Measurement of Thermal Conductivity and Diffusivity of Drill Core Samples from the Los Humeros Geothermal Field, Mexico, by a Line-Source Technique

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Keywords

Experimental method, line heat source, thermal properties, rocks, geothermal fields

ABSTRACT

The thermal conductivity and diffusivity and the specific heat drill core samples from the Los Humeros geothermal field was measured using a technique based on the temperature distribution around an ideal line heat source of constant strength embedded in an infinite medium. The thermal properties can be determined quickly from a relatively simple experiment which requires only the measurement of the temperature history at a point during a short heating period. An experimental system for application of this technique was implemented and procedures for testing and data reduction were developed. Accuracy and reproducibility of the method were evaluated through measurements on standard materials. Tests show that thermal conductivity and diffusivity can be determined with a reproducibility of 2% within an accuracy of ±3%. Thermal properties of 15 dry and water-saturated core samples from 13 wells of the Los Humeros geothermal field (LHGF), Mexico were determined. Dry sample thermal conductivity varied from 0.97 to 1.74 W/m-K while thermal diffusivity ranged from 0.40E-02 to 1.00E-02 cm^2/s and specific heat from 711.6 to 1046.5 J/kg-K. Saturation increased these properties by 59.0, 21.4 and 27.7 %, respectively.

Introduction

Knowledge of the thermal conductivity and diffusivity and specific heat of rocks is important for many applications related to the location, evaluation and exploitation of underground energy resources as well as other areas of scientific and technological interest in which the geological formations participate. Terrestrial heat flow determination requires knowledge of thermal conductivity of rocks and the local geothermal gradient. Thermal diffusivity controls transient heat conduction; for rocks it is fundamental to study and model phenomena associated with dike injection, intrusion of magmatic bodies, energy extraction from hot dry rock and disposal of radioactive materials in underground storage.

Thermal conductivity and diffusivity and specific heat are related by:

$$\alpha = \frac{k}{\rho Cp} \tag{1}$$

Assuming that sample total density is known, it is seen that any of the three thermal properties may be determined if the other two are known. Thus, direct experimental determination of any two of these properties leads to the indirect determination of the third. In this way, the indirect determination of specific heat from a primary knowledge of thermal conductivity and diffusivity has become a common practice which, in turn, determines the need of experimental capacity availability for direct measurement of these properties.

In this context, an experimental methodology for the simultaneous determination of the thermal properties of rocks and other low to medium thermal conductivity solids from a single quick experiment was implemented and validated using standard materials (Contreras and Garcia, 1996), based on the model of Jaeger (1959). Later on, the technique was applied to determine the thermal properties of 15 drill core samples from the Los Humeros GF. The classical line heat source technique is widely used for the measurement of the thermal conductivity k of a variety of solids such as mineral aggregates, rocks, bricks, concrete, polymers, thermal insulators, soils, etc. (Marovelli and Veith, 1964; Garcia, 1994; Woodisde and Messmer, 1961) and fluids (Nieto de Castro, 1990). It has also been adopted as a standard for soils by the IEEE (IEEE, 1981). In order to benefit from the basic advantages of the line heat source method and at the same time to be able to determine the exact location of the temperature sensor, two alternative approaches (Jaeger, 1959; Lanciani et al., 1993) have been proposed which allow simultaneous determination of the thermal conductivity and diffusivity and specific heat of solids. In both cases, the experimental set-up is similar and consists of a line heat source at the sample centerline and a temperature sensor at a known radial distance from the source. The main difference between both approaches is the analysis of data. In this work, the implementation of the line source technique for simultaneously determining thermal conductivity and diffusivity and specific heat together with a data reduction procedure based on an algorithm proposed by Jaeger (1959) are described.

Mathematical Model

The mathematical model on which the experimental method is based was developed by Jaeger (1959) using the analytical solution of the heat conduction equation for an infinite medium in which an ideal line source of constant uniform strength is immersed, Figure 1. In this system, the temperature rise T(a,t) at time t from the beginning of heating at a radial distance a from the heat source is given (Carslaw and Jaeger, 1959) by:

$$T(a,t) = \frac{Q}{4\pi K} E_1\left(\frac{a^2}{4\alpha t}\right) \tag{2}$$

where $E_1(x)$ is the exponential integral which is tabulated (Abramowitz and Segun, 1964). Following Jaeger's (1959) development, the relation for the temperature rise at a given point and times t and 2t is obtained from Eq. (2) as:

$$\frac{T(a,2t)}{T(a,t)} = \frac{E_1\left(\frac{a^2}{8\alpha t}\right)}{E_1\left(\frac{a^2}{4\alpha t}\right)} \tag{3}$$

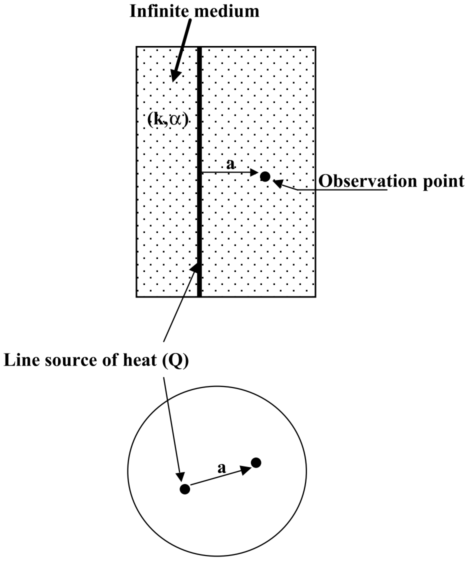

Figure 1. Schematic representation of physical model of Jaeger´s line-source method.

The right-hand side of Eq. (3) is of the form

$$G(x) = \frac{E_1(x)}{E_1(2x)}; \qquad x = \frac{a^2}{8\alpha t} \tag{4}$$

In practice, values of the relation T(2t)/T(t) for different times t are determined from an experimental record of temperature vs. time. Since these values are described by Eq. (3), use of the theoretical function $G(x) = E_1(x)/E_1(2x)$ allows determination of the argument $x = a^2/8\alpha t$ for each value of the T(2t)/T(t) relation. This is possible since the G(x) function is single valued, i.e., to each value of the argument x corresponds a unique value of the G(x) function and viceversa. G(x) can be obtained easily from tabulated values of the exponential integral). Once the argument x is obtained as described, the thermal diffusivity is calculated by solving for α form Eq. (4):

$$\alpha = \frac{a^2}{8xt} \tag{5}$$

Knowledge of the temperature-time history at a given point a also allows determination of the thermal conductivity of the material. In fact, solving for the thermal conductivity k from Eq. (2) and realizing that $(a^2/4\alpha t) = 2x$, according to the adopted notation, it follows that:

$$k = \frac{Q}{4\pi\, T(a,t)} E_1(2x) \tag{6}$$

Experimental Work

The complete experimental system includes two subsystems: (a) A heating system which applies a constant heat generation along a line source contained in the sample with simultaneous measurement of the heat generation per unit length while the temperature measurement, and (b) A recording system which is used to measure and register the temperature rise caused by the thermal perturbation as function of time at a point located at a known distance from the heat source. Details of the experimental system can be found in (Contreras and Garcia, 1996). A sample 10 cm in diameter and 12 cm long is used as the infinite medium. A 0.32 cm diameter (1/8") electrical resistance heater is tightly inserted in an axial borehole drilled at the sample cross section center. A thermocouple is inserted in a borehole 0.16 cm (1/16") in diameter by 5 cm deep at a radial distance of 1.3 cm approximately from the centerline of the heater borehole. The thermocouple is placed so that its sensing junction is in direct contact with the sample material at the borehole bottom. A constant intensity direct current is applied to the heater during a time period of 3 to 5 minutes and the temperature rise as function of time is continuously measured and recorded with a thermocouple sensing junction. Also, the voltage drop across the heater is monitored and the circulating current measured. Knowledge of these parameters along with the heater resistance and heated length, allows calculation of the source strength.

The procedure to apply the method and to determine the thermal properties from a temperature-time history is described with the aid of a real example from a test performed on a Berea sandstone.

1. From the temperature-time data, a column of $T(nt_0)$ values is generated for n=1,2,3..., up to $n_{max}=t_T/t_0$, where t_T is total time interval for data recording and t_0 is a reference time interval. A value of 2-5 sec for t_0 is adequate when total time is 3-5 min and data recording frequency is 60 data-points/min. The second column of Table 1 shows the values of $T(nt_0)$ generated. The experiment lasted 3 min

and recording frequency was 60 data-points/min. The time t_0 involved is 4.064 sec which is the time interval among four data-points.

2. The relation $T(2nt_0)/T(nt_0)$ for n=1 to $n=n_{max}/2$ is created to generate column 3 of this table.
3. The argument x for each value of $T(2nt_0)/T(nt_0)$ is determined from G(x), Eq. (4). This can be done by plotting the theoretical function G(x). A fourth column (see Table 1) is generated with the x-argument values thus obtained which represent the values of the $(a^2/8\alpha nt_0)$ group, Eq. (5).
4. Each value of x, fourth column, is multiplied times its corresponding n value from the first column, thus obtaining the $(a^2/8\alpha t_0)$ group, fifth column.
5. The fifth column is analyzed to identify the interval in which values of $(a^2/8\alpha t_0)$ remain constant within experimental fluctuation. A group average is calculated in this interval and is assigned to the $(a^2/8\alpha t_0)$ group and the thermal diffusivity is solved for. For the example case, the $(a^2/8\alpha t_0)$ group takes on essentially constant values from n=5 to n=22, with an average of 3.6246 and a standard deviation of 0.0465. From the average, the thermal diffusivity is 1.37 mm^2/sec since a=0.0127 m.
6. The values of column 4 are multiplied times 2 to generate $(a^2/4\alpha nt_0)$ values,column 6.
7. The E_1 function is evaluated using as argument the values of column 6. Column 7 is thus generated with the $E_1(a^2/4\alpha nt_0)$ relation obtained from tables of the exponential integral function of the first order (Abraowitz and Stegun, 1964) or a graph of $E_1(x)$.
8. Column 8 is generated by dividing each value of column 2 (n=1 to $n=n_{max}/2$) by the corresponding value of column 7, i.e., the same n. The column thus obtained contains values of $[T(nt_0)/E_1(a^2/4\alpha nt_0)$. According to Eq. (2), each such value equals $Q/(4\pi K)$, and these should be fairly coincident within a narrow margin of experimental fluctuation.
9. Analyis of column 8 reveals the interval where $[T(nt_0)/E_1(a^2/4\alpha nt_0)]$ remains constant within experimental fluctuation. An average value is calculated in this interval and assigned to $Q/(4\pi K)$, and the thermal conductivity is solved for. For the sample case, $[T(nt_0)/E_1(a^2/4\alpha nt_0)]$ is essentially constant from n=5 to n=22 and has a mean value of 3.1987 and a standard deviation of 0.0402. Using the heater length of 0.1143 m, R=153.1 ohm and V=40.25 volt, a thermal conductivity of 2.30 W/(m-°C) is obtained.
10. The specific heat is determined using Eq. (1). For this example, a total density of 2300 kg/m^3 was measured. This results in a value of 0.731 kJ/(kg-°C) for the specific heat.
11. As complementary to the data reduction, it is recommended to compare the experimentally measured temperature-time history with the corresponding theoretical history which results from Eq. (2) after substitution of the recently determined values of thermal conductivity and diffusivity. In this way, an objective evaluation can be performed of the degree to which the experiment developed according to the theoretical model on which the method is based. Figure 2 shows a comparison between the experimental and theoretical temperature-time histories of the sample case. An excellent agreement between both histories is readily observed.

Table 1. Example case used to illustrate the data reduction procedure of Jaeger´s method.

n	**$T(nt_0)$**	**$T(2nt_0)/T(nt_0)$**	**$x=a^2/(8\alpha nt_0)$**	$a^2/(8\alpha nt_0)$	$z=a^2/(4\alpha nt_0)$	**$E_1(z)$**	**$(Q/4\pi k)=T(nt_0)/(E_1(z))$**
1	0.0031	8.0000	1.5390	1.5390	3.0781	0.0118	0.2623
2	0.0248	7.9919	1.5381	30.7620	3.0762	0.0118	0.2623
3	0.0773	6.5149	1.3456	4.0367	2.6912	0.0194	3.9837
4	0.1982	4.1564	0.9318	3.7271	1.8636	0.0592	3.3504
5	0.3436	3.3417	0.7321	3.6607	1.4643	0.1055	3.2570
6	0.5036	2.8773	0.5994	3.5961	1.1987	0.1587	3.1728
7	0.6646	2.5960	0.5114	3.5797	1.0228	0.2112	3.1468
8	0.8238	2.4179	0.4526	3.6208	0.9052	0.2579	3.1946
9	0.9875	2.2612	0.3987	3.5887	0.7975	0.3120	3.1649
10	1.1482	2.1445	0.3574	3.5740	0.7148	0.3635	3.1591
11	1.2929	2.0714	0.3309	3.6403	0.6619	0.4021	3.2152
12	1.4490	1.9782	0.2966	3.5588	0.5931	0.4607	3.1451
13	1.5847	1.9240	0.2763	3.5915	0.5525	0.5007	3.1649
14	1.7253	1.8740	0.2573	3.6022	0.5146	0.5424	3.1806
15	1.8600	1.8332	0.2417	3.6251	0.4833	0.5805	3.2046
16	1.9919	1.7836	0.2225	3.5603	0.4450	0.6324	3.1496
17	2.1054	1.7100	0.2176	3.6991	0.4352	0.6486	3.2549
18	2.2329	1.7302	0.2016	3.6292	0.4032	0.6970	3.2037
19	2.3493	1.7077	0.1928	3.6630	0.3856	0.7272	3.2307
20	2.4623	1.6858	0.1841	3.6822	0.3686	0.7588	3.2451
21	2.5689	1.6665	0.1764	3.7051	0.3529	0.7885	3.2581
22	2.6781	1.6420	0.1667	3.6664	0.3333	0.8289	3.2307

Columns 1 and 2 continued from n=23 to n=44

n	$T(nt_0)$	n	$T(nt_0)$	n	$T(nt_0)$	n	$T(nt_0)$
23	2.7730	29	3.3319	35	3.6498	41	4.2061
24	2.8664	30	3.4097	36	3.7286	42	4.2811
25	2.9635	31	3.4885	37	3.8046	43	4.3391
26	3.0490	32	3.5528	38	3.8633	44	4.3975
27	3.1500	33	3.6498	39	3.9046		
28	3.2332	34	3.7286	40	4.1510		

Method Validation

The validation process consisted of evaluating the method accuracy and reproducibility by performing a series of tests using a fused quartz sample. Due to its purity and homogeneity, this material is widely used as a standard reference material since many of its physical, thermal, mechanical and optical properties are well characterized. Within a reasonable range of discrepancy, the values reported in the literature by several authors agree with values reported by the main material manufacturers, such as General Electric and United Fused Company who report: a thermal conductivity of 1.38 W/(m-°C), a specific heat of 0.753 kJ/(kg-°C) and a total density of 2200 kg/m^3 from which a thermal diffusivity of 0.84 mm^2/sec is obtained.

The average measured value for the present sample case was 0.82 mm^2/sec with a standard deviation of 0.03 mm^2/sec. Reported values of the thermal diffusivity of fused quartz include 0.85±0.06 mm^2/sec (Lindroth, 1974); 0.84 mm^2/sec at 20°C (Toulokian et al., 1981); 0.78- 0.82 mm^2/sec (Drury, 1984).

Using the data of the previous paragraph, the accuracy and reproducibility of the experimental method was evaluate by carrying out six independent experiments to determine the thermal conductivity and diffusivity and specific heat of a fused quartz sample were performed. The temperature history of the experiments revealed an excellent reproducibility of the physical phenomena. From this, also good reproducibility of the thermal properties values to which the primary data lead was expected and confirmed. The mean and standard deviation values in all six

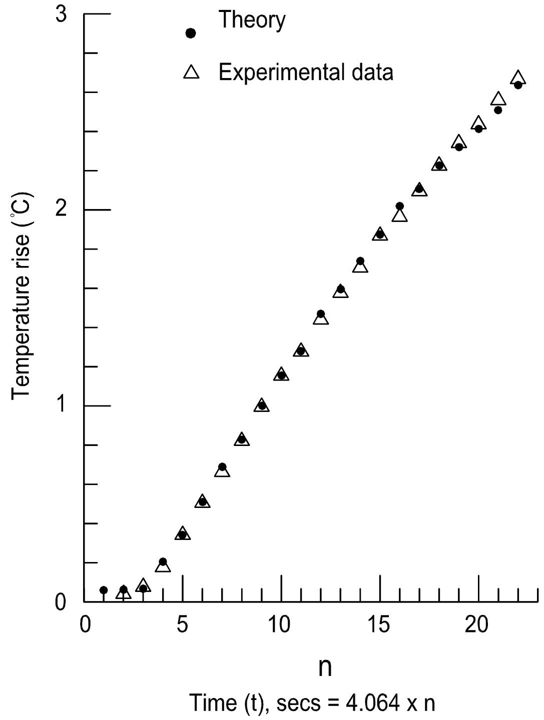

Figure 2. Comparison of the theoretical and experimental temperature-time histories of the sample case (Contreras and Garcia, 1996).

Table 2. Results of thermal conductivity, thermal diffusivity and specific heat of dry and saturated drill core samples from the Los Humeros geothermal field, Mexico.

Sample ID	Depth	Lithology (Arellano et al., 2003)	Thermal conductivity W/(m-°C)		Thermal diffusivity mm^2/sec		Specific heat J/(kg-°C)	
			Dry	Saturated	Dry	Saturated	Dry	Saturated
2.1.6	616-619	2	0.97	1.54	0.56	0.62	795.3	1046.5
4.2.2	907-910	4	1.02	1.96	0.62	0.78	753.8	1046.5
10.1.5	1469-1473	5	1.16	1.61	0.49	0.59	962.8	1088.4
10.2.6	1825-1830	6	1.54	2.29	0.69	0.92	879.1	962.8
17.1.x	2227-2230	7	1.24	2.74	0.50	0.80	962.8	1213.9
18.3.3	1750-1753	5	1.67	2.42	1.00	1.07	711.6	1188.0
19.3.8	1769-1771	5	1.03	1.91	0.40	0.63	1046.5	1172.1
20.4.17	1403-1406	5	1.23	2.19	0.66	0.87	837.2	1046.5
22.1.9	663-666	5	1.16	1.96	0.62	0.75	837.2	1088.4
23.3.4	1924-1927	5	1.42	1.82	0.67	0.68	879.1	1088.4
24.3.2	2297-2300	7	1.74	2.14	0.74	0.77	1004.6	1130.2
24.4.12	2844-2847	7	0.99	1.62	0.45	0.61	920.9	1046.5
26.3.2	1810-1813	5	1.62	1.95	0.62	0.70	962.8	1004.6
27.2.6	1500-1503	5	1.45	1.89	0.61	0.67	1004.6	1130.2
29.1.21	1200-1203	5	1.02	1.86	0.55	0.74	837.2	1046.5
Average and std. deviation values			1.28	1.99	0.61	0.74	879.1	1088.4
Increase due to water saturation			59.0%		21.4%		27.7%	

Lithology: **2:** Lithic tuffs and Zaragoza Ignimbrites; **4:** Intercalation of andesites and ignimbrites; **5:** Teziutlán augite andesites; **6:** Los Humeros vitreous tuff; **7:** Hornblende andesite

experiments of thermal diffusivity were 0.81 and 0.01 mm^2/sec; while those for thermal conductivity and specific heat were 1.39 and 0.02 W/(m-°C and 0.783 and 0.0196 kJ/(kg-°C), respectively. The excellent agreement among the thermal property values obtained from the six different measurement events indicates that the implemented method had a reproducibility of about 2% and an accuracy of ±3%.

Application to Drill Core Sample from the Los Humeros, Mexico Geothermal Field

The method described above was applied to the determination of the thermal properties of 15 drill cores from 13 geothermal wells from the Los Humeros, México geothermal field. Both dry and distilled water-saturated samples were employed. These determinations are part of a larger research program to characterize the physical, thermal and mechanical properties of a larger set of samples (Contreras and Garcia, 1989). Table 3 shows the results of this application. The data in the table include the sample identification, the sampling depth and the thermal conductivity and diffusivity and specific heat for both dry and saturated conditions. The samples ID consists of three numbers: the well number, the core number and the core fragment number. Also included is the lithology of each sample (Arellano et al., 2003). According to the validation results of the method, these results are accurate to within ±3% and their reproducibility is also within ±2%. They are also of the correct order of magnitude for similar rocks. From this table, it is possible to obtain average and standard deviation field property values and to infer the effect that water saturation of the samples has on the thermal properties of these rocks. These data are also shown at the bottom of Table 2.

Conclusions

A new technique to simultaneously determine the thermal conductivity and diffusivity and specific heat of rocks and other solids of low to medium conductivity was implemented and validated. The data is obtained from a single and fast experiment. The advantages of the method include the factibility to obtain all the thermal properties using only one sample and from a single experiment; reduced execution time; relative simplicity of application; the possibility to detect anomalies in the application of the method by analyzing the experimental data (self-testing method); excellent reproducibility and accuracy and the possibility to perform determinations on rocks at high pressures and temperatures using different saturation fluids. Validation tests results show that thermal diffusivity and conductivity may be determined with a reproducibility of 2% and within an accuracy of 3%. Application of the method to the determination of the thermal properties of dry and water saturated drill cores from the Los Humeros geothermal field proved successful.

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