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Improvements on Analytical Modeling for Vertical U-Tube Ground Heat Exchangers

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ABSTRACT

Improved procedures for performance prediction of vertical U-tube ground heat exchangers (GHEs) in Ground-Coupled Heat Pump (GCHP) systems are proposed using cylindrical source functions. Appropriate length of timesteps to simulate the performance of GHEs under conditions of variable heat loads is also studied for the practical application of the procedures.

Introduction

For the installation of Ground-Coupled Heat Pump (GCHP) systems, the determination of the minimum necessary length of GHEs is important for the improvement of the feasibility of the systems. The lengths of GHEs are determined on the basis of the simulation calculations of the heat transfer performance around GHEs. In general, numerical models or analytical models are applicable for the heat transfer calculations.

Numerical models are useful in case high resolution is required in the prediction since they can model complex reservoir and well conditions, including heterogeneity of the formation, effect of thermal conductivities of grouting materials, advection effects by groundwater flow, etc. Several researchers have developed numerical models of GCHP systems. Rottmayer et al. (1997) constructed a numerical model of a single vertical U-tube GHE in an infinite medium using the finite difference method (FDM). Morita and Tago (2000) developed a numerical model of a vertical coaxial GHE using FDM and demonstrated the reliability of the model using field data of a snow-melt system in Northern Japan. Kohl et al. (2002) applied finite element method for the modeling of deep vertical coaxial GHEs. The above studies showed the reliability of numerical models under various conditions. Numerical models, however, requires longer computation time than analytical models since grid sizes need to be sufficiently small in the vicinity of GHEs. Analytical models, therefore, will be useful when a prompt decision is required for the design of GCHP systems.

Analytical models of GHEs systems have been developed using the cylindrical source function (Ingersoll, et al. 1954) and superposition techniques. Deerman and Kavanaugh (1990) developed an analytical model for vertical U-tube GHEs. Gu and O'Neal (1995) developed a composite analytical model to be able to handle the different thermal properties of formation and grouting materials. Fujii et al. (2002) demonstrated the reliability of analytical models with a series of thermal response tests in various locations. In the above studies, analytical models showed sufficient reliability and usefulness.

In the actual design of GCHP systems, long-term performances of the system with variable heat load need to be predicted. Using superposition techniques, the computation time of analytical models will increase when small time steps are used to model the daily or seasonal changes in heat load. In this paper, the reliability of the analytical model is examined by the interpretation of thermal response tests (TRTs). Then, sensitivity studies are conducted for minimizing the computation time of analytical models for vertical U-tube GHEs.

Theory and Application of Analytical Model

The cylindrical source function G in Eq.(1) is derived by solving partial differential equations of heat conduction in a radial coordinate under the following conditions:

- constant heat load,
- uniform heat flux along GHE,
- homogeneous formation,
- no groundwater flow,
- no interference between adjacent wells,

$$G(Z,P) =$$

$$\frac{1}{}\int_0^{\infty}\frac{e^{-\beta^2 Z}-1}{J^2{}_1(\beta)+Y^2{}_2(\beta)}\left[J_0(P\beta)Y_1(\beta)\text{-}J_1(\beta)Y_0(P\beta\right]\frac{d\beta}{\beta^2} \qquad (1)$$

$Z= \alpha_s t/r^2$ (Fourier number)

(2)

$P=r/r_o$ (3)

where, J_0 and J_1 are the Bessel functions of the first-kind of order zero and one, respectively, Y_0,Y_1 and Y_2 are the Bessel functions of the second-kind of order zero, one and two, respectively, α_s is the thermal diffusivity of soil and ro is the outer radius of the GHE. In the analytical model, the temperature difference (ΔT_g) between farfield temperature (T_{ff}) and the outer wall of heat exchanger (T_{ro}) is calculated using the cylindrical source function G as shown in Eq. (4).

$$\Delta T_g = T_{ff} - T_{ro} = \frac{q_{gc}}{\lambda_s L}\ G(Z,P) \qquad (4)$$

where, q_{gc} is the heat exchange rate between formation and GHE, λ_s is the thermal conductivity of soil and L is the length of GHE. The procedures of calculating heat transfer in the U-tube are given by Deerman and Kavanaugh (1994). The integral part in Eq.(4) is calculated numerically using a 21-point Gauss-Kronrod rule (Visual Numerics Inc, 1994).

In GCHP systems, heat exchange rates between formation and heat exchangers are not constant but are transient due to the change in heat loads or with the change in formation temperatures. These changes in heat exchange rates are modeled by superposing heat exchanges in the past time steps as shown in Eq.(5).

$$\Delta T_g = \frac{1}{\lambda_s L}\sum_{i=1}^{nstep}\left\{q_{gci}\left[G(Z,P)_{nstep+1-i} - G(Z,P)_{nstep-i}\right]\right\} \qquad (5)$$

where,

nstep : total number of timesteps
i : timestep

To demonstrate the validity of the above analytical solutions, temperature performance data from TRTs conducted in Northern Japan, were interpreted as shown below.

The first TRT was conducted in a 50m GHE drilled in shallow deposit of silt and fine sand with relatively uniform properties in Akita City. From the observations while drilling the GHE, no significant groundwater flow was detected at the well location. The GHE was completed with a single polyethylene U-tube and grouted with cement. During the TRT, heated water was circulated through the U-tube with a constant well inlet temperature of 30°C for 72hours. After the circulation period, temperature recovery was monitored with a temperature sensor at –50m (bottom of hole) for 72 hours. Figure 1 shows the Horner plot (Horner, 1951) of temperature recovery, which gives an estimation of thermal conductivity of the formation. The slope m of straight line m fitted to the temperature data in the Horner time range less than 20 gives a soil thermal conductivity (λs) estimation of 1.18 W/(m.K). The analytical model calculated outlet temperatures and heat exchange rates as shown in Figure 2 using the same λs value of 1.18 W/(m.K). Timestep length used in the analytical model was 30 minutes. The calculated temperature matched the observation data quite well since conditions like homogeneous

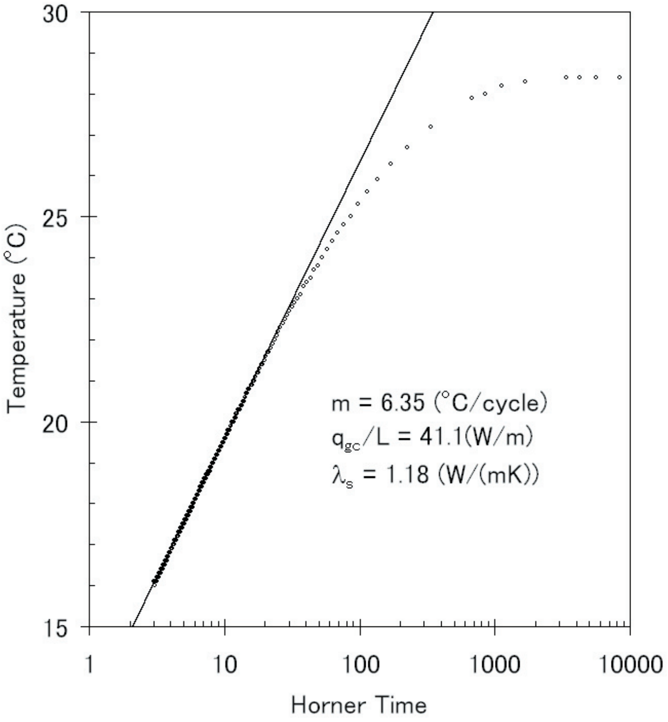

Figure 1. Horner plot of TRT in Akita City.

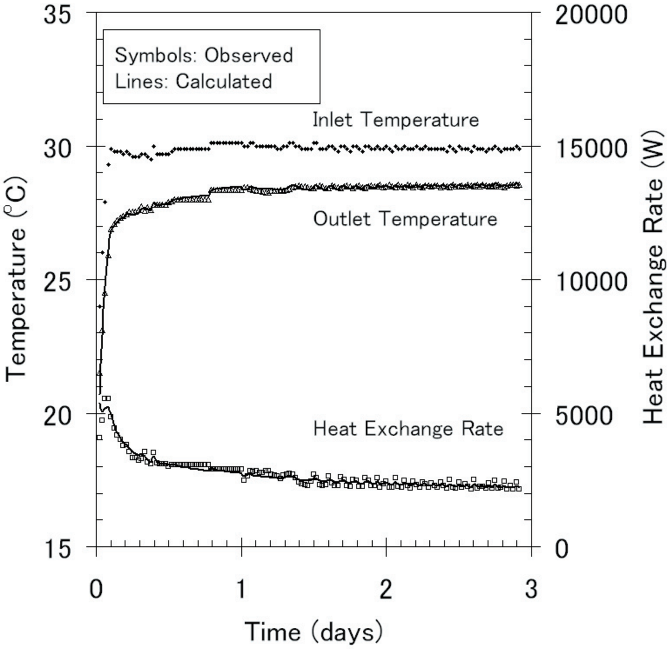

Figure 2. History matching results of TRT in Akita City.

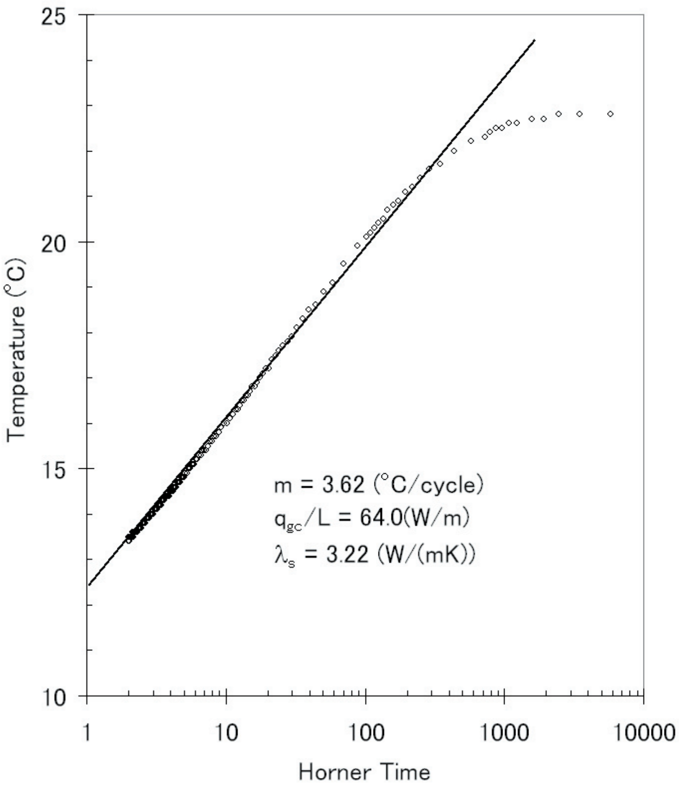

Figure 3. Horner plot of TRT in Hirosaki City.

formation, insignificant effect of groundwater flow and stable heat exchange rates were well satisfied in this TRT.

The second TRT was conducted in a deeper GHE of 90m drilled in a deposit of coarse sand and silt alternation with in Hirosaki City. In the drilling operation of the GHE, existence of active groundwater flow was observed. The GHE was completed with a single polyethylene U-tube and grouted with silica sand. During the TRT, heated water was circulated with a constant well inlet temperature of 30°C for 48 hours. After the circulation period, temperature recovery was monitored at –89m (bottom of hole) for 48hours. Figure 3 shows the Horner plot of temperature recovery. The straight line gives a λs estimation of 3.22W/(m.K). Considering λs of saturated sand is generally 1.0-2.0W/(m.K), the high λs in the interpretation would be due to the advection effect of groundwater flow. History matching results using λs of 3.22 W/(m.K) are shown in Figure 4. Timestep length used in the analytical model was 30 minutes. The model could simulate the observation data well even under the condition of active groundwater flow.

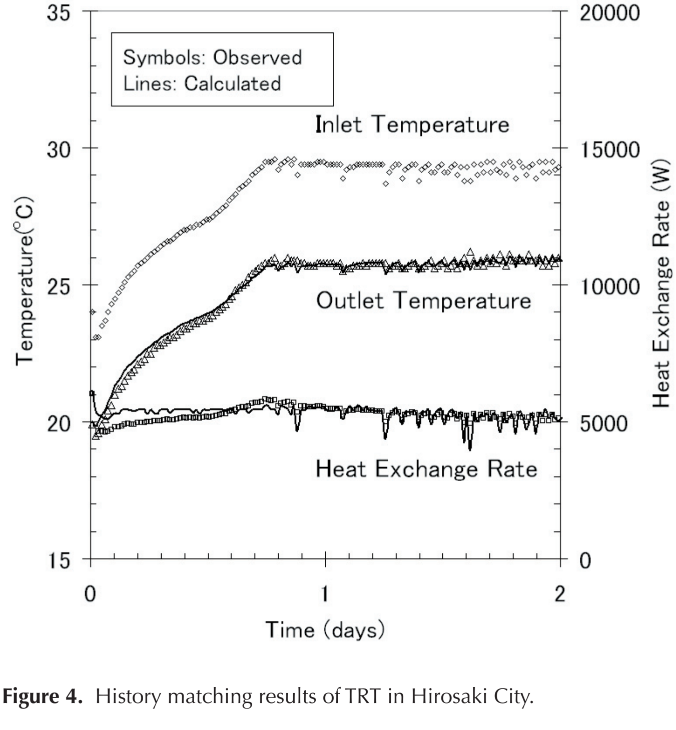

Figure 4. History matching results of TRT in Hirosaki City.

The above examples show that the analytical models using cylindrical model is reliable for analyzing the performance of shallow GHEs. For deeper GHEs, modeling needs further improvement since heat flux to the GHE may significantly vary along the wellbore.

Approximation of Cylindrical Source Function

For the design of GCHP systems, simulations should be conducted for a sufficiently long period to consider the effects of annual increase (decrease) of formation temperature due to the unbalanced amount of heat extraction from the formation and disposal into the formation. From Eq.(5), however, it is explicit that the amount of required computation increases with the increase of timesteps, which would make performance predictions quite time consuming. Using Eq.(1) and Eq.(5), a simulation run of 100 timesteps with a timestep length of 1 hour takes 2.0 seconds (total simulation period 4.2 days), while a run for 1000 timesteps (41.6 days) requires over 300 seconds using a PC with a CPU of Pentium4 2.6GHz. Since most of the computation time is spent on the calculation of cylindrical function G, approximated calculation of G will significantly reduce the CPU time.

In approximating G, dimensionless radius P was set equal to unity to evaluate the temperature at the outer surface of the heat exchanger. Fourier number Z was divided into three ranges to approximate G accurately. Approximated forms of G are obtained using lease-square-method as follows:

$$Z<1 \qquad G=0.1443Z^{0.3374}-0.0162 \tag{6}$$

$$1<Z<100 \qquad G=0.5414Z^{0.0986}-0.4166 \tag{7}$$

$$100<Z \qquad G=0.1827\log_{10}Z+0.0668 \tag{8}$$

Figure 5, overleaf, compares the exact (as expressed in Eq.(1)) and approximated (as expressed in Eqs.(6), (7), (8)) values of G. The average errors in approximated G are 0.30% for $Z<1$, 0.38% for $1<Z<100$ and 0.05% for $100<Z<10^5$, which are sufficiently small.

Assuming $\alpha s=1.0\text{x}10^{-6}\text{m}^2/\text{s}$ and r=0.02m, Z=1 gives t=400s, while Z=100 gives t=11.1hours. Hence, Eq.(8) will be practically used as approximated form of G for the performance prediction of GCHP systems except for the interpretation of short TRTs.

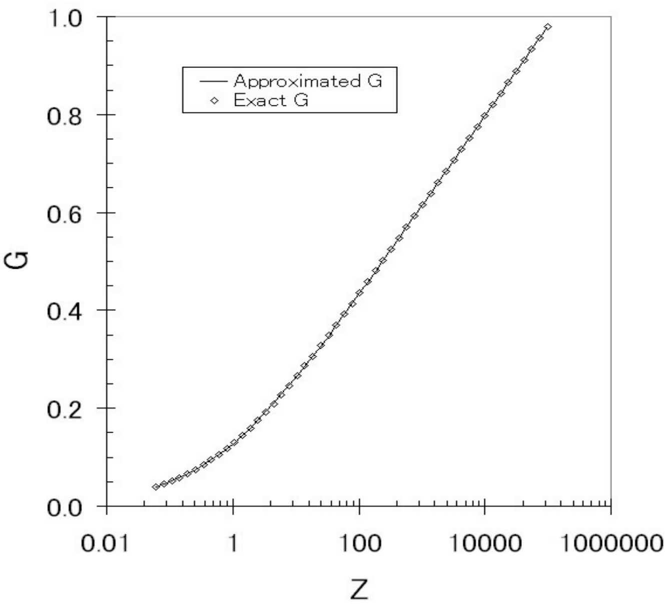

Figure 5. Comparison of exact and approximated cylindrical source function G.

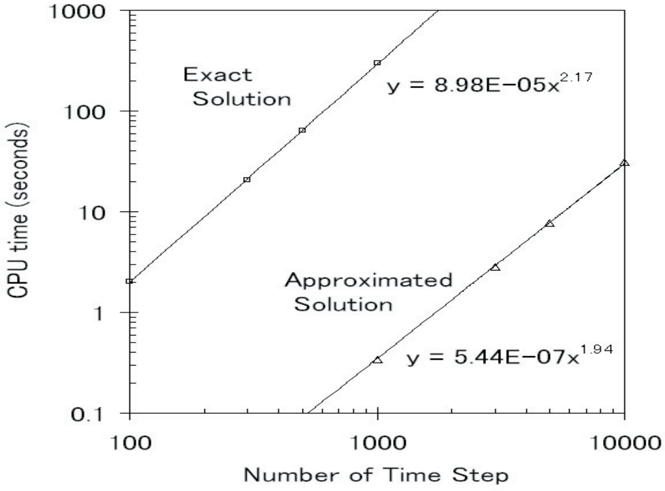

Figure 6. Comparison of CPU times.

The CPU time required for the calculation of GCHP systems using exact formula and approximated formula are compared in Figure 6, overleaf. A PC with a CPU of Pentium4 2.6GHz was used with a timestep length of 1 hour. The figure shows a remarkable reduction in CPU time with the use of approximated formula of G. The ratio of CPU time was 875 (exact): 1 (approximated) in the case of 1000 timesteps. A prediction run of 10 years with timestep length of 1 hour requires 87600 timesteps. From Figure.6, the CPU times are estimated 1325 hours and 0.56 hours using exact and approximated formula, respectively.

Sensitivity Studies for Optimum Timestep Length

The above calculations used a timestep length of 1 hour. Length of timesteps, however, needs to be determined carefully to eliminate the errors attributed to the size of timestep length. Using the approximated formulae of G (Eqs.(6)-(8)), optimum timestep length is investigated.

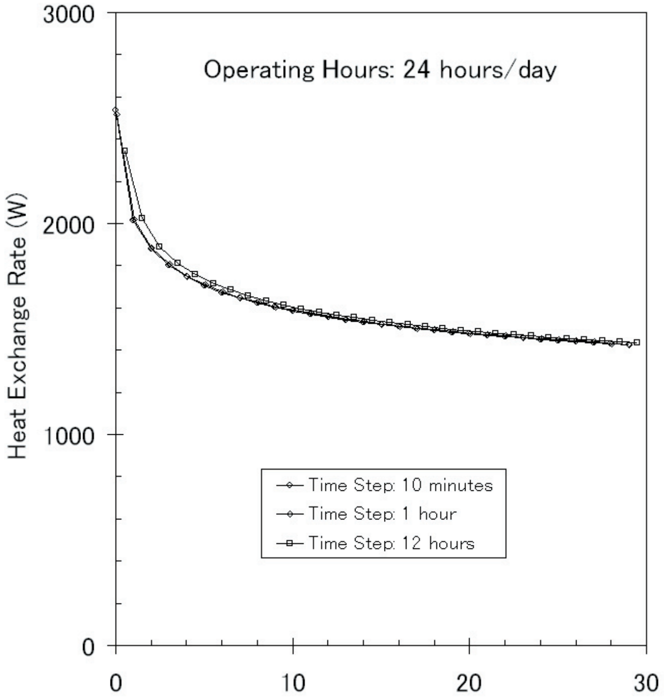

Figure 7. Comparison of heat exchange profiles under continuous operations.

Figure 7 compares the daily-average heat exchange rates under a continuous circulation (24 hours/day) of heated water in a GHE for 30 days using three different timestep lengths, namely, 10 minutes (Case 1-1), 1 hour (Case 1-2) and 12 hours (Case 1-3). Simulation conditions are summarized in Table 1. The good agreement of the temperature performance in Case 1-1 and Case 1-2 indicates that a timestep of 1 hour is small enough to maintain the reliability of simulation for cases with continuous operation. Case 1-3, however, showed different performances from other two cases, especially in the early period of calculation. The average difference in temperature between Case 1-1 and

Table 1. Simulation conditions.

Formation Temp. (°C)	15.0
Well Depth (m)	50
Type of U Tube	Single
OD of U Tube (m)	0.033
ID of U Tube (m)	0.027
Thermal Conductivity (W/(mK))	
Formation	1.50
Grout	1.50
U Tube	0.41
Heat Capacity (J/(m^3 K))	
Formation	3.0E+06
Grout	3.0E+06
Inlet Temperature (°C)	30.0
Circulation Rate (L/min)	25.0

Table 2. Effect of timestep length on simulation results under continuous operations.

Case	Time Step	Avg. Difference from Case 1-1
1-1	10 min	---
1-2	1 hour	0.19%
1-3	12 hour	0.86%

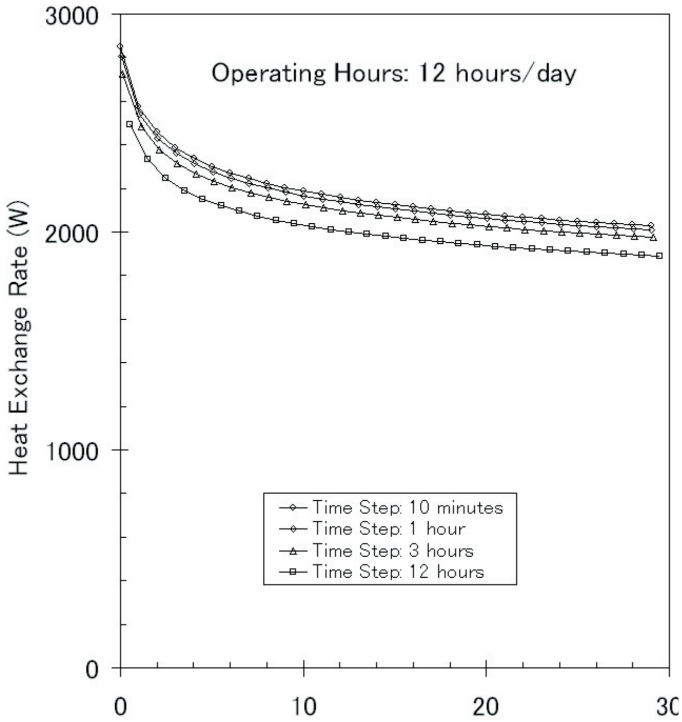

Figure 8. Comparison of heat exchange profiles under intermittent operations.

Table 3. Effect of timestep length on simulation results under intermittent operations.

Case	Time Step	Avg. Difference from Case 2-1
2-1	10 min	---
2-2	1 hour	0.99%
2-3	3 hour	2.89%
2-4	12 hours	7.59%

other cases are summarized in Table 2. The small difference between Case 1-1 and Case 1-2 indicates that a timestep of 1 hour is sufficiently short.

Figure 8 compares the daily-average heat exchange rates under an intermittent circulation (circulation of 12hour + shutdown of 12hours everyday) of heated water using four different timestep lengths, namely 10 minutes (Case 2-1), 1 hour (Case 2-2), 3 hours (Case 2-3) and 12 hours (Case 2-4). Conditions for the calculation are same as the ones used in Cases 1-1 through 1-3. Good agreement of the temperature performance was observed between Cases 2-1 and 2-2. Cases 2-3 and 2-4, however, showed significant differences in temperatures from Case 2-1 in the entire simulation period. The average differences between Case 2-1 and other cases are summarized in Table 3.

The above calculations show that a time step of 1 hour is small enough to model intermittent GCHP operations as wells as continuous operations. As discussed in the previous section, a prediction run of 10 years with timestep length of 1 hour requires 0.56 hour's CPU time using approximated formula, which would be practically acceptable. Considering the simulation of 10 years is sufficiently long to model the long-term performance of GCHP systems operation, 1 hour's timestep will be quite reasonable for the simulation of GCHP systems using analytical models.

Summary

Improvements on the analytical simulation procedures of ground heat exchangers in GCHP systems were investigated with sensitivity studies. Major findings of the studies are as follows:

1) Remarkable reduction in CPU time was achieved with the use of approximated formula of cylindrical source function G.
2) Timestep length of 1 hour leads to reasonable results for the simulation using analytical models.

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