Development of a New Holdup Correlation for Geothermal Wells: A Preliminary Report

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ABSTRACT

Simulation of two-phase flow in wellbores requires use of empirical correlations for liquid holdup. Use of currently available holdup correlations often yields widely differing results for geothermal wells. This paper reports results of an ongoing effort to develop new holdup correlations for geothermal applications using high-quality discharge and downhole data from flowing geothermal wells. The latter data set encompasses a wide range of discharge rates and flowing enthalpies. Development of a preliminary holdup correlation for the cased section of geothermal boreholes is described.

Introduction

The ability to predict both the quantity of fluid that can be produced and its thermodynamic state (pressure, temperature, enthalpy, gas content, salinity, *etc.*) is essential for estimating the total usable energy of a geothermal resource. Numerical reservoir simulators can be utilized to calculate the thermodynamic state of the fluid at the underground feedzone(s) at which the fluid enters the wellbore. The computation of the wellhead fluid properties from a given underground state (or vice-versa) requires the use of a wellbore simulator. The fluid flow in the wellbore is not amenable to strict analytical treatment. Existing methods for treating two-phase flow in a wellbore require use of empirical correlations for liquid hold-up (Because of slip between the gas and liquid phases, the flowing gas quality Q_f is generally different from the *in situ* gas quality Q_s. The liquid hold-up correlation provides a relationship between Q_f and Q_s.) and friction factor. Almost all of the existing holdup correlations (see *e.g.*, Ansari *et al.*, 1994; Aziz *et al.*, 1972; Beggs and Brill, 1973; Duns and Ros, 1963; Hagedorn and Brown, 1965; Hadgu, 1989; Hughmark, 1962; Hughmark and Pressburg, 1961; Orkiszewski, 1967) are based on flow in two-phase petroleum (oil and gas) systems. At present, there does not exist a sufficient basis for selecting one or another of these correlations to simulate two-phase flow in geothermal wellbores. Utilization of different correlations very often yields widely differing results (see *e.g.*, Finger *et al.*, 1999).

Recent availability of high-quality downhole pressure/temperature/spinner logs from flowing geothermal wells suggests that it may be worth taking a fresh look at the empirical correlations for liquid hold-up. The present research effort is designed to develop new hold-up correlations for geothermal applications using data from flowing geothermal wells. As a result of a detailed examination of well data made available by Unocal and various Japanese developers, Garg and Pritchett (2001) identified 32 wells with high-quality discharge (mass discharge rate and enthalpy) and downhole pressure and temperature data. The data set encompasses a wide range of discharge enthalpies (*i.e.*, moderate enthalpy wells producing from liquid feedzones, and wells with enthalpies approaching the enthalpy of saturated steam), and casing diameters (ID's ranging from 100 mm to 384 mm). As far as fluid composition is concerned, the data set is less comprehensive. The salinity and non-condensable gas content of most of the wells in the data set are less than 1.5% and 1% (mass fraction of the produced fluid), respectively. In any event, the present data set is eminently suitable for developing a new empirical liquid hold-up correlation for geothermal wells.

To make the problem tractable, it was decided to at first consider only the cased section of geothermal wells. Obviously, the fluid flow in the cased section is much simpler than in the open hole/slotted and blank liner section. With the exception of the work by Hadgu (1989), all of the published papers treat two-phase flow in a pipe (*i.e.*, the cased section). The present paper is in the nature of a progress report. Our methodology for developing a hold-up correlation consists of using an existing wellbore simulator with an adjustable holdup correlation to match the downhole pressure profiles in flowing wells. These simulation results are employed to generate a multi-parameter (*i.e.*, flowing and *in situ* qualities, *in situ* liquid and gas fractions, *etc.*) data set. The latter data set was then used to formulate a relationship between the *in situ* and flowing steam qualities.

Simulation of Fluid Flow

The downhole pressure and temperature profiles in the cased portion of flowing wells were simulated using a specially modified version (see below) of the wellbore computer simulation program WELBOR (Pritchett, 1985). The WELBOR code treats the steady flow of liquid water and steam up a borehole. The user provides parameters describing the well geometry (inside diameter and angle of deviation with respect to the vertical along the hole length), a stable formation temperature distribution with depth, and an "effective thermal conductivity" as a function of depth representing the effects of conductive heat transfer between the fluid in the wellbore and the surrounding formation. For boreholes with two-phase flow at the bottom of the cased portion, the fluid state is prescribed by specifying flowing pressure, flowing enthalpy, salinity, and gas content.

In two-phase water/steam flow, pressure and temperature are not independent of each other. For any reasonable value of effective thermal conductivity K_m, the downhole flowing enthalpy may be adjusted to yield the appropriate pressure, and hence temperature, distribution in the wellbore, and flowing wellhead enthalpy. Matching the pressure/temperature distribution in the wellbore and flowing wellhead enthalpy does not constrain the heat loss and downhole (*e.g.*, at the bottom of the cased section) enthalpy. Since the flowing downhole enthalpy is not a measured quantity, it is not possible to determine a unique value for the heat loss in the presence of two-phase flow. It is, therefore, appropriate to eliminate effective thermal conductivity as a free parameter, and use a constant value for K_m. For almost all of the cases considered herein, K_m was assumed to be 4 W/m·°C.

In WELBOR, the frictional pressure gradient is computed using the Dukler I (or Dukler II) correlation (Dukler *et al.*, 1964), and a user prescribed roughness factor, ε. The Dukler II correlation gives a much larger pressure drop than the Dukler I correlation. Numerical experiments (Garg and Combs, 2002; also present study) have shown that it is usually necessary to use the Dukler I correlation in order to match the reported discharge rate and enthalpy data from geothermal boreholes. It was, therefore, decided to use the Dukler I correlation for the present study. The roughness factor, ε, may vary with depth. For most of the pressure profiles considered herein, the roughness factor was assumed to be zero. In a few cases, it was found necessary to use a non-zero value for ε.

The relative slip between the liquid and gas phases is treated in WELBOR using a modified version of the Hughmark liquid holdup correlation (Hughmark, 1962). The slippage rate may vary between the value given by the Hughmark correlation and no slip at all, according to the value of a user specified parameter, η, which varies between zero (no slip) and unity (Hughmark). For the present application, the WELBOR code was modified so as to allow η to vary as a function of depth. For all of the pressure profiles considered herein, it was found that at most two values of η (and a small transition zone in between) were required to produce a satisfactory match between the measured and computed pressures.

The principal parameters that may be varied to match the measured conditions along the wellbore (pressure and temperature) and at the wellhead (pressure, temperature, steam and liquid flow rates, liquid salinity, gas content of steam) are (1) flowing enthalpy, salinity and gas content at the bottom of the cased interval, (2) holdup parameter, η and (3) interior roughness factor, ε.

To illustrate the computational procedure, it is useful to consider Unocal well A-1. This well is cased and cemented to a depth of 793.1 mTVD (TVD: total vertical depth). The following well geometry is assumed for the cased section of well A-1:

Measured Depth (meters)	Vertical Depth (meters)	Angle with Vertical (Degrees)	Internal Diameter (mm)
253.0	253.0	0.000	384
809.8	793.1	14.068	315

A pressure of ~18.34 bars (absolute), taken as the average of measured and saturation pressures, was recorded in the flowing well at 793.1 mTVD. The reported discharge rate and wellhead enthalpy were 107 (± 3) kg/s and 1093 (± 23) kJ/kg, respectively. Total dissolved solids content of the separated liquid was 14,100 (± 140) ppm; the non-condensable gas content of the steam was 1.0 (± 0.1) %.

The stable formation temperature (Garg and Pritchett, 2001) was approximated by the following temperature distribution using linear interpolations between tabulated data.

Vertical Depth (meters)	Temperature (Degrees Celsius)
0	26.67
305	87.3
606	201.7
793.1	228.7

The best match to the downhole pressure profile and wellhead fluid state (pressure, enthalpy, salinity, gas content) was obtained using the following values for the unknown model parameters:

Flowing enthalpy at 793.1 mTVD
= 1105 kJ/kg
Fluid (liquid + steam) salinity at 793.1 mTVD
= 0.0115 kg/kg
Fluid (liquid + steam) gas content at 793.1 mTVD
= 0.0018 kg/kg
Hughmark parameter, η
= 0.11 for depths < 400 m
= 0.11 + 0.0044 (depth – 400) for
400 m < depth < 450 m
= 0.33 for depths > 450 m
Roughness factor, ε
= 0.00 mm for all depths

The computed pressure profile is compared with the measurements in Figure 1; the agreement is excellent. The computed fluid state at the wellhead (fluid enthalpy: 1,093 kJ/kg, liquid-phase salinity: 13,990 ppm; steam phase gas content: 1.005 %) is very close to the measurements.

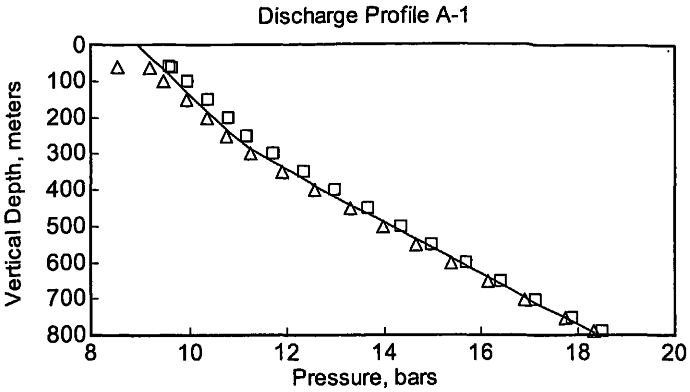

Figure 1. Pressure profile (triangles) recorded in discharging well A-1. The squares indicate the saturation pressure corresponding to the local measured temperature. The solid line is the computed pressure profile.

An essentially identical procedure was used to fit the downhole pressure and wellhead fluid state measurements for all of the thirty-two wells with high-quality downhole and wellhead data (Garg and Pritchett, 2001). The results of these computations were used to define the fluid state and associated quantities (*e.g.*, liquid and gas velocities) in the cased section of all the wells in the data set. Somewhat arbitrarily, it was decided to use 21 equally spaced points along each downhole profile to create a data set for formulating a new holdup correlation.

Holdup Correlation Parameters

Duns and Ros (1963) suggest that the various flow regimes that accompany two-phase flow in wells can be divided into three main regions depending on the gas throughput (Figure 2). The axis in Figure 2 denote the non-dimensional liquid, (N_l) and gas velocity (N_g) numbers:

$$\textit{Liquid velocity number, } N_l = v_l S_l \left(\frac{\rho_l}{g\sigma}\right)^{0.25}$$

$$\textit{Gas velocity number, } N_g = v_g S_g \left(\frac{\rho_g}{g\sigma}\right)^{0.25}$$

Here v_l (v_g) is the liquid (gas) velocity, ρ_l (ρ_g) is the liquid (gas) density, S_l (S_g) is the *in situ* liquid (gas) volume fraction, g is the acceleration due to gravity, and σ is the surface tension. Region I has a continuous liquid phase, and contains bubble flow, plug flow and part of froth-flow regimes. Liquid and gas phases alternate in region II covering the slug flow and remainder of the froth-flow regimes. Region III is characterized by a continuous gas phase, and contains the mist-flow regime.

Data from two-phase geothermal wells are shown as diamonds in Figure 2. Although the geothermal data lie in all the three regions, the bulk of these data are contained in Region II. It is apparent from Figure 2 that geothermal wells are characterized by relatively high liquid velocities such that only froth-flow (regions I and II) and mist-flow (region III) are encountered in geothermal wells. Bubble flow, plug flow and slug flow regimes appear to be of little practical interest in geothermal applications. Because of the limited range of flow regimes, it should be possible to describe two-phase flow in geothermal wells by a single (or at most a two-part) holdup correlation.

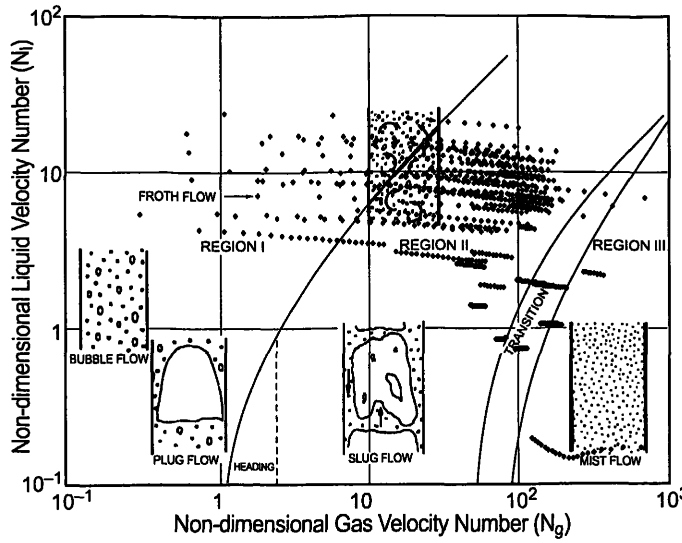

Figure 2. Two-Phase fluid flow regimes according to Duns and Ros (1963). Also shown (as diamonds) are the data from geothermal boreholes.

The flowing quality Q_f is defined as follows:

$$Q_f = \frac{AS_g\rho_g v_g}{M} \tag{1}$$

where A is the internal cross-sectional area of the pipe, and M is the total mass flow rate. Assuming a power law relationship for the velocity and *in situ* gas volume fraction, Bankoff (1960) derived a relation for flowing quality Q_f which is equivalent to:

$$Q_f = \frac{Q_s}{[1-Q_s]K + Q_s[1-\rho_l(1-K)/\rho_g]} \tag{2}$$

where Q_s is the *in situ* gas quality, ρ is the *in situ* mixture (gas plus liquid) density, and K is a flow parameter (see below).

$$Q_s = \frac{S_g\rho_g}{\rho} \tag{3}$$

$$\rho = S_l\rho_l + S_g\rho_g \tag{4}$$

For the case of homogeneous (*i.e.*, no slip) flow, the *in situ* quality Q_s is equal to the flowing quality Q_f; furthermore, flow parameter K is identically equal to unity. In general, one would expect the gas phase to rise more rapidly in the well than the liquid phase due to buoyancy; this implies that

$$Q_f \geq Q_s \tag{5a}$$

$$S_g \leq K \leq 1 \tag{5b}$$

The two-phase flow in a well is influenced by buoyancy, inertial, viscous and surface tension forces (Bankoff, 1960). Based on dimensional arguments, Hughmark (1962) concluded that the flow parameter K may be expected to depend on the flowing liquid volume fraction Y_l, and Reynolds (Rn), Froude (Fr), and Weber (We) numbers.

$$Y_l = \frac{S_l v_l}{(S_l v_l + S_g v_g)}$$
$$Rn = \frac{d_w M}{A\mu_m}$$
$$Fr = \frac{M^2}{(d_w A^2 g \rho_m^2)} \tag{6a–d}$$
$$We = \frac{d_w^2 g \rho_g Fr}{\sigma}$$

Here d_w is the inside well diameter, and σ is the liquid surface tension. Mixture (liquid plus gas) density ρ_m and mixture viscosity μ_m can be defined in a number of ways (see *e.g.*, Hughmark, 1962; Dukler *et al.*, 1964); in the following, the expressions given by Dukler *et al.* (1964) will be used.

$$\rho_m = \rho_l Y_l + \rho_g (1 - Y_l)$$
$$\mu_m = \mu_l Y_l + \mu_g (1 - Y_l) \tag{7a–b}$$

In Equation (7), μ_l (μ_g) denotes the liquid (gas) viscosity. Note that the above definitions for ρ_m and μ_m are different than those used by Hughmark (1962).

Holdup Correlation

To find a correlation for *K*, it is assumed that *K* can be expressed as a function of a single variable Z:

$$K = K\big(Z(Rn, Fr, Y_l, We, S_l)\big) \tag{8}$$

Hughmark (1962) investigated the dependence of Z on *Rn, Fr,* Y_l, and *We*, and found that Z (and hence K) did not appreciably depênd on the Weber number (*We*). In the following, Z is assumed to depend on all the five variables listed in Equation (8). Following Hughmark (1962), we introduce a particularly simple relationship for Z:

$$Z = Rn^\alpha Fr^\beta Y_l^\gamma We^\delta S_l^\omega \tag{9}$$

where α, β, γ, δ, and ω are as yet undetermined constants. Determination of the exponents in Equation (9) is straightforward when the functional relationship between *K* and Z is known; in this case, exponents can be estimated by minimizing the variance between the calculated (i.e., from the functional relationship *K*-*Z*) and measured (i.e., those derived from fitting the flow data) values for *K*. Since the functional relationship *K(Z)* is unknown, a different procedure was adopted to estimate the exponents in Equation (9).

Given a candidate set of exponents, we calculate *Z* for each point in the dataset for geothermal wells (Figure 2). Next, we sort the dataset in order of increasing Z, and calculate a pseudo-variance *S.V.* as follows:

$$S.V. = \sum_{i=1}^{n-1} (K_{i+1} - K_i)^2 \tag{10}$$

Here K_i denotes the value of *K* corresponding to Z_i, n is the number of points in the dataset, and Z_n is the largest value of Z.

The exponents in Equation (9) are obtained by minimizing this pseudo-variance.

Since a set of exponents may be multiplied by an arbitrary non-zero constant without changing the order of the sequence of *K's*, only four of the exponents need to be varied when minimizing the pseudo-variance. Note that very small changes in the exponents can leave the order of the sequence of *K's* unchanged and that when the order of the sequence is changed, the S.V. jumps discontinuously. Such functional behavior rules out the use of gradient methods for finding a minimum. The downhill simplex method (Press et al., 1986) is a procedure for finding minima of multidimensional functions that does not make use of derivatives, making it well suited to the present effort. A set of exponents for starting the procedure was obtained by assuming that log (*K*) is a linear function of log (Z) (and hence of log Rn, log *Fr*, log *We*, log Y_l, and log S_l), and using the least-squares method to determine an initial set of coefficients (α, β, γ, δ, and ω). The downhill simplex method finds smaller and smaller values for the *S.V.* until a region is reached which appears to be a broad local minimum. Since there is no reason to believe that this procedure yields a global minimum, various other sets of starting exponents were tried. While the downhill simplex method failed for some starting sets, it worked for a number of others. The function *K(Z)* was found to be monotonic in all cases, but with varying sign. By simply multiplying all the exponents in a set by -1, the order of the sequence of *K's* is inverted, giving the same *S.V.* and a graph with a slope of the opposite sign.

Changing the sign of the various exponents as needed to ensure that the exponent of the flowing liquid volume fraction is negative, and plotting *K* versus Z, it was found that the graphs always looked like a hyperbola with *K* = 1 as an asymptote (see e.g., Figure 3). Although a hyperbola for *K(Z)* can be made to yield a pseudo-variance that is close to the minimum, a more general functional form for *K(Z)* is needed in order to improve the fit in regions of Z that make little contribution to the pseudo-variance.

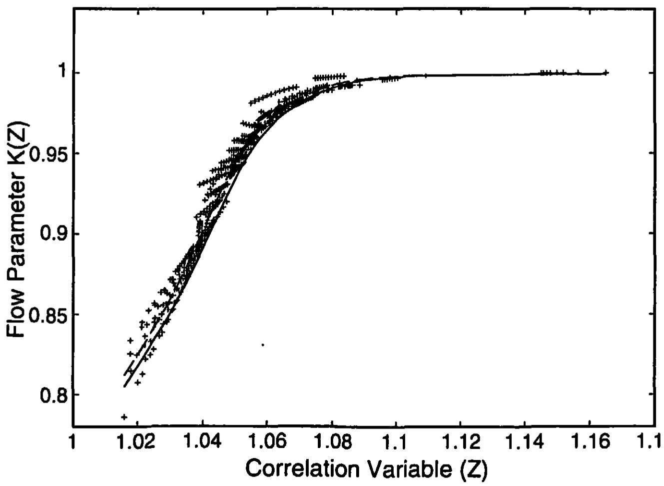

Figure 3. A plot of *K* versus Z. Data points (+), i.e. *K* versus *Z* values, in the figure were obtained by minimizing the pseudo-variance. The dashed line denotes the best fit to the *K-Z* data. The solid line was obtained by

Validation of Holdup Correlation

To verify that the holdup correlation (*i.e.*, $K(Z)$ relationship) developed in the preceding section can be used to simulate two-phase flow in geothermal wells, a special version of WELBOR was created; this version was configured to use the $K(Z)$ relation (dashed line) shown in Figure 3. The latter version of WELBOR was employed to simulate flow data for all the 32 wells in the dataset. Except for the holdup correlation, these simulations utilized the parameters used in the calculations described earlier (see *e.g.*, Figure 1).

As an example, we consider Unocal well A-4 which is cased and cemented to a depth of 888.5 mTVD. The cased section of well A-4 has the following geometry:

Measured Depth (meters)	Vertical Depth (meters)	Angle with Vertical (Degrees)	Internal Diameter (mm)
277.4	277.4	0.000	384
901.6	888.5	11.759	315

A pressure of ~23.49 bars (taken as the average of measured and saturation pressures) was recorded in the flowing well at 888.5 mTVD. The reported discharge rate and wellhead enthalpy were 135 (± 3) kg/s and 1089 (± 12) kJ/kg, respectively. Total dissolved solids content of the separated liquid was 14,600 (± 150) ppm; the non-condensable gas content of the steam was 0.68 (± 0.1)%.

The stable formation temperature was approximated by the following temperature distribution using linear interpolations between tabulated data.

Vertical Depth (meters)	Temperature (Degrees Celsius)
0	27
305	68
754	212
888.5	227

In the simulations used to create the data for Figures 2 and 3, the best match to the downhole pressure profile and wellhead fluid state (pressure, enthalpy, salinity, gas content) was obtained using the following values for the unknown model parameters:

Flowing enthalpy at 888.5 mTVD
$= 1102$ kJ/kg
Fluid (liquid + steam) salinity at 888.5 mTVD
$= 0.012$ kg/kg
Fluid (liquid + steam) gas content at 888.5 mTVD
$= 0.0011$ kg/kg
Hughmark parameter, η
$= 0.09$ for depths < 350 m
$= 0.09 + 0.0062\,(\text{depth} - 350)$ for $350\text{ m} < \text{depth} < 400\text{ m}$
$= 0.40$ for depths > 400 m
Roughness factor, ε
$= 0.00$ mm for all depths

For the present simulation, the modified Hughmark correlation was replaced by the $K(Z)$ relationship (dashed line) shown in Figure 3. The computed downhole pressure profile (solid line) is compared with the measurements in Figure 4. It is clear from Figure 4 that the simulated pressures are somewhat higher than the measured ones. Similar discrepancy between the measured and computed pressures was observed for other wells in the dataset.

The observation that the computed pressures are higher than the measured values suggests that the "best-fit" correlation for $K(Z)$ under-predicts the slip between the liquid and gas phases. Stated somewhat differently, the correlation yields too high values for K. It was, therefore, decided to modify the correlation for $K(Z)$; the modified correlation is shown as the solid curve in Figure 3.

Downhole pressure profiles for all the 32 wells in the dataset were again simulated using the modified correlation for $K(Z)$. The computed pressure profile for well A-4, shown as a dashed line in Figure 4, is in excellent agreement with the measurements. The modified correlation was also found to yield satisfactory results for all the high discharge-rate wells in the dataset. The computed pressures for low discharge-rate wells are, however, too high, and imply that the correlation for $K(Z)$ would need to be further modified.

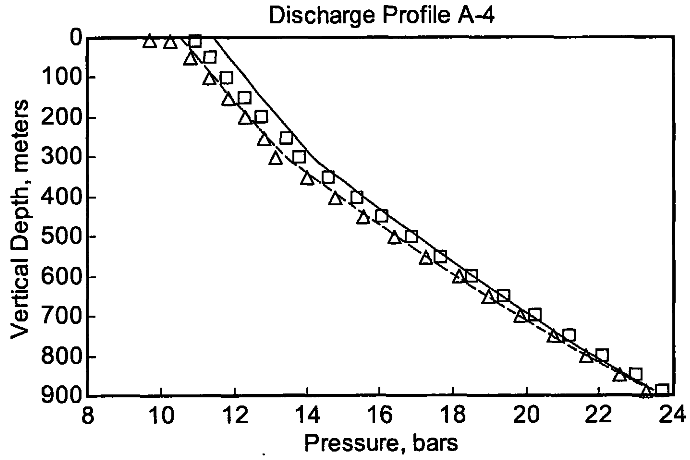

Figure 4. Pressure profile (triangles) recorded in discharging well A-4. The squares indicate saturation pressure corresponding to the local measured temperature. The solid (dashed) line is the computed pressure profile using the "best-fit" (modified) correlation for K(Z) shown in Figure 3.

Concluding Remarks

The principal goal of the present work is to use high-quality data from flowing geothermal wells (Garg and Pritchett, 2001) to devise new liquid holdup correlations for geothermal applications. To make the problem tractable, it was decided as a first step to develop a holdup correlation for only the cased section of geothermal wells. A methodology was formulated for constructing a correlation utilizing measurements in flowing wells. The preliminary correlation described here displays considerable promise for simulating two-phase flow from high-discharge rate geothermal wells. The latter correlation will, however, need to be modified for low-discharge rate wells. At present, it is not clear if it will be possible to formulate a single holdup correlation for both high- and low-discharge rate wells. The latter issue is currently being investigated; results will be reported in a future publication.

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