow results. EOS1 module in TOUGH2 was used to solve the tracer flow as well as the electric flow. Figure 1 shows the fracture network generated, where the computational grid was formed using the triangular mesh generator Triangle, developed by Shewchuk (1996).



Figure 1. A two-dimensional discrete fracture network.

The dimensions of the two-dimensional grid were  $30 \times 30 \times 1$  m<sup>3</sup> and closed (no-flow) boundary conditions were used. The porosity of the fractures was set to 0.9 and the width, *w*, was assigned as a function of the fracture length *L*,

$$w = L \cdot 10^{-4} \tag{4}$$

The corresponding permeability was determined by:

$$k = \frac{w^2}{12} \tag{5}$$

The matrix blocks were given a porosity value of 0.1 and a very low permeability value so the conductive fluid only flows through the fractures.

By using the DFN approach every element (both triangles and fracture segments) was given a transmissibility value which is related to the flow between two adjoining elements as,

$$Q_{ii} = T_{ii}(p_i - p_i) \tag{6}$$

where Q is the flow rate between gridblocks i and j, T is the transmissibility and p is the pressure. More details on the approach can be found in the reference by Karimi-Fard et al. (2003).

In Figure 1 an injection well is placed at the top of the figure and a production well at the bottom. Water was injected at the rate of  $5.6 \times 10^{-2}$  kg/s with enthalpy  $3.14 \times 10^{5}$  kJ/kg and the tracer injected was 0.1% of the water injected. The production well was modeled to deliver against a bottomhole pressure of  $10^{6}$  Pa with

productivity index of  $4 \times 10^{-12}$  m<sup>3</sup> (as specified for TOUGH2). The initial pressure was set to  $10^6$  Pa and the temperature to  $25^\circ$ C and the initial tracer mass fraction was set to  $5.42 \times 10^{-3}$ , which corresponds to ground-water. For the resistivity calculations the pores and fractures were modeled to be filled with ground-water before any tracer was injected into reservoir. The tracer was assumed to be a NaCl solution whose resistivity changes with temperature and concentration. Ucok et al. (1980) have established experimentally the resistivity of saline fluids over the temperature range 20-350°C and their results for resistivity of a NaCl solution calculated using a three-dimensional regression formula are shown in Figure 2.



Figure 2. Resistivity of NaCl solution as a function of temperature and concentration (Ucok et al., 1980).

Ucok et al. (1980) calculated that the dependence of resistivity is best represented by the formula:

$$\rho_w = b_0 + b_1 T^{-1} + b_2 T + b_3 T^2 + b_4 T^3 \tag{7}$$

where *T* is temperature and *b* are coefficients found empirically. The best fit for the concentration dependence was found to be:

$$\rho_w = 10/(\Lambda c) \tag{8}$$

where

$$\Lambda = B_0 - B_1 c^{1/2} + B_2 c \ln c + higher \ order \ terms \tag{9}$$

Coefficients B depend on the solution chemistry and c is the molar concentration.

In this project, the tracer concentration resulting from the flow simulation is changed into molar concentration and the following *B* coefficient matrix for the three-dimensional regression analysis of the data studied by Ucok et al. (1980) is used to calculate the resistivity of the NaCl solution,

Magnusdottir and Horne

	3.470	-6.650	2.633
	-59.23	198.1	64.80
B=	0.4551	-0.2058	0.005799
	-0.346E-5	7.368E-5	6.741E-5
	-1.766E-6	8.787E-7	<i>-2.136E-7</i>

Then, the resistivity of water saturated rock,  $\rho$ , is calculated using Archie's law,

$$\rho = a\varphi^{-b}\rho_w \tag{10}$$

where  $\phi$  is the porosity of the rock and *a* and *b* are empirical constants. Archie (1942) concluded that for typical sandstones of oil reservoirs the coefficient *a* is approximately 1 and *b* is approximately 2. Keller and Frischknecht (1996) showed that this power law is valid but with varying coefficients based on the rock type. In this case, *a* was set as 3.5 and *b* as 1.4. Based on Archie's law the resistivity value of each block depends on the tracer concentration in that block and the value decreases as more tracer flows into the block.

Figure 3 shows how the tracer concentration in the producer (green) changed with time as more tracer was injected into the reservoir.



Figure 3. Tracer history at the injector (blue) and at the producer (green).

The electrical resistivity method was used to examine how the potential difference history, which corresponds to the changes in resistivity, relates to the fracture network. The current was set as 1 A at the injector and as -1 A at the producer and the potential field calculated using EOS1 module in TOUGH2, see Figure 4.

The potential difference drops relatively quickly until about 0.25 days when it starts decreasing more slowly as a result of the entire fracture path from the injector to the producer becoming saturated with tracer. The relationship between the fractures and the time history of the electric potential can be made more visible by looking at the derivative of the potential difference, see Figure 5.

The first peak is after about 0.02 days when the conductive tracer reaches the production well. Figure 3 shows that the tracer concentration at the production well starts increasing at 0.02 days



Figure 4. Potential difference between the injector and the producer.



Figure 5. Derivative of the potential difference between the wells.

causing the resistivity to decrease and a low conductivity path to form between the injector and the producer, shown in Figure 6a). The electric current therefore flows through the low conductivity path, causing the electric potential difference between the wells to drop. Other peaks can be seen in Figure 5, for example after approximately 0.08 days and approximately 0.18 days. The peak after 0.08 days corresponds to a new low conductivity path formed to the left of the producer, see Figure 6c), and another path has been formed to the right after 0.18 days, see Figure 6d). The peaks of the derivative of the potential difference therefore correspond to the fracture network which verifies that the history of the electric potential could be used for fracture characterization.

## Fracture Network Analyzed as an Electric Circuit

The reservoir in Figure 1 acts in many ways like an electric circuit because the fractures form low-resistivity paths from the



Figure 6. Resistivity of the field after a) 0.01 days, b) 0.02 days, c) 0.08 days and d) 0.18 days.

injector to the producer. The electric current travels mainly through these paths due to the high resistivity of the reservoir. Figure 8 demonstrates the electric circuit that corresponds to the fracture network in Figure 7 which is the same network previously studied (Figure 1) except the width of the fractures was set as  $2 \times 10^{-3}$  m. All the fractures are assumed to be filled with ground-water with NaCl concentration equal to  $5.42 \times 10^{-3}$  and no conductive tracer has been injected into the reservoir.

The resistance, R [ohm], of the resistors in the electric circuit was calculated using the following relationship,

$$R = \frac{\rho L}{A} \tag{11}$$

where *L* [m] is the length and *A* [m<sup>2</sup>] is the cross sectional area of the corresponding water-filled fracture. The Y- $\Delta$  transformation theory published by Kennelly (1899) was used to simplify the resistors into a single equivalent resistor equal to R =  $1.2 \times 10^4$ ohm. The electric current at one end of the resistor was set as -1 A and as 1 A at the other end to simulate the



**Figure 7.** A fracture network with water-filled fractures.



**Figure 8.** An electric circuit that corresponds to the fracture network in Figure 7.

injector and the producer. The voltage drop in the electric circuit was calculated using Ohm's law (Equation 3.1) and compared to the voltage drop for the fracture network computed using module EOS1 in TOUGH2, see results in Table 2.

 Table 2. Voltage drop for a fracture network and corresponding electric circuit.

	Voltage drop [V]
Electric circuit	$1.1964 \times 10^4$
TOUGH2	$1.1980 \times 10^{4}$

The voltage drop calculated for the electric circuit is equivalent to the voltage drop computed using TOUGH2, so the EOS1 module in TOUGH2 can successfully be used to calculate the electric potential with sufficient accuracy for the procedure in this project. The difference is likely due to the pressure dependency of the viscosity, density and compressibility in EOS1, as previously analyzed, but should not affect the overall results of the fracture characterization.

# Fracture Network With Multiple Production Wells

Another Discrete Fracture Network (DFN) was modeled, this time with one injection well and three production wells, and the apparent resistivity between them was studied. The width of the fractures was set as  $10^{-2}$  m and the grid's dimensions were  $1000 \times 1000 \times 1 \text{ m}^3$ , see Figure 9.

A NaCl solution of 0.9 wt% was injected at the rate of  $1.1 \times 10^{-2}$  kg/s with enthalpy  $3.14 \times 10^{5}$  kJ/kg and the production wells modeled to deliver against a bottom hole pressure of  $10^{6}$  Pa with productivity index of  $4 \times 10^{-12}$  m<sup>3</sup>). The initial pressure was set to  $10^{6}$  Pa and the temperature to  $25^{\circ}$ C. Closed (no-flow) boundary conditions were used and the fractures were initially assumed to be filled with ground-water with  $5 \times 10^{-4}$  NaCl concentration. The electric potential differences between the wells were calculated and the apparent resistivity,  $\rho_{a}$  [ohm-m], solved using Ohm's law,

where  $\Delta \varphi$  [V] is the potential difference



**Figure 9.** A discrete fracture network with one injector and three producers.

between the wells, I [A] is the intensity of the current flowing through the network and k [m] is a geometric factor. In resistivity studies in geophysics, the total current is assumed to flow away from or toward each electrode across the surface of a half sphere, or a whole sphere if electrodes are placed underground. Here, the current flow is significantly different, because the rock is a good insulator so the current only flows through the thin fractures. Therefore, if a conventional geometric factor which only depends on the electrode spacing is used, the apparent resistivity values calculated would be very different from the true resistivity values. The volume considered for electrodes placed far apart (i.e. defined by the sphere shaped flow paths) would be much larger than for electrodes placed closer to each other, while the true increase in fracture flow path volume because of a larger distance between electrodes would be relatively small. Finding the true geometric factor is a very difficult task because the fracture characteristics are unknown, but in order to find a suitable geometric factor the potential differences between the wells before any tracer has been injected is used. It is assumed that all the wells are connected with fractures and that the resistivity of the fractures is  $\rho = 36.59$  ohm-m, corresponding to fractures with porosity 0.9 and filled with 9 ohm-m ground-water. Therefore, all the current flows through the fractures because of the high resistivity of the rock. The geometric factor, k [m], between each well pair is then calculated using Equation 12 as well as the assumed resistivity of the water-filled fractures, the known injected current, and the measured potential differences between the wells. If the fracture network was expressed as a simplified electric circuit, this geometric factor would represent the cross-sectional area of the wire, divided by its length, i.e. the length of the current path. Therefore, it corresponds to the cur-

rent flow path and could possibly be used to gain information about the fracture network. Here, it is used to calculate the apparent resistivity for the fractures, which is used for comparison at different time steps to locate where the conductive fluid is flowing.

The apparent resistivity was mapped by kriging and the general exponential-Bessel variogram was used to fit the data. Kriging is an optimal method for estimation of unknown values within known data points and was developed by Krige (1951). In this case, very few data points are known because of the few numbers of measurement points, i.e. few wells, but mapping by kriging helps illustrate the changes in resistivity as conductive tracer is injected into the reservoir. A test well is assumed to be located in a fracture in the middle of the reservoir, between all the wells to get more measurement points. First, the flow simulator TOUGH2 was used to calculate the flow of the tracer for 450 days and then to solve the electric field at different times as the tracer distributes through the network. Figure 10 shows the tracer return curves at the producers and the injected tracer concentration.

The conductive fluid travels fastest towards producer 1 because of the relatively straight path between the injector and producer 1, see Figure 9. The tracer return curves indicate more tortuous flow paths between the injector and producers 2 and 3. Figure 11 illustrates the changes in the apparent resistivity between the wells, mapped by kriging, as the conductive fluid flows through the fracture network.

At the beginning, all the fractures are filled with groundwater and therefore have the same resistivity, equal to 36.59 ohm-m. After 24 days of injection, the apparent resistivity has decreased in the upper part of the figure. Then, after 200 days



Figure 10. Tracer return curves.



**Figure 11.** Apparent resistivity [ohm-m] calculated from potential measurements between wells after a) 24 days, b) 70 days, c) 200 days and d) 450 days.



Figure 12. Resistivity [ohm-m] after 70 days of injection.

of injection, as well as after 450 days, the upper right corner has the lowest resistivity and it has changed significantly in the lower right corner as well. These changes in resistivity indicate good fracture connection from the injector to producer 2, then from producer 2 to producer 3, but lower connection towards producer 3.

Figure 12 shows the true resistivity distribution after 70 days, which is in accordance with previous results. The fracture path between producer 2 and producer 3 is the last one to fill up with conductive tracer, which causes high

potential difference between these producers and therefore high apparent resistivity in Figure 11. Considering that the changes in apparent resistivity with time gave good information about the connection between the wells, it has been demonstrated again that the time history of the electric potential should be useful for fracture characterization.

In reality, the produced fluid is likely to be reinjected into the reservoir, causing the injected fluid to be elevated in NaCl concentration with time due to the separation of steam. Therefore, the same case was studied but with the injected tracer concentration increasing in steps after 100 days. Figure 13 shows the injected tracer concentration and the tracer return curves for the three producers.

The tracer return curves indicate good connection between the injector and producer 1, but due to a weaker connection towards producers 2 and 3 the tracer concentration in these wells does not reach the injected concentration. As a result, the contrast in resistivity between the strongest and the weakest connections remains high, see Figure 14, because the majority of the tracer will always be flowing through the best connected flow path from the injector to the producers. In the previous case, where the injected water had a constant NaCl concentration, the weaker connected paths became greatly saturated as well once all the stronger connected paths were fully saturated with tracer. Therefore, the connection



Figure 13. Tracer return curves with increasing tracer injection.



**Figure 14.** Apparent resistivity [ohm-m] for reinjected fluid with elevated NaCl concentration, calculated from potential measurements between wells after 200 days (to the left) and 450 days (to the right).

between injector 1 and producer 2 could be observed after about 200 days of injection, while the mapped apparent resistivity at the same time for the reinjection case does not indicate the same connection, see Figure 14 (left). However, both examples gave some good information about the fracture connections between the wells and indicated that the time histories of the apparent resistivity ity between the wells could be used for fracture characterization.

## Conclusion

The TOUGH2 flow simulator was used successfully to calculate the electric field due to the flow of an electric current by utilizing the analogy between Ohm's law and Darcy's law. The flow simulator was both used to simulate the flow of a conductive tracer through different discrete fracture networks and to solve the electric field at each time step. The resulting changes in apparent resistivity between wells as the conductive fluid filled up fracture paths depend on the fracture network and gave promising possibilities for fracture characterization.

## **Future Work**

Future work of this project involves investigating ways to use inverse modeling with conductive tracer simulations and electric



Figure 15. The inverse analysis.

potential calculations to characterize fractures in reservoirs. In inverse modeling, the results of actual observations are used to infer the values of the parameters characterizing the system under investigation. In this study, the output parameters would be the tracer return curves at the producers and the history of the potential differences between the wells while the input parameters would include some of the fracture characteristics. The objective function measures the difference between the model calculation of the output parameters and the observed data, as illustrated in Figure 15, and a minimization algorithm proposes new parameter sets that improve the match iteratively.

Another future goal is to implement self-potential calculations into the model because the self-potential responds to fluid flow in the system. Thus, the change in measured potential difference due to self-potential could facilitate fracture characterization. It is also of interest to study the use of nanotracers and different chemical tracers. The objective is to develop a method which can be used to find where fractures are located and the character of their distribution.

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