Analytical Model for Thermal Response in a Fracture Due to a Change in Flow Rate

Egill Juliusson and Roland N. Horne

Stanford University, Department of Energy Resources Engineering, Stanford, CA, USA
egill.juliusson@gmail.com

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ABSTRACT

This paper describes an analytical model for thermal transport through a single fracture with a change in flow rate at a given point in time. The model can be used to assess the effects that a change in injection rate might have on thermal recovery in a fractured geothermal system. The model is also a useful benchmarking tool for hydrothermal reservoir simulators. Finally, the model may act as a stepping stone for the development of an analytical model for thermal transport with fully variable flow rate.

The paper describes the solution to the governing differential equation in detail. The final solution to the problem is provided in two-dimensional Laplace space. Although this solution could have been converted to real space, we chose to solve it using a numerical inversion code. The result was verified by comparison to results from the groundwater simulator FEFLOW. The two responses match with a small discrepancy that is likely due to numerical error.

Introduction

Juliusson and Horne (2011a) proposed a method for characterizing fractures using production data. The characterization was shown to be applicable to tracer transport at variable flow rate conditions, given the assumption that the interaction of the tracer with the matrix was negligible.

In Juliusson and Horne (2011b) it was shown how tracer and flow rate data could be utilized to optimize injection schedules for fractured reservoirs. The proposed model was used to predict the best set of constant injection rates for circulation through the reservoir. A possible expansion of the problem would be to search for the optimal injection schedule where the flow rates could change in time. A thermal transport model that works with variable flow rates would be needed to handle this problem. Having such a model would not only open up a wider range of possible injection configurations for exploration, but also allow more robust estimates of thermal recovery to be made after thermal decline had been observed in production wells. As a step towards this goal, we have derived an analytical thermal transport model that can handle a single change in flow rate.

Thermal Response in a Fracture Due to a Change in Flow Rate

In this section an analytical model for thermal transport through a fracture is formulated. The fracture has aperture $2b$ and is bounded by infinitely large matrix blocks. The flow rate is assumed to have a constant value, q_1, from time $t = 0$ until $t = t_c$. After that, the flow rate changes to $q_2 = q_1/\lambda$, where λ is a positive scalar. The solution to the problem before time t_c was derived by Lauwerier (1955). The solution for $t > t_c$ is similar to the one given by Kocabas (2010), although his work focused on injection-backflow tests, whereas this model is designed for tests on injector-producer doublets. The time before the change in injection is referred to as t_1 and the time after change in injection is referred to as t_2. A schematic diagram of the problem is given in Figure 1. The nomenclature for the parameters displayed in the diagram is given in Table 1.

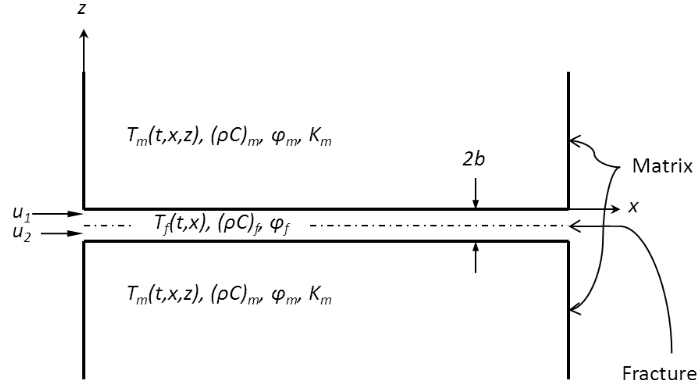

Figure 1. Schematic diagram of the thermal transport problem through a single fracture with two flow velocities.

Thermal transport within the fracture in the x-direction occurs only by advection, not conduction. Conversely, infinite thermal conductivity is assumed in the z-direction, and thus a constant temperature at any given x within the fracture. The fracture wall is also assumed to be at the same temperature as the fluid in the fracture and thermal transport from the matrix to the fracture occurs through conduction. Heat transport within the matrix occurs only through conduction in the z-direction.

Solution Before Change in Flow Rate

The solution derived in this section is very similar to the one derived by Lauwerier in 1955. The details of how that solution was obtained are not very well recorded in the 1955 publication. Laying out that process helps to explain the solution for thermal breakthrough after a change in flow rate and therefore the details are given here.

The governing equation for the variation in temperature within the fracture before the change in flow rate, i.e., before time t_c is:

$$(\rho C)_f \frac{\partial T_f}{\partial t_1} + (\rho C)_w \phi_f u_1 \frac{\partial T_f}{\partial x} - \frac{K_m}{b} \left. \frac{\partial T_m}{\partial z} \right|_{z=0} = 0 \tag{1.1}$$

Similarly, the governing equation for the matrix is:

$$(\rho C)_m \frac{\partial T_m}{\partial t_1} - K_m \frac{\partial^2 T_m}{\partial z^2} = 0 \tag{1.2}$$

where the volumetric heat capacity for the fracture is:

$$\left(\rho C\right)_f = \phi_f \left(\rho C\right)_w + (1-\phi_f)\left(\rho C\right)_r \tag{1.3}$$

and for the maxtrix:

Table 1. Nomenclature for parameters used in the thermal transport problem.

Symbol	Description
$T_f(t,x)$	Fracture temperature [C]
$T_m(t,x,z)$	Matrix temperature [C]
ρC	Volumetric heat capacity [J/m^3/C]
K	Thermal conductivity [W/m/C]
q	Volumetric flow rate [m^3/s]
u	Interstitial flow velocity, $u = q/(2bH\,\phi_f R)$, [m/s]
b	Half of fracture aperture [m]
H	Fracture height [m]
λ	Ratio between flow rates before and after t_c [-]
ϕ	Porosity [-]
R	Retardation factor, $R = 1 + \phi_m(1-\phi_f)/\phi_f$
x	Distance in the x-direction [m]
z	Distance in the z-direction [m]
t	Time [s]
s	Laplace transform variable
Subscript	**Description**
f	Bulk property of the fracture
m	Bulk property of the matrix
w	Property of liquid water
r	Property of reservoir rock
D	Dimensionless parameter
1	Variable before time t_c
2	Variable after time t_c

$$\left(\rho C\right)_m = \phi_m \left(\rho C\right)_w + (1-\phi_m)\left(\rho C\right)_r \tag{1.4}$$

All other parameters are defined in Table 1.

Initially the fracture and matrix are all at temperature T_0

$$T_f = T_m = T_0 \quad at \quad t_1 = 0 \tag{1.5}$$

At the injection point we assume a constant temperature:

$$T_f = T_i \quad at \quad x = z = 0 \tag{1.6}$$

and the temperature infinitely far away from the fracture remains at the initial temperature:

$$T_m \to T_0 \quad as \quad z \to \infty \tag{1.7}$$

Finally the requirement that the fracture wall must be at the same temperature as the fluid within the fracture gives the boundary condition:

$$T_f = T_m \quad at \quad z = 0 \tag{1.8}$$

To simplify further derivations we nondimensionalize with the following parameters:

$$T_{fD} = \frac{T_0 - T_f}{T_0 - T_i}, \qquad T_{mD} = \frac{T_0 - T_m}{T_0 - T_i},$$

$$x_D = \frac{K_m x}{\left(\rho C\right)_w \phi_f b^2 u_1}, \qquad z_D = \frac{z}{b},$$

$$t_{1D} = \frac{K_m t_1}{\left(\rho C\right)_f b^2}, \qquad \theta = \frac{\left(\rho C\right)_m}{\left(\rho C\right)_f} \tag{1.9}$$

This leads to the dimensionless governing equation for the fracture and matrix, respectively:

$$\frac{\partial T_{fD}}{\partial t_{1D}} + \frac{\partial T_{fD}}{\partial x_D} - \left. \frac{\partial T_{mD}}{\partial z_D} \right|_{z_D=0} = 0 \tag{1.10}$$

$$\theta \frac{\partial T_{mD}}{\partial t_{1D}} - \frac{\partial^2 T_{mD}}{\partial z_D^2} = 0 \tag{1.11}$$

The initial conditions become:

$$T_{fD} = T_{mD} = 0 \quad at \quad t_{1D} = 0 \tag{1.12}$$

and the boundary conditions become:

$$T_{fD} = 1 \quad at \quad x_D = z_D = 0 \tag{1.13}$$

$$T_{mD} \to 0 \quad as \quad z_D \to \infty \tag{1.14}$$

$$T_{fD} = T_{mD} = 0 \quad at \quad z_D = 0 \tag{1.15}$$

Taking the Laplace transform with respect to time converts the Equations (1.10) and (1.11) to:

$$s_1 \overline{T}_{fD} + \frac{\partial \overline{T}_{fD}}{\partial x_D} - \left. \frac{\partial \overline{T}_{mD}}{\partial z_D} \right|_{z_D=0} = 0 \tag{1.16}$$

and

$$s_1 \theta \overline{T}_{mD} - \frac{\partial^2 \overline{T}_{mD}}{\partial z_D^2} = 0 \tag{1.17}$$

The initial conditions are integrated into the governing equations but the boundary conditions become:

$$\overline{T}_{fD} = \frac{1}{s_1} \quad at \quad x_D = 0 \tag{1.18}$$

$$\overline{T}_{mD} \to 0 \quad as \quad z_D \to \infty \tag{1.19}$$

$$\overline{T}_{fD} = \overline{T}_{mD} \quad at \quad z_D = 0 \tag{1.20}$$

Equation (1.17) is a second order, linear, homogeneous ordinary differential equation which has the general solution:

$$\overline{T}_{mD} = A\left(s_1, x_D\right) e^{-\sqrt{s_1\theta} z_D} + B\left(s_1, x_D\right) e^{\sqrt{s_1\theta} z_D} \tag{1.21}$$

where A and B are unknown coefficients. Boundary condition (1.19) requires that the second term be eliminated by setting $B = 0$, so $\overline{T}_{mD} = A\left(s_1, x_D\right) e^{-\sqrt{s_1\theta} z_D}$. Differentiating with respect to z_D and setting $z_D = 0$ gives:

$$\left.\frac{\partial \overline{T}_{mD}}{\partial z_D}\right|_{z_D=0} = -A(s_1, x_D)\sqrt{s_1\theta} \tag{1.22}$$

Inserting this expression into Equation (1.16) yields a first order, linear, inhomogeneous ordinary differential equation:

$$\frac{\partial \overline{T}_{fD}}{\partial x_D} + s_1 \overline{T}_{fD} = -A(s_1, x_D)\sqrt{s_1\theta} \tag{1.23}$$

Multiplying this equation with the integrating factor $e^{s_1 x_D}$ and integrating on both sides leads to:

$$\overline{T}_{fD} = e^{-s_1 x_D}\left(\int -\sqrt{s_1\theta} A\left(s_1, \tilde{x}_D\right) e^{s_1 \tilde{x}_D} d\tilde{x}_D + C(s_1)\right) \tag{1.24}$$

Now we use boundary condition (1.20) to obtain:

$$e^{s_1 x_D} A\left(s_1, x_D\right) = \int -\sqrt{s_1\theta} A\left(s_1, \tilde{x}_D\right) e^{s_1 \tilde{x}_D} d\tilde{x}_D + C(s_1) \tag{1.25}$$

Differentiating with respect to x_D then leads to a first-order, linear, homogeneous differential equation for A:

$$\frac{\partial A}{\partial x_D} + \left(s_1 + \sqrt{s_1\theta}\right) A = 0 \tag{1.26}$$

The general solution is:

$$A\left(s_1, x_D\right) = D\left(s_1\right) e^{-\left(s_1 + \sqrt{s_1\theta}\right) x_D} \tag{1.27}$$

With this we can simplify Equation (1.24) to:

$$\overline{T}_{fD} = D\left(s_1\right) e^{-\left(s_1 + \sqrt{s_1\theta}\right) x_D} + C\left(s_1\right) e^{-s_1 x_D} \tag{1.28}$$

Referring again to boundary condition (1.20) leads to:

$$D\left(s_1\right) e^{-\left(s_1 + \sqrt{s_1\theta}\right) x_D} + C\left(s_1\right) e^{-s_1 x_D} = D\left(s_1\right) e^{-\left(s_1 + \sqrt{s_1\theta}\right) x_D} \tag{1.29}$$

and thus $C\left(s_1\right) = 0$. Finally Equation (1.18) leads to $D\left(s_1\right) = 1/s_1$, and we have the full solution in to the problem, in Laplace space, for time $t_1 < t_c$. For the fracture it is:

$$\overline{T}_{fD}\left(s_1, x_D\right) = \frac{1}{s_1} e^{-\left(s_1 + \sqrt{s_1\theta}\right) x_D} \tag{1.30}$$

and for the matrix:

$$\overline{T}_{mD}\left(s_1, x_D, z_D\right) = \frac{1}{s_1} e^{-\left(s_1 + \sqrt{s_1\theta}\right) x_D} e^{-\sqrt{s_1\theta} z_D} \tag{1.31}$$

The inverse Laplace transform of Equations (1.30) and (1.31) can be found using the following two inversion rules:

$$L\left\{\frac{1}{s} e^{-a\sqrt{s}}\right\} = erfc\left(\frac{a}{2\sqrt{t}}\right) \quad if \quad a, t > 0 \tag{1.32}$$

$$L\left\{e^{-sa}\overline{F}\left(s\right)\right\} = F\left(t-a\right) U(t-a) \tag{1.33}$$

where $erfc$ is the complementary error function, and U is the Heaviside step function. These lead to the real space solution to the problem before the change in flow rate, which is:

$$T_{fD}(t_{1D}, x_D) = erfc\left(\frac{x_D\sqrt{\theta}}{2\sqrt{t_{1D} - x_D}}\right) U(t_{1D} - x_D) \tag{1.34}$$

for the fracture, and:

$$T_{mD}\left(t_{1D}, x_D, z_D\right) = erfc\left(\frac{(x_D + z_D)\sqrt{\theta}}{2\sqrt{t_{1D} - x_D}}\right) U(t_{1D} - x_D) \tag{1.35}$$

for the matrix.

Solution After Change in Flow Rate

The governing equations for the time after t_c are similar to the equations posed by Kocabas (2010). Kocabas was investigating thermal injection-backflow tests, i.e., the case where the flow rate is reversed, and so he had a negative sign on λ. His solution focused only on the response at $x = 0$, and that lead to a different boundary condition at that point (i.e. Equation (1.40)).

After time t_c we assume that the flow rate changes from q_1 to $q_2 = q_1/\lambda$. We call the time variable starting after the change $t_2 = t_1 - t_c$. The nondimensionalization given in Equation (1.9) is applied again to the problem after time t_c and thus the governing equations become:

$$\frac{\partial T_{fD}}{\partial t_{2D}} + \frac{1}{\lambda}\frac{\partial T_{fD}}{\partial x_D} - \left.\frac{\partial T_{mD}}{\partial z_D}\right|_{z_D=0} = 0 \tag{1.36}$$

$$\theta\frac{\partial T_{mD}}{\partial t_{2D}} - \frac{\partial^2 T_{mD}}{\partial z_D^2} = 0 \tag{1.37}$$

The initial conditions are now determined by the state of the fracture and matrix at time t_c. The initial conditions for the fracture are therefore:

$$T_{fD} = T_{fD}\left(t_{1D} = t_{cD}, x_D\right) \quad at \quad t_{2D} = 0 \tag{1.38}$$

Likewise, for the matrix we have:

$$T_{mD} = T_{mD}\left(t_{1D} = t_{cD}, x_D, z_D\right) \quad at \quad t_{2D} = 0 \tag{1.39}$$

The boundary conditions are:

$$T_{fD} = 1 \quad at \quad x_D = z_D = 0 \tag{1.40}$$

$$T_{mD} \to 0 \quad as \quad z_D \to \infty \tag{1.41}$$

$$T_{fD} = T_{mD} \quad at \quad z_D = 0 \tag{1.42}$$

Now taking the Laplace transform with respect to t_1 t_1 yields the transformed governing equations:

$$\frac{\partial \overline{T}_{fD}}{\partial t_{2D}} + \frac{1}{\lambda}\frac{\partial \overline{T}_{fD}}{\partial x_D} - \left.\frac{\partial \overline{T}_{mD}}{\partial z_D}\right|_{z_D=0} = 0 \tag{1.43}$$

$$\theta \frac{\partial \overline{T}_{mD}}{\partial t_{2D}} - \frac{\partial^2 \overline{T}_{mD}}{\partial z_D^2} = 0 \tag{1.44}$$

The initial conditions become:

$$\overline{T}_{fD} = \frac{1}{s_1} e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D} \quad at \quad t_{2D} = 0 \tag{1.45}$$

and:

$$\overline{T}_{mD} = \frac{1}{s_1} e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D} e^{-\sqrt{s_1\theta}z_D} \quad at \quad t_{2D} = 0 \tag{1.46}$$

The boundary conditions change to:

$$\overline{T}_{fD} = \frac{1}{s_1} \quad at \quad x_D = z_D = 0 \tag{1.47}$$

$$\overline{T}_{mD} \to 0 \quad as \quad z_D \to \infty \tag{1.48}$$

$$\overline{T}_{fD} = \overline{T}_{mD} \quad at \quad z_D = 0 \tag{1.49}$$

A second Laplace transform, now with respect to t_2, gives:

$$s_2 \overline{\overline{T}}_{fD} + \frac{1}{\lambda}\frac{\partial \overline{\overline{T}}_{fD}}{\partial x_D} - \left.\frac{\partial \overline{\overline{T}}_{mD}}{\partial z_D}\right|_{z_D=0} = \frac{1}{s_1} e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D} \tag{1.50}$$

for the fracture. For the matrix we get:

$$s_2 \theta \overline{\overline{T}}_{mD} - \frac{\partial^2 \overline{\overline{T}}_{mD}}{\partial z_D^2} = \frac{1}{s_1} e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D} e^{-\sqrt{s_1\theta}z_D} \tag{1.51}$$

The boundary conditions become:

$$\overline{\overline{T}}_{fD} = \frac{1}{s_1 s_2} \quad at \quad x_D = z_D = 0 \tag{1.52}$$

$$\overline{\overline{T}}_{mD} \to 0 \quad as \quad z_D \to \infty \tag{1.53}$$

$$\overline{\overline{T}}_{fD} = \overline{\overline{T}}_{mD} \quad at \quad z_D = 0 \tag{1.54}$$

Equation (1.51) is a second-order, linear, inhomogeneous ordinary differential equation. A general solution to the homogeneous equation is:

$$\overline{\overline{T}}_{mD,h} = A\left(s_1,s_2,x_D\right) e^{-\sqrt{s_2\theta}z_D} + B\left(s_1,s_2,x_D\right) e^{\sqrt{s_2\theta}z_D} \tag{1.55}$$

but we immediately see that $B = 0$ from boundary condition (1.53). A particular solution for (1.51) can be found from the method of undetermined coefficients. Thus, we guess a particular solution of the form $\overline{\overline{T}}_{mD,p} = Ke^{-\sqrt{s_1\theta}z_D}$. Inserting this particular solution into Equation (1.51) gives:

$$K = \frac{e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D}}{s_1\theta(s_2-s_1)} \tag{1.56}$$

The total solution is the sum of the homogeneous and the particular solution.

$$\begin{aligned}\overline{\overline{T}}_{mD} &= A\left(s_1,s_2,x_D\right) e^{-\sqrt{s_2\theta}z_D} + \frac{e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D} e^{-\sqrt{s_1\theta}z_D}}{s_1\theta\left(s_2-s_1\right)} \\ &= A\left(s_1,s_2,x_D\right) e^{-\sqrt{s_2\theta}z_D} + \frac{\overline{T}_{fD} e^{-\sqrt{s_1\theta}z_D}}{\theta\left(s_2-s_1\right)}\end{aligned} \tag{1.57}$$

Now apply (1.54) to get:

$$A = \overline{\overline{T}}_{fD} - \frac{\overline{T}_{fD}}{\theta\left(s_2-s_1\right)} \tag{1.58}$$

So for the matrix we have:

$$\overline{\overline{T}}_{mD} = \overline{\overline{T}}_{fD} e^{-\sqrt{s_2\theta}z_D} + \frac{\overline{T}_{fD}}{\theta\left(s_2-s_1\right)}\left(e^{-\sqrt{s_1\theta}z_D} - e^{-\sqrt{s_2\theta}z_D}\right) \tag{1.59}$$

For the fracture equation (1.50) we need

$$\left.\frac{\partial \overline{\overline{T}}_{mD}}{\partial z_D}\right|_{z_D=0} = -\sqrt{s_2\theta}\,\overline{\overline{T}}_{fD} + \frac{\overline{T}_{fD}}{\sqrt{\theta s_1}+\sqrt{\theta s_2}} \tag{1.60}$$

Moving this expression into Equation (1.50) gives:

$$\begin{aligned}s_2 \overline{\overline{T}}_{fD} + \frac{1}{\lambda}\frac{\partial \overline{\overline{T}}_{fD}}{\partial x_D} + \sqrt{s_2\theta}\,\overline{\overline{T}}_{fD} - \frac{\overline{T}_{fD}}{\sqrt{\theta s_1}+\sqrt{\theta s_2}} \\ = \frac{1}{s_1} e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D} = \overline{T}_{fD}\end{aligned} \tag{1.61}$$

Rearrange to obtain:

$$\frac{\partial \overline{\overline{T}}_{fD}}{\partial x_D} + \lambda\left(s_2 + \sqrt{s_2\theta}\right)\overline{\overline{T}}_{fD} = \lambda\left(1 + \frac{1}{\sqrt{\theta s_1}+\sqrt{\theta s_2}}\right)\overline{T}_{fD} \tag{1.62}$$

The general solution to this first order, linear, inhomogeneous ordinary differential equation is:

$$\begin{aligned}\overline{\overline{T}}_{fD} = e^{-\lambda\left(s_2+\sqrt{s_2\theta}\right)x_D} \\ \left(\begin{aligned}&\int e^{\lambda\left(s_2+\sqrt{s_2\theta}\right)\tilde{x}_D}\lambda\left(1+\frac{1}{\sqrt{\theta s_1}+\sqrt{\theta s_2}}\right) \\ &\overline{T}_{fD}(s_1,\tilde{x}_D)d\tilde{x}_D + C(s_1,s_2)\end{aligned}\right)\end{aligned} \tag{1.63}$$

Evaluate the integral as:

$$\int e^{\lambda\left(s_2+\sqrt{s_2\theta}\right)\tilde{x}_D}\lambda\left(1+\frac{1}{\sqrt{\theta s_1}+\sqrt{\theta s_2}}\right)\overline{T}_{fD}(s_1,\tilde{x}_D)\,d\tilde{x}_D$$

$$= F(s_1,s_2)e^{\lambda\left(s_2+\sqrt{s_2\theta}\right)x_D}e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D} \tag{1.64}$$

where

$$F\left(s_1,s_2\right)=\frac{\lambda}{s_1}\left(1+\frac{1}{\sqrt{\theta s_1}+\sqrt{\theta s_2}}\right)$$

$$\left(\frac{1}{\lambda\left(s_2+\sqrt{s_2\theta}\right)-\left(s_1+\sqrt{s_1\theta}\right)}\right) \tag{1.65}$$

then Equations (1.63) and (1.64) give:

$$\overline{\overline{T}}_{fD}=F\left(s_1,s_2\right)e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D}+C(s_1,s_2)e^{-\lambda\left(s_2+\sqrt{s_2\theta}\right)x_D} \tag{1.66}$$

Finally we use boundary condition (1.52) to find:

$$C\left(s_1,s_2\right)=\frac{1}{s_1s_2}-F\left(s_1,s_2\right) \tag{1.67}$$

This gives the full solution for the temperature transient after the change in flow rate, in Laplace space. The solution for the fracture is:

$$\overline{\overline{T}}_{fD}=\frac{1}{s_1s_2}e^{-\lambda\left(s_2+\sqrt{s_2\theta}\right)x_D}$$

$$+F\left(s_1,s_2\right)\left(e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D}-e^{-\lambda\left(s_2+\sqrt{s_2\theta}\right)x_D}\right) \tag{1.68}$$

and the solution for the matrix is:

$$\overline{\overline{T}}_{mD}=\overline{\overline{T}}_{fD}e^{-\sqrt{s_2\theta}z_D}+\frac{e^{-\left(s_1+\sqrt{s_1\theta}\right)x_D}}{s_1\theta\left(s_2-s_1\right)}\left(e^{-\sqrt{s_1\theta}z_D}-e^{-\sqrt{s_2\theta}z_D}\right) \tag{1.69}$$

Verification and Testing of the Solution

The solutions given by (1.68) and (1.69) were verified by making sure they satisfied Equations (1.50) and (1.51) and the boundary conditions given by Equations (1.52) through (1.54).

Expression (1.68) was inverted to real space using a number of rather tedious inversion rules. The solution had to be divided into nine different terms. Four of these terms involved a single numerical integration, and four other terms required a double numerical integration, which in some cases had nonsmooth integrands. This made the evaluation of the real space solution very inefficient and prone to error. After considerable work on computing the response this way, with limited success, we decided to abandon the real space inversion approach.

An alternative way of obtaining the real space response was to use a numerical Laplace inversion code for Equation (1.68). Valkó and Abate (2005) provide one such inversion code that is designed to invert two-dimensional functions in Laplace space. Their code is written in Mathematica and relies on some of the inner workings of that software (i.e. multiprecision computing). Inversion algorithms by Stehfest (1970) and Den Iseger (2005) were also applied, but they seemed to be poorly qualified for the inversion of this particular function.

The solution was tested by comparing predictions from the analytical model to results from a full-physics finite-element simulation using a single discrete fracture in the groundwater simulator FEFLOW. In this specific case we modeled a fracture with an aperture of 1 m, height 500 m and length 600 m. The porosity of the fracture was 0.05 and the porosity of the matrix was 0.001. The flow rate started at 2,500 m^3/day, but was reduced to 1,500 m^3/day after 3,000 days. The initial temperature in the reservoir was 150 C and the injection temperature was 50 C. The volumetric heat capacity for the water was 4.2×10^6 J/m^3/C and for the rock it was 2.5×10^6 J/m^3/C. The thermal conductivity of water was 0.65 W/m/C and for the rock it was 3 W/m/C. Figure 2 gives a snapshot of the temperature in the FEFLOW model after 10,000 days.

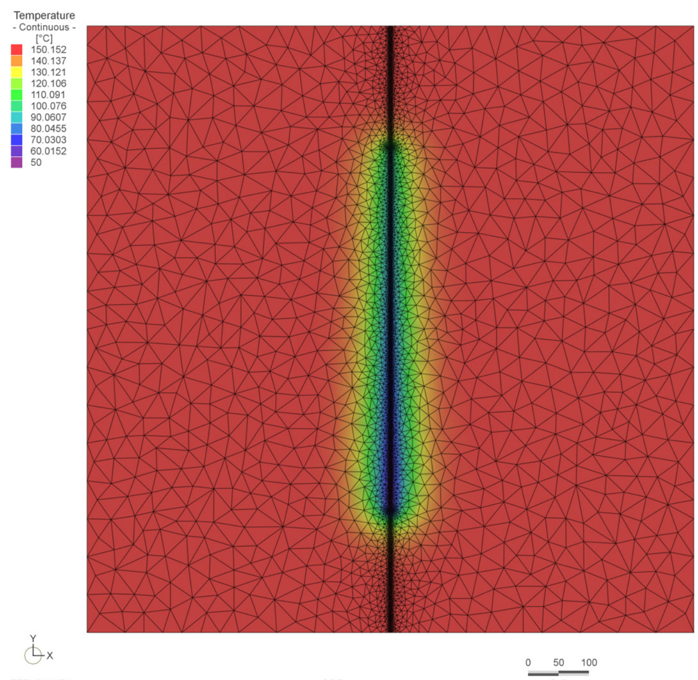

Figure 2. A snapshot of the fracture-matrix model created in FEFLOW, at the end of 10,000 days.

A comparison of the FEFLOW computation and the analytical model results is shown in Figure 3. It shows that the analytical solution is quite close the results from FEFLOW although not identical. The difference may well be attributable to minor discrepancies between our theoretical model and the actual setup of the problem in FEFLOW. There may also have been numerical errors in the FEFLOW calculations, or the numerical Laplace inversion, or both.

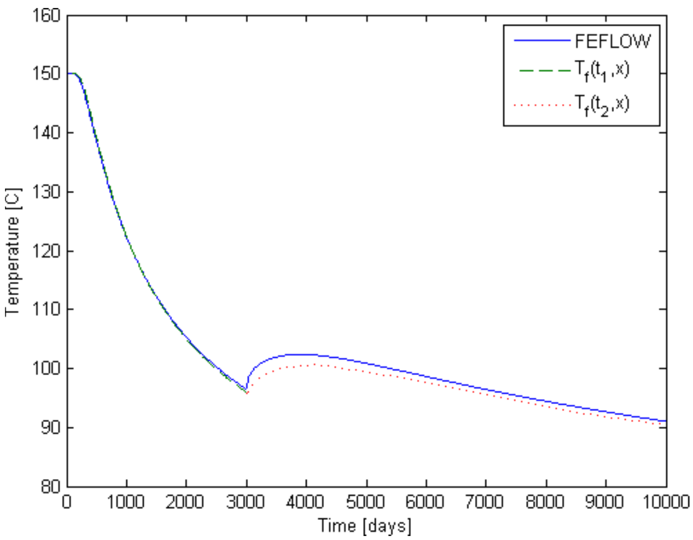

Figure 3. A comparison of the analytical solutions provided in this report and the simulated response to a change in flow rate, using the groundwater simulator FEFLOW. The green dashed line is computed from Equation (1.34) while the red dotted line is computed from a numerical inversion of Equation (1.68).

Discussion

An unfortunate drawback to the numerical solution method applied in this work is that it takes a relatively long time to evaluate the response after time t_c. For the example shown in Figure 3, it took about 20 minutes to compute the numerical inversion of Equation (1.68), but it took only around 15 minutes compute the response using the FEFLOW model (using a PC Desktop with 8 GB RAM and an Intel Core i7 processor). This would make optimizations based on this function quite slow, because the optimization would require a large number of function evaluations.

The thermal transport model presented in this paper is a function of two constant flow rate values. Although this is a step beyond solutions that have been derived previously, it would be ideal if a solution could be found for a fully variable flow rate function. Perhaps this could be done by use of a convenient change of variables, e.g., by viewing the flow rate in terms of cumulative flow, $Q(t) = \int_0^t q(\tau)\,d\tau$. With this type of solution, the thermal transient might become more useful as a characterization tool for the fracture network.

Direct measurements of temperature are not the only signals that could be used to infer thermal transport properties for interwell flow paths. It may be possible to infer the heat transfer area using either reactive tracers or thermally degrading tracers. There are some practical problems with reactive tracers because they will react with different minerals at severely different rates. This problem is avoided with thermally degrading tracers, where all one needs to estimate is the initial temperature in the system. Williams et al. (2010) discuss methods of this type in more detail.

Conclusions

An analytical model was derived for thermal transport through a fracture with a change in flow rate at a given instance in time. The model builds on assumptions similar to those used by Lauwerier (1955) and Kocabas (2010).

The analytical solution is given in two-dimensional Laplace space. Although the solution can be converted to real space, it was found more favorable to invert it numerically using the inversion code developed by Valkó and Abate (2005). The solution was verified by comparison to the results of a full-physics flow simulation in FEFLOW.

The model can be used to assess the effects that a change in injection rate will have on thermal recovery in a fractured geothermal system. For example, the production temperature may increase temporarily if the injection rate is reduced. The model would also be useful as a benchmarking tool for hydrothermal reservoir simulators. Finally, the derivation presented in this paper may act as a stepping stone for the development of an analytical model for thermal transport with fully variable flow rates.

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References

Den Iseger, P.: Numerical transform inversion using Gaussian quadrature. *Probability in the Engineering and Informational Sciences*, **20(01)**, (2005), 1–44.

Juliusson, E., and R.N. Horne: Analyzing Tracer Tests during Variable Flow Rate Injection and Production. *Proceedings of the Thirty-Sixth Workshop on Geothermal Reservoir Engineering,* Stanford University, Stanford, California, (2011a).

Juliusson, E., and R.N. Horne: Optimization of Reinjection Scheduling in Fractured Reservoirs based on Tracer Tests. *Transactions of the Geothermal Resources Council,* San Diego, California, (2011b).

Kocabas, I.: Designing Thermal and Tracer Injection Backflow Tests, *World Geothermal Congress,* (2010).

Lauwerier, H. A.: The transport of heat in an oil layer caused by the injection of hot fluid. *Applied Scientific Research*, 5(2), (1955) 145-150.

Stehfest, H.: Numerical inversion of Laplace transforms. Communications of the ACM, 13(1), (1970), 47-49.

Valkó, P. P., and Abate, J.: Numerical inversion of 2-D Laplace transforms applied to fractional diffusion equations, Applied numerical mathematics, 53(1), (2005), 73–88.

Williams, M.D., Newell, D., Vermeul, V.R., Watson, T., and Reimus, P.W.: Development of Models to Simulate Tracer Behavior in Enhanced Geothermal Systems Status Report, Pacific Northwest National Laboratory (2010), URL: http://www.pnl.gov/main/publications/external/technical_reports/PNNL-19523.pdf.