Geothermal Resources Council Transactions, Vol. 28, August 29 - September 1, 2004

Averaging Model for Cuttings Transport in Horizontal Drilling

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Keywords

Two-phase, cuttings transport, solid-liquid flow, drilling modeling, volume averaging

ABSTRACT

This paper presents a theoretical analysis of the problem of cuttings transport for a two-region system composed of a fluid bed (ω-region) and a stationary bed of drill cuttings, which is considered as a porous medium (η-region) in the two-phase system. The ω-region is made up of a solid phase (σ-phase) dispersed in a continuous fluid phase (β-phase), while the η-region consists of a stationary solid phase (σ-phase) and a fluid phase (β-phase). The volume averaging method was applied in this study. Volume-averaged transport equations were derived for both the fluid bed and the porous medium regions. These equations are based on the non-local form of the volume-averaged momentum transport equation that is valid within the bounded region. Outside this region, the non-local form reduces of the equation reduces to the classic volume-averaged transport equation. From these equations, a one-equation model was obtained, and the constraints that the one-equation model must satisfy were applied.

Introduction

Due to the presence of two phases (solid and liquid) where the solid particles tend to settle at the bottom of the pipe (Doron and Barnea, 1993), the hydraulic transport of solid particles in horizontal pipes is a very complex physical phenomenon. Such phenomenon is relevant in several areas, such as the chemical, mining, geothermal and oil industries.

In the oil industry, horizontal drilling is used to exploit reservoirs exhibiting thin pay zones, to resolve the problems related to water and gas coning, to obtain greater drainage area, and to maximize the productive potential in naturally fractured reservoirs. However, a major deterrent in horizontal drilling is the reduction in performance of the transport of rocks solid fragments called cuttings transport (Cho et al., 2000).

Therefore, numerous mathematical and empirical models for the prediction of cuttings transport in horizontal and directional wells have been developed by several researchers (Cho et al., 2002; Cho et al., 2000; Kamp and Rivero, 1999; Sanchez et al., 1999; Santana et al., 1998; Nguyen and Rahman, 1998; Leising and Walton, 1998; Doron et al., 1997; Azar and Sanchez, 1997; Pilehvari et al., 1996; Nguyen and Rahman, 1996; Martins et al., 1996).

Tomren et al (1986) and Ford et al. (1990) carried out experimental work on cuttings transport in inclined and horizontal wellbores and observed the existence of different layers that might occur during the flow of mud and cuttings in a wellbore: a stationary bed, a sliding bed, and a heterogeneous suspension or clear mud.

The aim of this paper is to derive a mathematical model of cuttings transport in horizontal wellbores using the concept of a two-layer solid-liquid flow with a stationary bed and the method of volume averaging to predict the flow performance and to evaluate the effects of some important parameters which affect the mechanics of cuttings transport during horizontal well drilling. In order to accomplish the objective of this study, the model is developed using the method of volume averaging (Whitaker, 1999) which is a technique used to rigorously derive transport equations for multiphase systems and one of the main approaches in two-phase flow modeling (Espinosa-Paredes et al., 2002).

Model Description

The system under consideration is illustrated in Figure 1 (overleaf), where the fluid bed system is identified as the ω-region and the porous medium as the η-region. An exploded view of the ω-region that is made up of the solid phase (σ-phase) dispersed in a continuous fluid phase (β-phase) is also shown in Figure 1. Additionally, an exploded view of the η-region that consists of a stationary solid phase (β-phase) and the

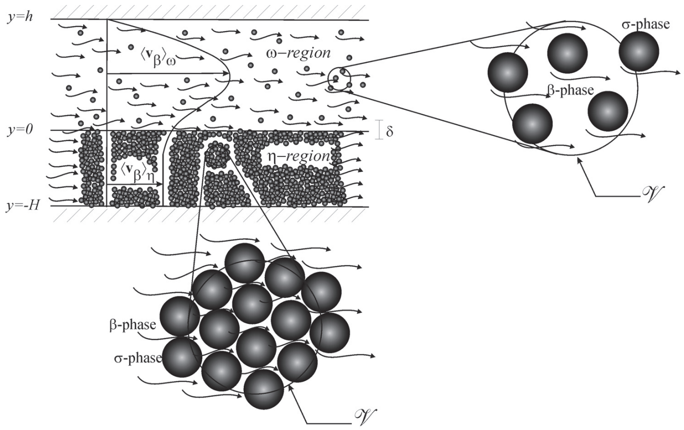

Figure 1. Cutting transport system and averaging volume.

fluid phase (β-phase) is shown there. Note that the β-phase is flowing in both the ω and the η regions.

The governing point equations, boundary (BC) and initial (IC) conditions that describe the process of momentum transfer in both the ω and the η- regions are given by:

i) ω -region

$$\nabla \cdot \mathbf{v}_\beta = 0 \qquad \text{in the } \beta\text{-phase} \tag{1}$$

$$\rho_\beta \left[\frac{\partial \mathbf{v}_\beta}{\partial t} + \nabla \cdot (\mathbf{v}_\beta \mathbf{v}_\beta) \right] = -\nabla p_\beta + \nabla \cdot \mathbf{T}_\beta + \rho_\beta \mathbf{g} \qquad \text{in the } \beta\text{-phase} \tag{2}$$

$$\nabla \cdot \mathbf{v}_\sigma = 0 \qquad \text{in the } \sigma\text{-phase} \tag{3}$$

$$\rho_\sigma \left[\frac{\partial \mathbf{v}_\sigma}{\partial t} + \nabla \cdot (\mathbf{v}_\sigma \mathbf{v}_\sigma) \right] = -\nabla p_\sigma + \nabla \cdot \mathbf{T}_\sigma + \rho_\sigma \mathbf{g} \qquad \text{in the } \sigma\text{-phase} \tag{4}$$

$$\text{I.C. 1} \qquad \mathbf{v}_\beta(\mathbf{r}, t=0) = f(\mathbf{r}) \tag{5}$$

$$\text{I.C. 2} \qquad \mathbf{v}_\sigma(\mathbf{r}, t=0) = g(\mathbf{r}) \tag{6}$$

$$\text{B.C. 1} \qquad \mathbf{v}_\beta = \mathbf{v}_\sigma \qquad \text{at the } \beta - \sigma \text{ interface} \tag{7}$$

$$\text{B.C.2} \qquad \mathbf{v}_\beta = 0 \qquad y = h \tag{8}$$

$$\text{B.C.3} \qquad (\mathbf{T}_\beta - p_\beta \mathbf{I}) \cdot \mathbf{n}_{\beta\sigma} + (\mathbf{T}_\sigma - p_\sigma \mathbf{I}) \cdot \mathbf{n}_{\sigma\beta} = 0 \qquad \text{at the } \beta - \sigma \text{ interface} \tag{9}$$

ii) η-region

$$\nabla \cdot \mathbf{v}_\beta = 0 \quad \text{in the } \beta\text{-phase} \tag{10}$$

$$0 = -\nabla p_\beta + \rho_\beta \mathbf{g} + \mu_\beta \nabla^2 \mathbf{v}_\beta \qquad \text{in the } \beta\text{-phase} \tag{11}$$

$$\text{B.C.4} \qquad \mathbf{v}_\beta = 0 \qquad \text{at the } \beta - \sigma \text{ interface} \tag{12}$$

$$\text{B.C.5} \qquad \mathbf{v}_\beta = 0 \qquad \text{y} = -\text{H} \tag{13}$$

$$\text{B.C.6} \qquad \mathbf{j} \cdot \langle \mathbf{v}_\beta \rangle = 0 \qquad \text{y} = -\text{H} \tag{14}$$

where $\mathbf{v}$, ρ, $\mathbf{T}$ are local variables representing the velocity vector, the density and the total stress tensor (laminar and turbulent), respectively, p is the pressure and $\mathbf{g}$ is the gravity acceleration vector. In the jump condition given by Eq. (9), $\mathbf{n}_{\beta\sigma}$ is the unit vector normal to the interface pointing out of the β-phase. In the solid phase, the term p_σ in Eq. (9) represents the solid phase pressure, which is due to collisional and kinetic effects (Huilin and Gidaspow, 2003).

Ochoa-Tapia and Whitaker (1995) established the set equations given by (10)-(14). These authors considered that inertial effects ($\rho_\beta \nabla \cdot \mathbf{v}_\beta \mathbf{v}_\beta$) in the η-region were negligible, while in the present work these effects were included in the ω-region. Also, in the η-region Newtonian flow is postulated.

The boundary condition at $y = -H$ has been expressed by Eq. (14) in a form that is suitable for use with Darcy´s law, thus we have used $\langle \mathbf{v}_\beta \rangle$ to represent the superficial volume-averaged velocity. The boundary conditions given by Eq. (8) and (14) are an indication of the mismatch of the length-scales that one often encounters in transport problems that involve porous media. The point boundary condition given by Eq. (8) is based on the idea that the point velocity is continuous, while the volume-averaged boundary condition given by Eq. (14) is an approximation based on the idea that the interface at $y = -H$ is impermeable.

As stated previously, two homogeneous regions are distinguished: a homogeneous ω-region and a homogeneous η- region both constituted by the σ-phase and the β-phase (Figure 1). We denominate homogeneous regions those portions of the system that are not influenced by the rapid changes that occur in the boundary between regions. Then, to study the process that takes place in two-phase flow, volume-averaged transport equations that are valid within both homogeneous regions need to be developed. Thus, the averaging model for the ω-region that describes cuttings transport at the macroscopic level is developed in this paper.

In order to describe the process of cuttings transport illustrated in Figure 1, the volume-averaged form of the equations (1)-(4) are developed (ω-region). Development of such mathematical model for this system is relatively straightforward when classic length-scales constraints are satisfied (Zanotti and Carbonell, 1984; Carbonell and Whitaker, 1984), however difficulties arise in the neighborhood of the ω-η boundary where there exist rapid changes in the liquid volume fraction and the length-scale constraints fail.

The Method of Volume Averaging

The superficial volume average of some function Ψ_β associated with the β-phase is defined as:

$$\langle \psi_\beta \rangle \big|_{\mathbf{x}} = \frac{1}{\mathscr{V}} \int_{V_\beta(\mathbf{x},t)} \psi_\beta(\mathbf{x} + \mathbf{y}_\beta, t) dV_y \tag{15}$$

where $V_\beta(\mathbf{x}t)$ is the volume of the β-phase contained within the averaging volume $\mathscr{V}$ illustrated in Figure 2. In this figure, it is indicated that $\mathbf{x}$ represents the position vector locating the centroid of the averaging volume, while $\mathbf{y}_\beta$ represents the position vector locating points in the β-phase relative to the centroid. In Eq. (15) dV_y is used to indicate that the integration is carried out with respect to the components of $\mathbf{y}_\beta$, and the nomenclature used in Eq. (15) clearly indicates that volume-averaged quantities are associated with the centroid. In Eq. (15), $\mathscr{V} = V_\sigma + V_\beta$ and is independent of space and time, however V_σ and V_β depend on $\mathbf{x}$ and on t.

In order to simplify the notation, the precise nomenclature used in Eq. (15) will be avoided and the superficial average of Ψ_β will be represented as:

$$\langle\psi_\beta\rangle = \frac{1}{\mathscr{V}}\int_{V_\beta}\psi_\beta\,dV \tag{16}$$

while the intrinsic average is expressed in the form:

$$\langle\psi_\beta\rangle^\beta = \frac{1}{V_\beta}\int_{V_\beta}\psi_\beta\,dV \tag{17}$$

Both of these averages will be used in the theoretical development of this paper. They are related by:

$$\langle\psi_\beta\rangle = \varepsilon_\beta\langle\psi_\beta\rangle^\beta \tag{18}$$

The liquid volume fraction ε_β is defined explicitly as:

$$\varepsilon_\beta(\mathbf{x},t) = \frac{V_\beta(\mathbf{x},t)}{\mathscr{V}} \tag{19}$$

It should be clear that the liquid volume fraction ε_β is a function of position and depends of the sampling point located by $\mathbf{x}$.

In addition to the definitions given by Eqs. (16)-(19), use is made of the spatial averaging theorem and general transport theorem given by Gray and Lee (1977)

$$\langle\nabla\psi_\beta\rangle = \nabla\langle\psi_\beta\rangle + \frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\psi_\beta dA \tag{20}$$

$$\left\langle\frac{\partial\psi_\beta}{\partial t}\right\rangle = \frac{\partial\langle\psi_\beta\rangle}{\partial t} - \frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\psi_\beta\mathbf{w}\cdot\mathbf{n}_{\beta\sigma}dA \tag{21}$$

where $\mathbf{w}$ is the interface velocity.

Two-fluid Model for the ω-Region

Following the nomenclature given by Eqs. (16) to (21), the intrinsic averages of Eqs. (1)-(4) are expressed as:

$$\frac{\partial\varepsilon_\beta}{\partial t} + \langle\mathbf{v}_\beta\rangle^\beta\cdot\nabla\varepsilon_\beta + \varepsilon_\beta\nabla\cdot\langle\mathbf{v}_\beta\rangle^\beta = 0 \tag{22}$$

$$\begin{aligned}&\underbrace{\rho_\beta\frac{\partial}{\partial t}(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta)}_{accumulation} + \underbrace{\rho_\beta\nabla\cdot\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle}_{convection} = \\ &-\underbrace{\nabla\langle p_\beta\rangle}_{pressure} + \underbrace{\nabla\cdot\langle\mathbf{T}_\beta\rangle}_{stress\ tensor} + \underbrace{\varepsilon_\beta\rho_\beta\mathbf{g}}_{gravity} \\ &+\underbrace{\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot\mathbf{T}_\beta dA - \frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}p_\beta dA}_{interfacial\ force}\end{aligned} \tag{23}$$

$$\frac{\partial\varepsilon_\sigma}{\partial t} + \langle\mathbf{v}_\sigma\rangle^\sigma\cdot\nabla\varepsilon_\sigma + \varepsilon_\sigma\nabla\cdot\langle\mathbf{v}_\sigma\rangle^\sigma = 0 \tag{24}$$

$$\begin{aligned}&\underbrace{\rho_\sigma\frac{\partial}{\partial t}(\varepsilon_\sigma\langle\mathbf{v}_\sigma\rangle^\sigma)}_{accumulation} + \underbrace{\rho_\sigma\nabla\cdot\langle\mathbf{v}_\sigma\mathbf{v}_\sigma\rangle}_{convection} = \\ &-\underbrace{\nabla\langle p_\sigma\rangle}_{pressure} + \underbrace{\nabla\cdot\langle\mathbf{T}_\sigma\rangle}_{stress\ tensor} + \underbrace{\varepsilon_\sigma\rho_\sigma\mathbf{g}}_{gravity} \\ &+\underbrace{\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\sigma\beta}\cdot\mathbf{T}_\sigma dA - \frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\sigma\beta}p_\sigma dA}_{interfacial\ force}\end{aligned} \tag{25}$$

On the basis that the cuttings (σ- phase) are rigid, then the interface between β-phase and σ- phase is rigid, i.e., $\mathbf{w}\cdot\mathbf{n}_{\beta\sigma} = \mathbf{v}_\beta\cdot\mathbf{n}_{\beta\sigma}$ and this result was used to simplify the Eqs. (22)-(25).

It is important to note that no length-scale constraints have been imposed on the volume-averaged transport equations. The absence of any length-scale constraint simply means that Eqs. (22)-(25) are also valid in the boundary between the ω-and η-regions.

To eliminate the average of a product, we make use of the velocity decomposition given by Gray (1975)

$$\mathbf{v}_\beta = \langle\mathbf{v}_\beta\rangle^\beta + \tilde{\mathbf{v}}_\beta \tag{26}$$

Following the work of Carbonell and Whitaker (1983), we neglect the variation of average quantities within the averaging volume so that the convective term takes the form

$$\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle = \varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta\langle\mathbf{v}_\beta\rangle^\beta + \langle\tilde{\mathbf{v}}_\beta\tilde{\mathbf{v}}_\beta\rangle \tag{27}$$

Use of Eq. (27) require the imposition of length-scale constraints (Carbonell and Whitaker, 1984; Quintard and Whitaker, 1994), which are not valid within the boundary region between the ω η regions. To avoid imposing length-scale constrains, the excess convective terms are defined in a way similar to that of Ochoa-Tapia and Whitaker (1997):

$$\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle_{exc} = \langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle - \varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta\langle\mathbf{v}_\beta\rangle^\beta - \langle\tilde{\mathbf{v}}_\beta\tilde{\mathbf{v}}_\beta\rangle \tag{28}$$

$$\langle\mathbf{v}_\sigma\mathbf{v}_\sigma\rangle_{exc} = \langle\mathbf{v}_\sigma\mathbf{v}_\sigma\rangle - \varepsilon_\sigma\langle\mathbf{v}_\sigma\rangle^\sigma\langle\mathbf{v}_\sigma\rangle^\sigma - \langle\tilde{\mathbf{v}}_\sigma\tilde{\mathbf{v}}_\sigma\rangle \tag{29}$$

based on the idea that $\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle_{exc}$=0, $\langle\mathbf{v}_\sigma\mathbf{v}_\sigma\rangle_{exc}$=0 in the homogeneous regions, i.e., when the length-scale constraints l_β, $l_\sigma << r_o << L$ (where l_β are the characteristic lengths associated with the β, σ-phases, r_o is the radius of the averaging volume and L is the large length-scale) are fulfilled. The excess convective term will not be equal to zero in the $\omega-\eta$ boundary, where the length-scale constraints developed by Carbonell and Whitaker (1984) are not valid.

Then, the convective term in Eq. (23) can be expressed as:

$$\nabla\cdot\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle = \underbrace{\nabla\cdot(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta\langle\mathbf{v}_\beta\rangle^\beta)}_{convection}$$

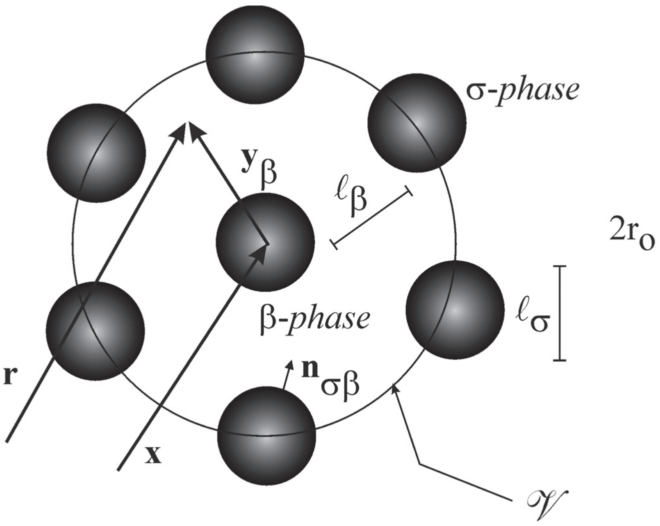

Figure 2. Position vectors associated with the averaging volume.

$$+\underbrace{\nabla\cdot\langle\tilde{\mathbf{v}}_\beta\tilde{\mathbf{v}}_\beta\rangle}_{dispersion}+\underbrace{\nabla\cdot\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle_{exc}}_{nonlocal\ dispersion} \tag{30}$$

Here, $\nabla\cdot\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle_{ex}$ is identified as a non-local term since it indirectly involves values of $\langle\mathbf{v}_\beta\rangle^\beta$ that are not associated with the centroid of the averaging volume illustrated in Figure 2. Substituting Eq. (30) into Eq. (23) leads to a form that contains the traditional convective and dispersive transport terms in addition to the excess dispersion:

$$\underbrace{\rho_\beta\frac{\partial}{\partial t}(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta)}_{accumulation}+\underbrace{\rho_\beta\nabla\cdot(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta\langle\mathbf{v}_\beta\rangle^\beta)}_{convection}$$
$$+\underbrace{\rho_\beta\nabla\cdot\langle\tilde{\mathbf{v}}_\beta\tilde{\mathbf{v}}_\beta\rangle}_{dispersion}+\underbrace{\rho_\beta\nabla\cdot\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle_{exc}}_{non-local\ dispersion}=$$
$$-\underbrace{\nabla(\varepsilon_\beta\langle p_\beta\rangle^\beta)}_{pressure}+\underbrace{\nabla\cdot\langle\mathbf{T}_\beta\rangle}_{stress\ tensor}$$
$$+\underbrace{\varepsilon_\beta\rho_\beta\mathbf{g}}_{gravity}+\underbrace{\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot[-\mathbf{I}p_\beta+\mathbf{T}_\beta]dA}_{interfacial\ force} \tag{31}$$

In order to obtain a volume-averaged transport equation that contains only average quantities and spatial deviations, the interfacial area integral in this equation is developed with the same idea given by Ochoa-Tapia and Whitaker (1998)

Using the volume-averaging theorem given by Eq. (20) and letting $\Psi_\beta=1$, the following result is obtained:

$$\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}dA=-\nabla\varepsilon_\beta \tag{32}$$

which can be used to write:

$$\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\langle p_\beta\rangle^\beta\Big|_{\mathbf{x}}dA=-\mathbf{I}\langle p_\beta\rangle^\beta\Big|_{\mathbf{x}}\cdot\nabla\varepsilon_\beta \tag{33}$$

$$\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot\langle\mathbf{T}_\beta\rangle^\beta\Big|_{\mathbf{x}}dA=-\langle\mathbf{T}_\beta\rangle^\beta\Big|_{\mathbf{x}}\cdot\nabla\varepsilon_\beta \tag{34}$$

The convention used here is that terms evaluated at the centroid can be removed from the integral. Use of this result, together with Eq. (31), leads to the following expression for the volume-averaged momentum equation:

$$\rho_\beta\frac{\partial}{\partial t}(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta)+\rho_\beta\nabla\cdot(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta\langle\mathbf{v}_\beta\rangle^\beta+\underbrace{\rho_\beta\nabla\cdot\langle\tilde{\mathbf{v}}_\beta\tilde{\mathbf{v}}_\beta\rangle}_{dispersion}$$
$$+\underbrace{\rho_\beta\nabla\cdot\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle_{exc}}_{non-local\ dispersion}=-\varepsilon_\beta\nabla\langle p_\beta\rangle^\beta+\varepsilon_\beta\nabla\cdot(\varepsilon_\beta^{-1}\langle\mathbf{T}_\beta\rangle)+\varepsilon_\beta\rho_\beta\mathbf{g}$$
$$+\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot[-\mathbf{I}(p_\beta\big|_{\mathbf{x}+\mathbf{y}}-\langle p_\beta\rangle^\beta\big|_{\mathbf{x}})+(\mathbf{T}_\beta\big|_{\mathbf{x}+\mathbf{y}}-\langle\mathbf{T}_\beta\rangle^\beta\big|_{\mathbf{x}})]dA \tag{35}$$

Using the decompositions $P_\beta=\langle P_\beta\rangle^\beta+\tilde{P}_\beta$ and $\mathrm{T}_\beta=\langle\mathrm{T}_\beta\rangle^\beta+\tilde{\mathrm{T}}_\beta$ in Eq. (35) yields the following result:

$$\rho_\beta\frac{\partial}{\partial t}(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta)+\rho_\beta\nabla\cdot(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta\langle\mathbf{v}_\beta\rangle^\beta$$
$$+\underbrace{\rho_\beta\nabla\cdot\langle\tilde{\mathbf{v}}_\beta\tilde{\mathbf{v}}_\beta\rangle}_{dispertion}+\underbrace{\rho_\beta\nabla\cdot\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle_{exc}}_{non-local\ dispertion}=$$
$$-\varepsilon_\beta\nabla\langle p_\beta\rangle^\beta+\varepsilon_\beta\nabla\cdot(\varepsilon_\beta^{-1}\langle\mathbf{T}_\beta\rangle)+\varepsilon_\beta\rho_\beta\mathbf{g}$$
$$+\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot[-\mathbf{I}\,\tilde{p}_\beta\big|_{\mathbf{x}+\mathbf{y}}+\tilde{\mathbf{T}}_\beta\big|_{\mathbf{x}+\mathbf{y}}]dA$$
$$+\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot[-\mathbf{I}(\langle p_\beta\rangle^\beta\big|_{\mathbf{x}+\mathbf{y}}-\langle p_\beta\rangle^\beta\big|_{\mathbf{x}})]dA$$
$$+\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot[(\langle\mathbf{T}_\beta\rangle^\beta\big|_{\mathbf{x}+\mathbf{y}}-\langle\mathbf{T}_\beta\rangle^\beta\big|_{\mathbf{x}})]dA \tag{36}$$

The second and third integral on the right-hand side of this equation are identified as a non-local term since it involves values of $\langle p_\beta\rangle^\beta\big|_{\mathbf{x}+\mathbf{y}}$ and $\langle\mathbf{T}_\beta\rangle^\beta\big|_{\mathbf{x}+\mathbf{y}}$ that are evaluated at points within the averaging volume not located at the centroid. These terms could be negligible in homogeneous regions if some appropriate length-scale constrains were imposed. A key point to remember about Eq. (36) is that no length-scale constraints have been imposed and this means that it is valid everywhere in the system illustrated in Figure1.

At this point, it is convenient to express the result given by Eq. (36) in compact form:

$$\rho_\beta\frac{\partial}{\partial t}(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta)+\rho_\beta\nabla\cdot(\varepsilon_\beta\langle\mathbf{v}_\beta\rangle^\beta\langle\mathbf{v}_\beta\rangle^\beta$$

$$+\rho_\beta\nabla\cdot\langle\tilde{\mathbf{v}}_\beta\tilde{\mathbf{v}}_\beta\rangle+\rho_\beta\nabla\cdot\langle\mathbf{v}_\beta\mathbf{v}_\beta\rangle_{exc}=$$

$$-\varepsilon_\beta\nabla\langle p_\beta\rangle^\beta+\varepsilon_\beta\nabla\cdot(\varepsilon_\beta^{-1}\langle\mathbf{T}_\beta\rangle)+\varepsilon_\beta\rho_\beta\mathbf{g}+\mathbf{M}_{\beta\sigma} \qquad (37)$$

in which the vector $\mathbf{M}_{\beta\sigma}$ is defined as

$$\mathbf{M}_{\beta\sigma}=\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot[-\mathbf{I}\,\tilde{p}_\beta\big|_{\mathbf{x}+\mathbf{y}}+\tilde{\mathbf{T}}_\beta\big|_{\mathbf{x}+\mathbf{y}}]dA$$
$$+\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot[-\mathbf{I}(\langle p_\beta\rangle^\beta\big|_{\mathbf{x}+\mathbf{y}}-\langle p_\beta\rangle^\beta\big|_{\mathbf{x}})]dA$$
$$+\frac{1}{\mathscr{V}}\int_{A_{\beta\sigma}}\mathbf{n}_{\beta\sigma}\cdot[(\langle\mathbf{T}_\beta\rangle^\beta\big|_{\mathbf{x}+\mathbf{y}}-\langle\mathbf{T}_\beta\rangle^\beta\big|_{\mathbf{x}})]dA \qquad (38)$$

and it represents the interfacial force per unit volume applied on phase β.

The procedure leading to the β-phase continuity equation can be repeated for the σ-phase beginning with Eq. (25), and the result is given by:

$$\rho_\sigma\frac{\partial}{\partial t}(\varepsilon_\sigma\langle\mathbf{v}_\sigma\rangle^\sigma)+\rho_\sigma\nabla\cdot(\varepsilon_\sigma\langle\mathbf{v}_\sigma\rangle^\sigma\langle\mathbf{v}_\sigma\rangle^\sigma)$$
$$+\rho_\sigma\nabla\cdot\langle\tilde{\mathbf{v}}_\sigma\tilde{\mathbf{v}}_\sigma\rangle+\rho_\sigma\nabla\cdot\langle\mathbf{v}_\sigma\mathbf{v}_\sigma\rangle_{exc}$$
$$=-\varepsilon_\sigma\nabla\langle p_\sigma\rangle^\sigma+\varepsilon_\sigma\nabla\cdot(\varepsilon_\sigma^{-1}\langle\mathbf{T}_\sigma\rangle)+\varepsilon_\sigma\rho_\sigma\mathbf{g}+\mathbf{M}_{\sigma\beta} \qquad (39)$$

where the vector $\mathbf{M}_{\sigma\beta}$ represents the interfacial force per unit volume applied on phase σ.

Averaging Model for the Homogeneous η-Region

The averaging momentum transport equations that describe the transport phenomena in the η-region is given by Ochoa-Tapia and Whitaker (1995):

$$\nabla\cdot\langle\mathbf{v}_\beta\rangle=0 \qquad (40)$$

$$0=-\varepsilon_\beta\nabla\langle p_\beta\rangle^\beta+\varepsilon_\beta\rho_\beta\mathbf{g}+\mu_\beta\nabla^2\langle\mathbf{v}_\beta\rangle$$
$$-\mu_\beta(\nabla\varepsilon_\beta)\cdot[\nabla(\varepsilon_\beta^{-1}\langle\mathbf{v}_\beta\rangle)]-\mu_\beta\mathbf{\Phi}_\beta \qquad (41)$$

where the first viscous term is known as the first Brinkman correction, the viscous term involving the gradient of the porosity is known as the second Brinkman correction, and $\mathbf{\Phi}_\beta$ is a vector defined by:

$$\mu_\beta\mathbf{\Phi}_\beta=-\frac{1}{V_\beta}\int_{A_{\beta\sigma}}\mathbf{n}\cdot[-\mathbf{I}(p_\beta\big|_{\mathbf{x}+\mathbf{y}}-\langle p_\beta\rangle^\beta\big|_{\mathbf{x}})]dA$$
$$-\frac{1}{V_\beta}\int_{A_{\beta\sigma}}\mathbf{n}\cdot[\mu_\beta(\nabla\mathbf{v}_\beta\big|_{\mathbf{x}+\mathbf{y}}-\nabla\langle\mathbf{v}_\beta\rangle^\beta\big|_{\mathbf{x}})]dA \qquad (42)$$

Whitaker (1986) derived an expression for $\mathbf{\Phi}_\beta$ which is given by:

$$\mathbf{\Phi}_\beta=\mathbf{K}_\beta^{-1}\cdot\langle\mathbf{v}_\beta\rangle \text{ for the homogeneous } \eta\text{-region} \qquad (43)$$

and it is valid when the following three length-scale constraints are imposed (Ochoa-Tapia and Whitaker, 1995):

$$\frac{r_o}{L_\varepsilon L_{p1}}<<1;\quad \frac{r_o}{L_\varepsilon L_{v2}}<<1;\quad l_\beta<<r_o; \qquad (44)$$

In these equations, $\mathbf{K}_\beta$ represents the Darcy´s law permeability tensor, $\mathbf{L}_\varepsilon$ is the characteristic length associated with $\nabla\varepsilon_\beta$, Lp_1 is the characteristic length associated with $\nabla\langle P_\beta\rangle^\beta$, and L_{v2} is the characteristic length associated with $\nabla^2\langle\mathbf{v}_\beta\rangle$. When the constraints indicated by Eq. (44) are valid, the second Brinkman correction is negligible, compared with the first Brinkman correction:

$$\mu_\beta(\nabla\varepsilon_\beta)\cdot[\nabla(\varepsilon_\beta^{-1}\langle\mathbf{v}_\beta\rangle)]=0 \qquad (45)$$

Use of Eq. (43) and Eq. (45) in Eq. (41) leads to:

$$\langle\mathbf{v}_\beta\rangle=-\frac{\mathbf{K}_\beta}{\mu_\beta}\cdot(\nabla\langle p_\beta\rangle^\beta-\rho_\beta\mathbf{g}-\varepsilon_\beta^{-1}\mu_\beta\nabla^2\langle\mathbf{v}_\beta\rangle)$$
$$\text{in the homogeneous } \eta\text{-region} \qquad (46)$$

The Blake-Kozeny equation is used to express permeability as (Bird et al., 2002):

$$K_{zz}=\frac{d_p^2\varepsilon_\beta^3}{\lambda(1-\varepsilon_\beta)^2} \qquad (47)$$

where $\mathbf{K}_\beta=K_{zz}\mathbf{e}_z\mathbf{e}_z$, d_p is the effective particle diameter, and λ is a constant which is obtained from experimental tests. Ochoa-Tapia and Whitaker (1995) and Whitaker (1996) reported that λ=180 but Bird et al. (2002) reported that λ=150.

A One-Equation Model for the Homogeneous ω-Region

In the concept of a one-equation model, the mixture is considered as a whole, rather than as two separate phases. It is evident that the one-equation model is simple with respect to the two-phase model. The one-equation model consists of a reduced number of the total averaged equations and closure relationships required in the complete formulation. In order to obtain such a one-equation model for the region occupied by the fluid bed, the principle of local hydrodynamic equilibrium is introduced together with the next definitions:

$$\{p\}_\omega=\varepsilon_\sigma\langle p_\sigma\rangle^\sigma+\varepsilon_\beta\langle p_\beta\rangle^\beta \qquad (48)$$

$$\{\mathbf{v}\}_\omega=\langle\mathbf{v}_\sigma\rangle^\sigma=\langle\mathbf{v}_\beta\rangle^\beta \qquad (49)$$

$$\rho_\omega=\varepsilon_\sigma\rho_\sigma+\varepsilon_\beta\rho_\beta \qquad (50)$$

$$\{\mathbf{T}\}_\omega=\langle\mathbf{T}_\sigma\rangle+\langle\mathbf{T}_\beta\rangle \qquad (51)$$

When Eq. (49) is applied in Eqs. (37) and (39), the dispersion terms are zero, i.e.,

$$\langle\tilde{\mathbf{v}}_\beta\tilde{\mathbf{v}}_\beta\rangle=0 \qquad (52)$$

$$\langle\tilde{\mathbf{v}}_\sigma\tilde{\mathbf{v}}_\sigma\rangle=0 \qquad (53)$$

The interfacial force terms in Eqs. (37) and (39) are equal and opposite, and they cancel if these two transport equations can be added to obtain a one-equation model. With these considerations, the one equation model for the homogeneous ω-region can be written as:

$$\nabla \cdot \{\mathbf{v}\}_{\omega} = 0 \tag{54}$$

$$\rho_{\omega} \frac{\partial}{\partial t}\{\mathbf{v}\}_{\omega} + \rho_{\omega}\nabla\cdot(\{\mathbf{v}\}_{\omega}\{\mathbf{v}\}_{\omega}) = -\nabla\{p\}_{\omega} + \nabla\cdot\{\mathbf{T}\}_{\omega} + \rho_{\omega}\mathbf{g} \tag{55}$$

The ω subscript indicates the homogeneous region where the Eqs. (54) and (55) are strictly valid. When the following length-scale constraints are imposed

$$\ell_{\sigma} \ll r_o; \qquad \ell_{\beta} \ll r_o; \qquad r_o^2 \ll L^2 \tag{56}$$

then the excess terms $\langle \mathbf{v}_{\beta}\mathbf{v}_{\beta}\rangle_{exc}$ and $\langle \mathbf{v}_{\sigma}\mathbf{v}_{\sigma}\rangle_{exc}$ of Eqs. (37) and (39), respectively, are negligible compared with the other terms of these equations.

Coupling Of The Averaging Models of the ω and η-Regions

The final formulation for the two-region model is represented by a set of four equations: equations (57) to (60), given below. The model was obtained adding terms for the coupling of the averaging models of the ω and η regions according the force balances made by Doron and Barnea (1993) and Doron et al. (1987).

$$\nabla \cdot \{\mathbf{v}\}_{\omega} = 0 \quad \text{in the } \omega\text{-region} \tag{57}$$

$$\rho_{\omega} \frac{\partial}{\partial t}\{\mathbf{v}\}_{\omega} + \rho_{\omega}\nabla\cdot(\{\mathbf{v}\}_{\omega}\{\mathbf{v}\}_{\omega}) = -\nabla\{p\}_{\omega} + \nabla\cdot\{\mathbf{T}\}_{\omega} + \rho_{\omega}\mathbf{g} - \left(\frac{\{\mathbf{T}\}_{\omega w}\cdot\mathbf{n}_{\omega w}}{D_{H\omega}}\right) - \left(\frac{\{\mathbf{T}\}_{\omega\eta}\cdot\mathbf{n}_{\omega\eta}}{D_{H\omega\eta}}\right) \tag{58}$$

$$\nabla\cdot\langle\mathbf{v}_{\beta}\rangle_{\eta} = 0 \qquad \text{in the } \eta\text{- region} \tag{59}$$

$$0 = -\varepsilon_{\beta}\nabla\langle p_{\beta}\rangle_{\eta}^{\beta} + \varepsilon_{\beta}\rho_{\beta}\mathbf{g} + \mu_{\beta}\nabla^2\langle\mathbf{v}_{\beta}\rangle_{\eta} - \mu_{\beta}\mathbf{K}_{\beta\eta}^{-1}\cdot\langle\mathbf{v}_{\beta}\rangle_{\eta} + \left(\frac{\{\mathbf{T}\}_{\omega\eta}\cdot\mathbf{n}_{\eta\omega}}{D_{H\omega\eta}}\right) \tag{60}$$

where $\{\mathrm{T}\}_{\omega w}$ is the wall stress in the ω-region, $\{\mathrm{T}\}_{\omega\eta}$ is the stress in the inter-region between the ω and η-regions, $\mathrm{n}_{\omega w}(=-\mathrm{n}_{w\omega})$ is the unit vector normal to the wall pointing out of the ω-region and $\mathrm{n}_{\eta\omega}(=-\mathrm{n}_{\omega\eta})$ is the unit vector normal to the inter-region pointing out of the η-region, as illustrated in Figure 3; $D_{H\omega}$ is the hydraulic diameter in the ω-region and $D_{H\omega\eta}$ is the hydraulic diameter in the inter-region.

This set of equations is complemented by the inter-regional boundary condition, which is given by

$$\text{B.C. 1} \quad \{p\}_{\omega} = \langle p_{\beta}\rangle_{\eta}^{\beta}, \quad \text{at} \quad A_{\omega\eta} \tag{61}$$

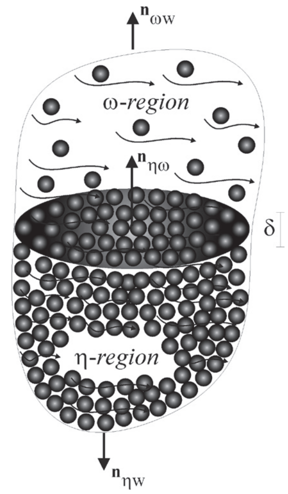

Figure 3. Unit vectors.

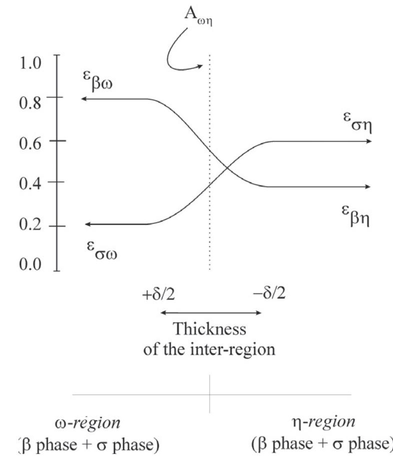

Figure 4. Volume fraction variation in the neighborhood of the non-homogeneous zone.

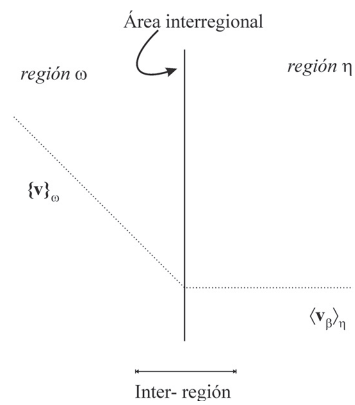

Figure 5. Continuity of the global spatial average velocity.

On the other hand, it is noted that in the $\omega-\eta$ boundary, ε_{β} and ε_{σ} undergo significant changes over a distance equal to the radius of the averaging volume $\mathbf{r}_0$, as illustrated in Figure 4. In this figure, δ represents the thickness of the interfacial region where there exist rapid changes in ε_{β} and ε_{σ}. Also in the $\omega-\eta$ boundary, Figure 5 shows the continuity of the velocity of each region. Thus further contributions will deal with the development of the momentum jump conditions between the ω- and η-regions for cutting transport.

Conclusions

In this work, the process of cutting transport for a system composed by two regions has been described: the ω-region which is a fluid bed, and the η-region which is a porous medium system. A rigorous mathematical model in which each variable is precisely defined has been derived. To do this, volume-averaged transport equations were derived for the fluid bed and the porous medium regions. From these equations, a one-equation model was obtained and the constraints that the model must satisfy are identified. Specifically, the coupling conditions between the homogeneous ω-region (σ-phase and β-phase) and the homogeneous η-region (σ-phase) were identified. Further contributions include development of the jump conditions needed for solving the model and application of the model to practical cases of cuttings transport. Work is underway in this direction.

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