Article 149

DISSIMILARITY BETWEEN SPATIAL AND VELOCITY-WEIGHTED SEDIMENT CONCENTRATIONS

By H. P. GUY and D. B. SIMONS, Fort Collins, Colo.

Abstract.—Theoretical and measured differences between spatial and velocity-weighted concentrations show (1) that the spatial concentration is normally the greater and increases as the velocity and concentration gradients increase, (2) that velocity-weighted concentration must be used to compute sediment discharge, and (3) that spatial concentration must be used to compute the pressure or specific weight on the bed.

Sediment-concentration data in natural streams are used mostly to determine the amount of solids, either by weight or by volume, moving with streamflow. Such concentration data must be discharge weighted; that is, they must be a mean of velocity-weighted concentrations at many points in the stream cross section. The depth-integrating suspended samplers give a velocity-weighted concentration when a uniform vertical transit rate is used at evenly spaced verticals in the cross section. A discharge-weighted concentration is also obtained by traversing the nappe of flow from a flume at a uniform transit rate with an interception device having a uniform width of slot.

The sediment concentration computed from a spatial-collection procedure is defined as the relative quantity of sediment contained in an immobilized prism of water-sediment mixture over a specific area of the channel. The chief distinction between velocity-weighted and spatial concentrations is that one is based on sediment and water discharged through a cross section and the other on sediment and water in motion above an area of streambed at a particular instant. The difference between the two concentration measures (velocity-weighted and spatial) has been understood, to some degree, by a few sedimentologists for several years. Much confusion still exists, however, and the quantitative differences between the measures have seldom been determined even approximately. The purposes of this article are (1) to explain the differences between, and some uses of, these concentrations and (2) to show differences between the two concentrations as determined experimentally by use of plastic pellets transported by water in a small flume.

THEORETICAL DIFFERENCES

Differences between the velocity-weighted and spatial concentrations can be evaluated theoretically by consideration of the four equations that follow.

Equation 1 is the Prandtl-von Karman relation used by Einstein (1950)

$$\frac{U_y}{U_*}=5.75 \log \frac{30.2\, yx}{K_s}, \qquad (1)$$

where U_y=average point velocity at distance y above the streambed;

U_*=shear velocity, $\sqrt{gDS}$;
g=acceleration due to gravity;
D=depth of flow;
S=slope of the energy grade line;
x=a corrective parameter; and
K_s=grain roughness.

For the distribution of sediment concentration with respect to depth,

$$\frac{C_y}{C_a}=\left[\frac{D-y}{y}\cdot\frac{a}{D-a}\right]^z, \qquad (2)$$

where C_y=concentration of particles at distance y above the bed,
C_a=concentration of particles having the settling velocity w at distance a from the bed, and
$z=w/0.40\, U_*$.

The concentration, C, obtained by velocity-weighted samples, and the water discharge in the sampled zone, Q_m, are used to compute the sediment discharge per unit of time, q_s, through the sampled zone

$$q_s=Q_mC. \qquad (3)$$

ART. 149 *IN* U.S. GEOL. SURVEY PROF. PAPER 475–D, PAGES D134–D137. 1964.

Equation 3 is also equivalent in concept to the sediment discharge, q_s, determined by the relation

$$q_s = \int_a^D C_y U_y dy, \tag{4}$$

where C_y and U_y are defined by equations 1 and 2.

Figure 149.1 shows the approximate variation of velocity, sediment concentration, and sediment discharge with depth. This illustration and the preceding equations show that the mean concentration as weighted with velocity is considerably lower than the spatial concentration. The relatively low velocity-weighted concentration is due to the integration with depth ranging from low concentration and high velocity near the stream surface to high concentration and low velocity near the streambed. The spatial concentration, on the other hand, is the mean of the concentration from top to bottom and thus is considerably greater than the velocity-weighted concentration. Under the rare condition when all the sediment being transported is very fine, the concentration may be uniform from top to bottom. Under this condition, spatial and velocity-weighted concentrations are equal.

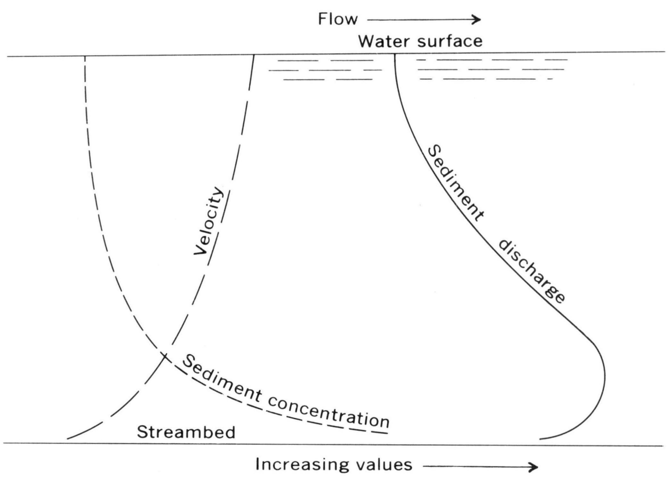

FIGURE 149.1.—Relative gradients of velocity, sediment concentration, and sediment discharge with depth.

Streamflow data show that neither U_y nor C_y are spatially or temporally consistent as defined in the equations; therefore, a sample of sediment or velocity at a point in a stream vertical or cross section cannot define the concentration or velocity distribution in such a vertical or cross section. To overcome this difficulty, the "precise method" was developed. This method, as described by the U.S. Inter-Agency Committee on Water Resources, Report 1 (1940, p. 63)

> involves collection of a relatively large number of point sediment samples simultaneously with velocity measurements. Sufficient data are collected to construct accurate vertical velocity and sediment distribution curves, the corresponding abscissas of which are multiplied to obtain a sediment-velocity curve. The area under this curve represents the sediment discharge in the vertical.

Needless to say, the "precise method" was too laborious for routine investigations of sediment movement in streams. The velocity-weighting technique, commonly called the depth-integration method, was developed to facilitate sediment-data collection and analysis. The technique, as described by the U.S. Inter-Agency Committee on Water Resources, Report 6 (1952, p. 14) is as follows:

> The discharge of the depth-integrating sampler was predicated on the hypothesis that an integrated sample of the water-sediment mixture existing at the place and time of sampling would be obtained if the filling rate were such that the velocity at the point of intake is equal to the local stream velocity while the sampler is moved at a uniform vertical speed in the stream.

The spatial concentration may be used to determine the actual load (weight per unit area) in transport over the streambed. It is also the correct concentration for determining the specific weight of the water-sediment complex. The spatial concentration can be obtained in streamflow by averaging the concentration of several equally spaced point samples in the stream cross section. In the laboratory, Bagnold (1955) used a mechanical device for isolating a rectangular slug of flow in a flume (fig. 149.2). A representative sample of spatial concentration is generally difficult to obtain because of the unsteady motion of particles near the bed. Most point samplers do not operate closer than within 0.3 or 0.4 foot of the bed. The mechanical-isolation device cannot distinguish between stationary and moving particles and thus cannot be used on an alluvial bed.

EXPERIMENTAL RESULTS

The difference between the velocity-weighted and spatial concentrations was studied by a series of runs in an 8-inch-wide recirculating flume. Plastic pellets with a specific gravity of 1.04 to 1.06 and with a median diameter of about 3 mm were used to represent the sediment grains. Samples were taken with a mechanical-isolation device after the velocity was sufficient to insure that all sediment over the bed was in motion, even though some particles were moving very slowly. The variation of sediment concentration and velocity with depth appeared to approximate that indicated by equations 1 and 2 and as illustrated in figure 149.1.

Sediment concentration was sampled by both methods for six different series of runs whereby each series contained a different amount of sediment circulating in the flume. To maintain the same amount of sediment in circulation, an equivalent volume of sample was returned to the flume for each sample removed. Two of these series were sampled at three widely different velocities. Velocity-weighted samples were taken by collecting in a nylon-mesh basket all particles discharged at the tail end of the flume. Sampling periods ranged from 8 to 15 seconds. Spatial-concentration samples were taken by quickly isolating with a sharp-edged frame having wire mesh ends, the water and sediment moving in a 1-foot length of the flume. This device is similar to the sampler illustrated in figure 149.2.

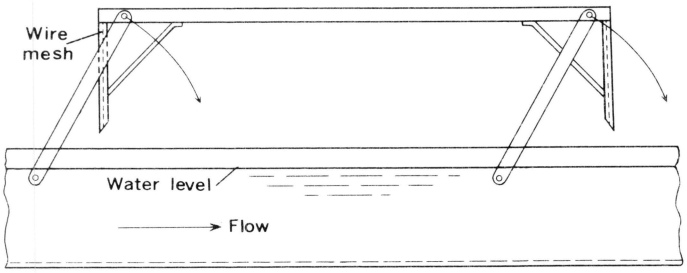

FIGURE 149.2.—Side view of sampling device for determining the spatial concentration of sediment moving in an open channel (after Bagnold, 1955).

The flow was stopped immediately after isolating the sample, and the particles were removed by siphoning from the known volume of water.

The concentrations in parts per million by weight for both methods of sampling are summarized in the accompanying table. This shows the concentration mean, the standard deviation, and the limit of probable error of measurement at the 90-percent confidence level for each run. The sediment concentration ranges from a velocity-weighted value of 85,000 ppm and a spatial value of 110,600 ppm on the flume bed for run 1 to a velocity-weighted value of 3,440 ppm and a spatial value of 5,250 ppm on the flume bed for run 6. The error of measurement is appreciably less at the tailbox because of the greater time of sampling; hence, only 3 samples were taken at the tailbox for each run, whereas 5 were taken on the flume bed for most runs.

Concentration of velocity-weighted and spatial samples of plastic pellets from 8-inch flume

Flume run	Mean velocity (ft/sec)	Velocity-weighted concentration at tailbox (ppm)			Spatial concentration on flume bed (ppm)		
		Mean	Standard deviation	Limit of probable error [1]	Mean	Standard deviation	Limit of probable error [1]
1	2.09	85,000	1,350	2,290	110,600	6,920	6,830
2	1.72	63,200	6,950	11,700	92,100	10,300	12,200
3	1.66	41,300	1,900	3,220	58,700	7,310	7,050
4	1.18	20,900	890	2,610	40,300	4,450	7,530
4	1.67	24,800	1,140	2,690	29,400	5,140	4,920
4 [2]	1.67	24,800	1,140	2,690	21,400	950	1,161
4	2.46	24,400	750	1,280	26,800	2,500	4,220
5	1.81	10,800	560	940	13,400	2,760	3,260
6	1.13	3,500	130	220	5,760	890	850
6	1.42	3,440	260	440	5,250	970	1,140
6	2.87	3,890	105	180	3,870	670	640

[1] Limit of probable error of measurement at 90-percent confidence level.
[2] Spatial samples collected near headbox of flume.

The sample concentrations compared in figure 149.3 illustrate the differences between the two methods of determining concentration. As expected, the spatial concentration is greater than the velocity-weighted concentration. The limits of the probable error of concentration measurement (90-percent confidence level) are shown (fig. 149.3) in the form of a rectangle approximately centered over each concentration mean. These rectangles indicate that the variability in the movement of sediment, and consequently the measurement error, is greater for the spatial means than for the velocity-weighted means.

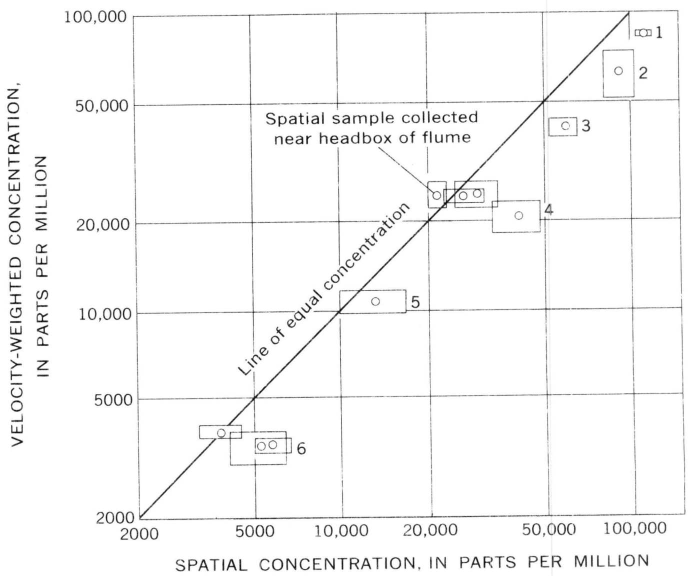

FIGURE 149.3.—Relation of velocity-weighted concentration to spatial concentration. The rectangle centered approximately around each concentration shows the limit of probable error of measurement at the 90-percent confidence level. Numbers correspond to flume runs described in table.

The larger the variation of velocity and sediment concentration with respect to depth (comparatively high V near the surface and a comparatively large C near the bed) the greater the difference between the two concentrations. As indicated by the highest velocities sampled (runs 4 and 6) the two methods may be comparable when the sediment is thoroughly suspended. In fact, the reverse of the normal result was found by comparing spatial concentrations from near the headbox with the velocity-weighted concentrations at the tailbox. The lower mean concentration of the spatial samples (see table and fig. 149.3) is caused by an inversion of the normal concentration gradient due to centrifugal force on the sediment particles.

CONCLUSIONS

Velocity and sediment-concentration distributions in a cross section of streamflow can be predicted only within rough limits. Theoretical and measured differences between spatial and velocity-weighted concentrations show that:

1. The dissimilarity between the two measures of concentration widens as the velocity and concentration gradients increase with respect to depth.
2. Sediment discharge in a stream should be computed from a velocity-weighted concentration obtained by sampling at several points or verticals in the stream cross section. Routine measurements of velocity-weighted concentration are made with several kinds of depth-integrating samplers.
3. Spatial concentration is needed when either the actual load or amount of sediment exerting pressure on the bed or the average specific weight of the sediment-water mixture over the bed is desired. Measurement of spatial concentration is usually difficult because immobile particles cannot be distinguished from moving particles.

REFERENCES

Bagnold, R. A., 1955, Some flume experiments on large grains but little denser than the transporting fluid, and their implications: Civil Engineers Proc., Excerpt pt. III, London, p. 192.

Einstein, H. A., 1950, The bed-load function for sediment transport in open channels: U.S. Dept. Agriculture Tech. Bull. 1026, p. 7–17.

U.S. Inter-Agency Committee on Water Resources, 1940, Report 1—Field practice and equipment used in sampling suspended sediment: U.S. Army Corps of Engineers, Iowa City, Iowa.

U.S. Inter-Agency Committee on Water Resources, 1952, Report 6—The design of improved types of suspended sediment samplers: U.S. Army Corps of Engineers, St. Paul, Minn.