Time-Dependent Shape Factors for Uniform and Non-Uniform Pressure Boundary Conditions

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ABSTRACT

Matrix-fracture transfer functions are the backbone of any dual-porosity or dual-permeability formulation. The chief feature within them is the accurate definition of shape factors. To date, there is no completely accepted definition of matrix-fracture transfer function. Many definitions of shape factors for instantly-filled fractures with uniform pressure distribution have been presented and used; however, they differ by up to five orders of magnitude.

Based on a recently presented definition of transfer function, time-dependent shape factors for pressure driven flow and for water imbibition are proposed. Also new matrix-fracture transfer pressure-based shape factors for instantly-filled fractures with non-uniform pressure distribution are presented in this paper. These are the boundary conditions for a case for porous media with clusters of parallel and disconnected fractures, for instance.

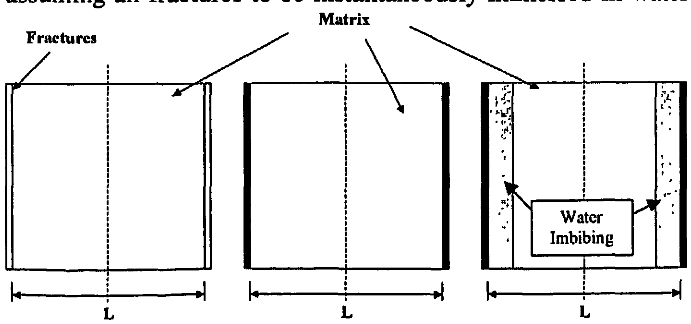
These new pressure-based shape factors were obtained by solving the pressure diffusivity equation using non-uniform boundary conditions. This leads to time-dependent shape factors because of the transient part of the solution for pressure. However, approximating the solution with an exponential function, one obtains constant shape factors that can be easily implemented in current dual-porosity reservoir simulators. They provide good results for systems where the transient behavior of pressure is short (a case commonly encountered in fractured reservoirs).

Introduction

Modeling multiphase flow in fractured porous media relies on the accurate description of matrix to fracture transfer of water. The rate of mass transfer between the rock matrix and fractures is significant, and calculation of this rate, within dual-continuum models, depends on matrix-fracture transfer functions incorporating the shape factor.

Typically, matrix-to-fracture transfer functions are obtained by assuming all fractures to be instantaneously immersed in water (instantly-filled), with a uniform fracture pressure distribution under pseudo-steady state conditions. The result is constant, time-independent, shape factors. Clearly, this is not necessarily true. Partially immersed fractures and other unsteady-state conditions do not lead to constant shape factors. The current formulations for modeling flow in fractured porous media need to be reconsidered.

Decades of controversy exist regarding the appropriate shape of relative permeability and capillary pressure curves for multiphase flow in fractures (for instance refer to Horne *et al.*, 2000; Akin, 2001). Transfer functions and shape factors have not received sufficient attention. Enormous discrepancies also exist for the value of shape factors proposed by different investigators. For blocks of size L, for example, values range from $4/L^2$ (Kazemi *et al.*, 1976) to $12/L^2$ (Warren and Root, 1963; de Swaan, 1990) for one-dimensional systems, and from $12/L^2$ (Kazemi *et al.*, 1976) to $60/L^2$ (Warren and Root, 1963; de Swaan, 1990) for three-dimensional systems. A full discussion of shape factors and transfer functions can be found in Rangel-German (2002). Typically, matrix-to-fracture transfer functions are obtained by assuming all fractures to be instantaneously immersed in water



$$p_m = p_i,\ 0 \le x \le L, \quad t = 0\ ; \qquad p_m = p_f,\ x = 0, \quad t > 0 \qquad p_m = p_f,\ x = L, \quad t > 0$$

Figure 1. Representation of the initial and boundary conditions of a matrix block surrounded by equidistant parallel fractures that are filled instantly.

(instantly-filled), with a uniform fracture pressure distribution under pseudo-steady state conditions, as shown in Figure 1. The result is constant, time-independent, shape factors. Clearly, this is not necessarily true. Partially immersed fractures and other unsteady-state conditions do not lead to constant shape factors. The current formulations for modeling flow in fractured porous media need to be reconsidered.

Physically Correct Transfer Functions

Rangel-German (2002) presented extensive experimental and analytical information to clarify the understanding of matrix-fracture transfer. Rangel-German and Kovscek (2003) pointed out the need for expressions for transfer functions that account for realistic boundary conditions: partially covered or totally immersed boundaries; unsteady and pseudo-steady state; and uniform and non-uniform pressure distribution in the fractures, as a function of parameters that can be obtained either in the laboratory or the field with high certainty.

They found two different modes of matrix and fracture fill-up. Relatively slow flow through fractures is found when fracture to matrix fluid transfer is rapid, fracture aperture is wide, and/or water injection is slow. In this regime, fractures fill slowly with fluid and the regime is referred to as a "filling fracture" (the recovery scales linearly with time). On the other hand, relatively low rates of fracture to matrix transfer, narrow apertures, and/or high water injection rates lead to rapid flow through fractures. This regime is labeled "instantly filled," and recovery scales with the square-root of time.

They presented a transfer function that includes the matrix-to-fracture mass transfer due to fluid-flow as a result of the pressure difference created by the step-change at the fractures, and the mass transfer due to imbibition, where capillary pressure is the only driving force. Considering both processes in multiphase flow:

$$\tau_w = \sigma_p V k \frac{k_{rw}}{\mu_w}\left(\overline{p_{wm}} - p_{wf}\right) - \sigma_S V \alpha_h \left(S_{w\,max} - \overline{S_{wm}}\right) \qquad (1)$$

where τ_w is the flow rate in an element V of bulk reservoir volume, p_m is the volumetric average matrix pressure and p_f is the pressure in the fracture. The shape factor, σ, reflects the geometry of the matrix elements (traditionally in pseudo-steady state, single-phase flow) at all times. σ_p is the shape factor based on pressure such as those presented by Chang (1993) and Lim and Aziz (1995), whereas σ_S is a time-dependent shape factor due to imbibition, presented in a different paper (Rangel-German and Kovscek, 2003), including both filling- and instantly-filled fracture regimes. A plot of σ_S is shown in Figure 2. It is important to emphasize that both σ_s, and σ_p, are time-dependent.

This definition of transfer function leads to a better understanding of the discrepancy among the different values for shape factors. It indicates that authors have considered different physical processes while calculating such factors. For instance, although Chang's (1993) solution to the pressure diffusivity equation is mathematically correct (and complete), it accounts exclusively for the mass transfer due to expansion of single-phase fluids as a result of the pressure difference created by the step-change at the instantly-filled fracture. On the other hand, Kazemi *et al.*'s (1976) shape factors were derived while obtaining dimensionless scaling times for imbibition-dominated oil recovery processes, where capillary pressure is the only driving fluid force. Each process has different weight depending on the location, properties, and characteristics of the reservoir; however, both processes have to be considered in order to achieve accurate modeling of the matrix-fracture mass transfer in fractured porous media.

Unsteady State Flow With Non-Uniform Boundary Conditions

The pressure diffusivity equation for a three-dimensional anisotropic flow between the rock matrix and the fracture in the x, y, and z Cartesian coordinates is:

$$k_x \frac{\partial^2 p_m}{\partial x^2} + k_y \frac{\partial^2 p_m}{\partial y^2} + k_z \frac{\partial^2 p_m}{\partial z^2} = \phi \mu c_t \frac{\partial p_m}{\partial t} \qquad (2)$$

where k_i is the permeability in the $i = x$, y, and z directions, ϕ is the porosity, μ is the viscosity and c_t is the compressibility. With the initial conditions shown in Figure 1, Eq. 2 can be solved for both the transient and the pseudo-steady state flow periods under different boundary conditions. Chang (1993) and Lim and Aziz (1995) found the solutions for uniform pressure distribution in instantly-filled fractures. A plot of the shape factor versus dimensionless time is shown in Figure 2.

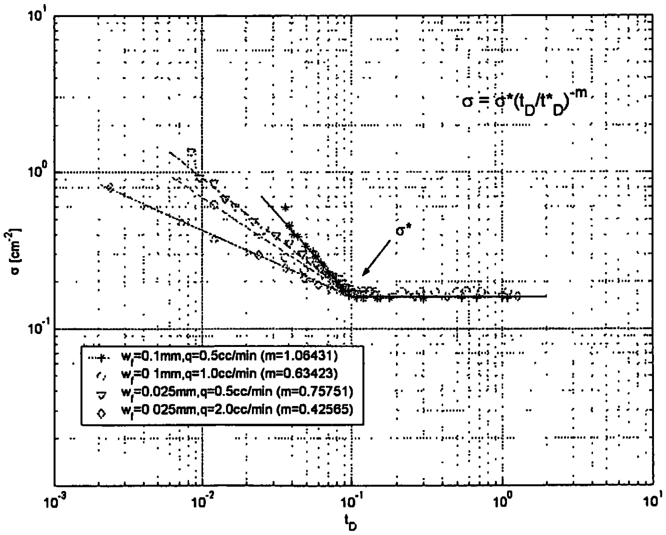


Figure 2. Comparison of the imbibition-based shape factor, σs, versus dimensionless time for both filling-fracture and instantly-filled fracture experiments with the analytical approximation (log-log coordinates). (from Rangel-German and Kovscek, 2003).

A more general situation of the cases above is that of fractures filling instantly (totally immersed) but having non-uniform pressure distribution; that is, the pressure within the fractures in Figure 1 is constant initially, but a gradient exists for any time greater than zero ($t > 0$). This is the case for porous media with clusters of parallel and disconnected fractures. Thus, the initial and boundary conditions for this case are

$$p_m = p_i, \; -0 \le x \le L, \quad t = 0 \qquad (3)$$

$$p_m = p_{f1}, x = 0, \quad t > 0$$
$$p_m = p_{f2}, x = L, \quad t > 0 \tag{4}$$

To solve this case, the solution for the diffusion equation (Eq. 2) was obtained for the boundary conditions described by Eqs. 3 and 4. For one-dimensional flow, it reads (Crank, 1975)

$$\frac{M_t}{M_\infty} = 1 - \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}exp\left\{\frac{-D(2n+1)^2\pi^2 t}{L^2}\right\} \tag{5}$$

where M_t is the total amount of mass that has entered the system at time t, and M_∞ is the amount of mass after infinite time. In this case $M_\infty = l\{1/2(p_{f1} + p_{f2}) - p_i\}$. Eq. 5 is similar to Eq. 4.23 in Crank (1975) with the proviso that L signifies the whole thickness of the membrane (Crank, 1975). Because Eq. 5 is written on a unit volume basis, the left-hand side can be replaced with the ratio of the density difference at time t compared to the initial state and the expected increment at infinite time (Lim and Aziz, 1995):

$$\frac{M_t}{M_\infty} = \frac{\overline{\rho_m} - \rho_i}{\rho_f - \rho_i} \tag{6}$$

The assumption of a small and constant compressibility fluid implies

$$\rho(p) = \rho^o\left[1 + c(p - p^o)\right] \tag{7}$$

where the superscript "o" refers to a standard or reference condition. Combining Eqs. 5, 6, and 7, we obtain:

$$\frac{\overline{p_m} - p_i}{\overline{p_f} - p_i} = 1 - \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}exp\left\{\frac{-(2n+1)^2\pi^2 kt}{\phi\mu c_t L^2}\right\} \tag{8}$$

where $\overline{p_f}$ is equal to $(p_{f1} + p_{f2})/2$.

Strictly, the pressure-based shape factor, σ_p, should be obtained by taking the ratio of the flow rate at $x = L$ over the pressure difference $(\overline{p_f} - \overline{p_m})$. This procedure leads to a very complex form of the shape factor, especially for two- and three-dimensional flow. However, it is important to point out that the dimensionless solution of the total amount of mass that has entered the system for non-uniform boundary conditions has the same form as that for uniform boundary conditions, so taking the ratio of these solutions we obtain:

$$\frac{\left(\frac{M_t}{M_\infty}\right)_{uniform}}{\left(\frac{M_t}{M_\infty}\right)_{non-uniform}} = \frac{\left[\frac{\overline{p_m} - p_i}{p_f - p_i}\right]_{uniform}}{\left[\frac{\overline{p_m} - p_i}{\overline{p_f} - p_i}\right]_{non-uniform}} \tag{9}$$

This ratio indicates that the solution for non-uniform boundary conditions is directly proportional to that for uniform boundary conditions. Thus, following a procedure similar to that of Lim and Aziz (1995), the solution is approximated by taking the first term in the infinite series in Eq. 8 to eliminate the time dependence of the function, and the term inside the exponential term (dimensionless group for time) can be modified to incorporate easily the effect of the reservoir properties, block size or boundary conditions, so for non-uniform boundary conditions that cause a block mass intake equivalent to that of uniform boundary conditions and half-sized blocks, one obtains:

$$\frac{\overline{p_m} - p_i}{\overline{p_f} - p_i} = 1 - 0.81057\,exp\left[\frac{-4\pi^2 kt}{\phi\mu c_t L^2}\right] \tag{10}$$

Taking the derivative of Eq. 10 with respect to t and simplifying yields

$$\frac{\partial\overline{p_m}}{\partial t} = \frac{4\pi^2 k}{\phi\mu c_t L^2}\left(\overline{p_f} - \overline{p_m}\right) \tag{11}$$

For a strictly dual-porosity model, τ_w can be expressed as:

$$\tau_w = -V\phi c\frac{\partial\overline{p_m}}{\partial t} \tag{12}$$

Substituting Eq. 11 into Eq. 12 results in the following matrix-fracture transfer function:

$$\tau_w = \frac{4\pi^2}{L^2}\frac{kV}{\mu}\left(\overline{p_m} - \overline{p_f}\right) \tag{13}$$

As Lim and Aziz (1995) pointed out, this kind of derivation leads to an equation similar to the typical expression for matrix-fracture transfer function (Warren and Root, 1963):

$$\tau_w = \frac{\sigma kV}{\mu}\left(\overline{p_m} - \overline{p_f}\right) \tag{14}$$

but the assumption of pseudo-steady state was not made in its derivation. Here, we extend the unsteady state solution to a set of parallel fractures with non-uniform pressure distribution. Comparing Eq. 13 and Eq. 14, the shape factor for one set of orthogonal fractures is:

$$\sigma = \frac{4\pi^2}{L^2} \tag{15}$$

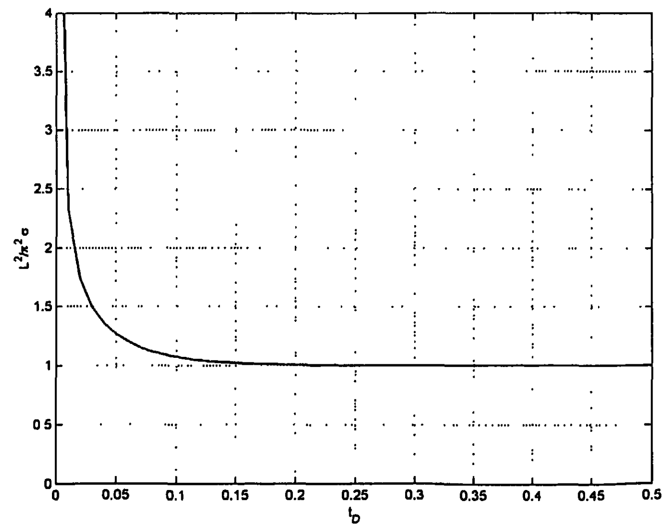


Figure 3. Behavior of the pressure change-driven shape factor, σp, for instantly-filled fracture regime. One-dimensional and uniform pressure boundary conditions.

Figure 4, oerleaf, shows dimensionless pressure versus dimensionless time $(t_D = \frac{\alpha_h t}{L_x^2})$ for both the uniform and variable pressure distribution. The lines with symbols in Figure 4 are the

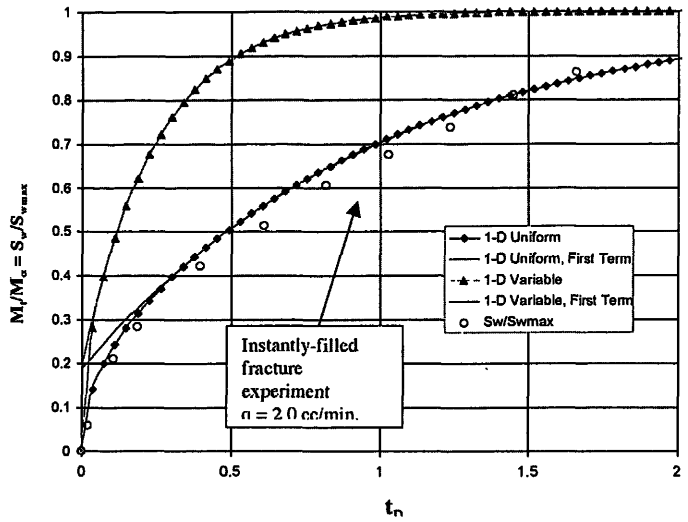


Figure 4. Analytic solution of the one-dimensional diffusion in a plane and its approximations.

results for the analytical solution, whereas the solid lines are the exponential approximations to the analytical solution derived by taking the first term of the summation. From Eq. 6, it is apparent that Figure 4 is analogous to a plot of the standardized water saturation in the matrix block, S_w/S_{wmax}, against time. The results obtained for variable pressure in the fractures represent a faster recovery process. Correspondingly, the shape factor shown in Eq. 15 is larger than that obtained for constant pressure distribution within fractures. Figure 4 also includes experimental data for the imbibition of water into matrix from a fracture aperture of 0.025 mm and flow rate of 2 cm³/min (Rangel-German and Kovscek, 2002). The block water saturation is standardized to the maximum value (S_{wmax} = 62%). This experiment behaved in a 1-D fashion. The constant shape factor formulation, leads to a good match for times greater than roughly t_D equal to 0.25.

Satisfactory agreement is not necessarily found for every instantly-filled fracture case. Cases where the transient behavior takes a relatively long time to develop are not well matched by the exponential approximation. The exponential approximation in Figure 4 never reaches zero water saturation (or dimensionless pressure, in the analogous pressure distribution problem).

Similarly, results for both two and three sets of orthogonal fractures under unsteady state conditions with different pressure distributions are derived (Crank, 1975):

$$\frac{M_t}{M_\infty} = 1 - \frac{4}{\left(b^2 - a^2\right)} \sum_{n=0}^{\infty} \frac{J_0(a\alpha_n) - J_0(b\alpha_n)}{\alpha_n^2\{J_0(a\alpha_n) + J_0(b\alpha_n)\}} exp\left\{-D\alpha_n^2 t\right\} \quad (16)$$

and

$$\frac{M_t}{M_\infty} = 1 - \frac{6}{\pi^2\left(a^2 + ab + b^2\right)} \sum_{n=0}^{\infty} \left(\frac{b\cos n\pi - a}{n}\right)^2 exp\left\{\frac{-Dn^2\pi^2 t}{(b-a)^2}\right\} \quad (17)$$

where the α_n's are the roots of $J_0(a\alpha_n) = 0$. $J_0(x)$ is the Bessel function of the first kind of order zero, and a and b are the inner and outer limits of the cylinder or sphere approximation, respectively. Note that for two-dimensional flow and $a = 0$ (solid cylinder), the solution reduces to Eq. 5.23 in Crank (1975) and the plane sheet solution is obtained for $b/a = 1$. Similarly, for three-dimensional flow and $a = 0$ (solid sphere), the solution reduces to Eq. 6.20 in Crank (1975), and also the plane sheet solution is obtained for $b/a = 1$.

Assuming that the equivalent radius, a, of the cylinder is the radius that yields the same volume as a bar with an L x L square-shaped cross-section, i.e. $L \approx (\pi a^2)^{1/2}$, then the volume is used as a basis for equating the two geometries (Lim and Aziz, 1995), therefore, a = 0.564L; and similarly a = 0.620L for a spherical system. Following the procedure described in Eq. 6 through 14, the following shape factors result for two and three sets of orthogonal fractures:

$$\sigma = \frac{8\pi^2}{L^2} \quad (18)$$

$$\sigma = \frac{12\pi^2}{L^2} \quad (19)$$

Figure 5 shows plots of S_w/S_{wmax}, against dimensionless time for both uniform and variable pressure distributions. The lines with symbols are the results for the analytical solutions, whereas the solid lines are the exponential approximations to the analytical solutions. Again, the results obtained for variable pressure in the fractures represent a faster recovery process. Correspondingly, the shape factor shown in Eq. 19 is larger than that obtained for constant uniform pressure within fractures by Lim and Aziz (1995): $3\pi^2/L^2$.

Discussion

The analytical solutions to the diffusion equation, their approximations, as well as the shape factors obtained for unsteady state and non-uniform pressure distribution within the fractures were developed with the idea of obtaining an exhaustive set of shape factors (and therefore transfer functions) covering one- to three-dimensional system and for different boundary conditions. It appears unlikely, however, that this approach yields shape factors approximating the behavior for filling fractures. Figure 6 shows a comparison among the experiment with a fracture aperture of 0.1 mm and flow rate of 1 cm³/min with analytic solutions and approximations of the diffusion equation for different dimensions and boundary conditions. This experimental data lay well within the filling-fracture regime (Rangel-German and Kovscek, 2002). For purposes of clarity, solutions for three sets of orthogonal fractures (shown in Figure 5) are not included. These curves fall above those shown below. None of the solutions in Figure 6 match the case of a filling-fracture. For such cases, the method recently presented by Rangel-German and Kovscek (2003) should be used.

Conclusions

An exhaustive set of matrix-fracture transfer functions for instantly-filled fractures under pseudo-steady and unsteady state

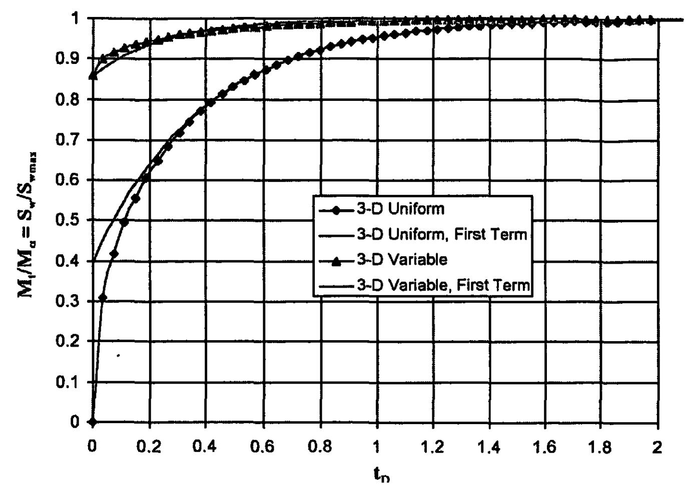


Figure 5. Analytic solutions of the diffusion in a three-dimensional system and its approximations.

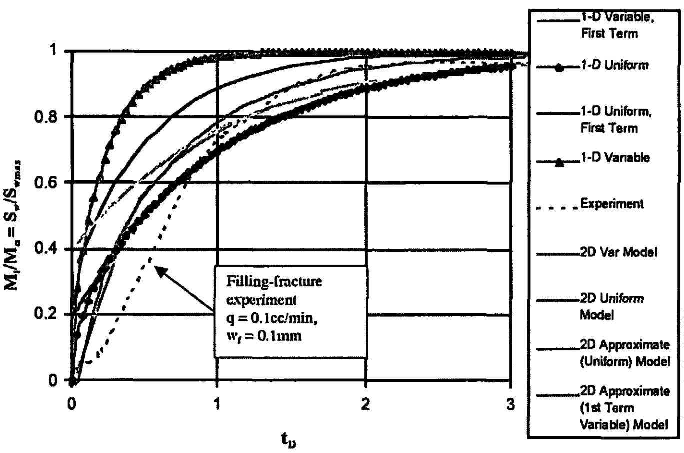


Figure 6. Analytic solutions and approximations of diffusion equation for different dimensions and boundary conditions.

conditions with uniform and non-uniform fractures conditions were derived for one-, two- and three- sets of orthogonal fractures. These transfer functions were verified with experimental data for a wide range of flow rates and fracture apertures, considering both filling- and instantly-filled fractures presented by Rangel-German and Kovscek (2002). Good agreement was found for the instantly-filled fracture cases. However, satisfactory agreement is not necessarily found for every instantly-filled fracture case. Cases where the transient behavior takes a relatively long time to develop will not be well matched by the exponential approximation. The exponential approximations never reach zero dimensionless pressure.

Shape factors in Eq. 1 are both time-dependent. We recognize that the shape factors corresponding to the pressure-driven expansion presented here are simple (single-phase flow) approximation to complex problems. However, in cases where the transient behavior of pressure is short, these constant (time-independent) shape factors lead to reasonable results, and their implementation in current dual-porosity models is rather simple. Shape factors corresponding to the imbibition part of Eq. 1 and those for partially-covered fractures should be obtained by means of the method presented by Rangel-German and Kovscek (2003).

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References

Akin, S., Schembre, J.M., Bhat, S.K., and Kovscek, A.R. (2000), "Spontaneous Imbibition Characteristics of Diatomite," *Journal of Petroleum Science and Engineering*, 25, 149-165.

Chang, M.: "Deriving the Shape Factor of a Fractured Rock Matrix," Technical Report NIPER-696 (DE93000170), NIPER, Bartlesville OK (September 1993).

de Swaan, A. (1990), "Influence of Shape and Skin of Matrix-Rock Blocks on Pressure Transients in Fractured Reservoirs, *SPE Formation Evaluation*, 344-352.

Horne, R.N., Satik, C., Mahiya, G., Li, K., Ambusso, W., Tovar, R. Wang, C., and Nassori, H.: "Steam-Water Relative Permeability," paper presented at the 2000 World Geothermal Congress, Kyushu-Tohoku, Japan, June.

Kazemi, H., Merrill, Jr., L.S., Porterfield, K.L., and Zeman, P.R. (1976), "Numerical Simulation of Water-Oil Flow in Naturally Fractured Reservoirs," *SPE Journal*, 317-326.

Lim, K.T. and Aziz, K.: "Matrix-Fracture Transfer Shape Factors for Dual-Porosity Simulators," *Journal of Petroleum Science and Engineering* (1995) 13, 169.

Rangel-Germán, E. (2002), "Water Infiltration in Fractured Porous Media: In-situ Imaging, Analytical Model, and Numerical Study," Ph.D. Dissertation, Stanford University, Stanford, CA.

Rangel-Germán, E.R. and Kovscek, A.R. (2002), "Experimental and Analytical Study of Multidimensional Imbibition in Fractured Porous Media," *Journal of Petroleum Science and Engineering*, 36, 45-60.

Rangel-Germán, E. and Kovscek, A. (2003), "Time-Dependent Matrix-Fracture Shape Factors for Partially and Complete Immersed Fractures", SPE Paper 84411, presented at the SPE Annual Technical Conference and Exhibition, Denver, USA. October.

Warren, J.E. and Root, P.J. (1963), "The Behavior of Naturally Fractured Reservoirs," *SPE Journal*, 245-55.