Geothermal Resources Council Transactions, Vol. 24, September 24-27, 2000

Analysis of Interference Test in Fractured Reservoir —Application of Simulated Annealing—

M. Yoshida, Toshiaki Tanaka, Ryuichi Itoi and Michihiro Fukuda

**Department of Earth Resources Engineering,
Faculty of Engineering, Kyushu University,
6-10-1 Hakozaki, Higashiku, Fukuoka, Japan**

ABSTRACT

Simulated annealing algorithm has been applied to analyze interference well test data for fractured geothermal reservoirs. Effects of a magnitude of cooling coefficient (α) on accuracy of estimated parameters as well as computational time are examined by analyzing model data for a simple well configuration. Cooling coefficient of α=0.999 yields good estimates and an acceptable computational time. Analysis of field data from Sumikawa, Japan, provides good estimates of transmissivity ratio $\lambda=2.27\times10^{-7}$, storativity ratio $\omega=1.14\times10^{-5}$, and transmissivity of fracture medium $T_f=1.81\times10^{-7}$ (m^3/Pa•s), and storativity of the total system $S_t=7.93\times10^{-9}$ (m/Pa).

Introduction

Geothermal reservoirs are generally modeled into two types: porous reservoir and fractured reservoir. In a fractured reservoir, fractures mainly play a path of fluid with high permeability and low storativity whereas rock matrix block stores fluid due to its high storativity. In order to evaluate hydraulic properties of the reservoir system, well tests such as pressure transient tests and/or interference test are commonly conducted. Hydraulic properties of fractured reservoir are featured by two parameters (ω: storativity ratio, λ: transmissivity ratio) on the basis of Warren and Root model (1963).

There are analyzing methods for interference test: e.g. nonlinear least squares methods (Arellano et al.,1990) and Kalman filter method (R. Itoi et al.,1993). Conventional least squares methods may lead to false estimates when several local minimums present with the related problems. Simulated annealing (SA) that we applied to interference test analysis is classified as one of the least squares methods. SA has advantages to reach the global minimums even if there are several local minimums in the space of squares of residuals. This method is hard to be captured at local minimum but requires relatively long computational time compared with the other methods. In SA, computational time strongly depends on a magnitude of coefficient of cooling. We analyze model data of the interference test to examine influences of the magnitude of the coefficient on accuracy of estimated parameters and computational time. Then, we analyze field data from the Sumikawa geothermal field, Japan, with the method and estimated for ω and λ as well as other two parameters simultaneously; T_f: transmissivity of fractures, S_t: storativity for a total system.

Pressure Response for Fractured Reservoirs

Reservoir pressure under conditions of infinite extent and constant flow rate at an active well for the Warren and Root model (1963) can be expressed by (Da Prat, 1990):

$$\Delta P_f = \frac{q}{2\pi}\frac{1}{T_f}\frac{K_0\left[\sqrt{sf(s)}r_D\right]}{s\sqrt{sf(s)}K_1\left[\sqrt{sf(s)}\right]} \tag{1}$$

where ΔP_f is the pressure difference (Pa), q is the flow rate (m^3/s), and K_0 and K_1 are modified Bessel functions of the second kind of order zero and one, respectively. Subscript f denotes fractured medium. $f(s)$, T_f and r_D are expressed as:

$$f(s) = \frac{\omega(1-\omega)s+\lambda}{(1-\omega)s+\lambda} \tag{2}$$

$$T_f = \frac{k_f h}{\mu} \tag{3}$$

$$r_D = \frac{r}{r_w} \tag{4}$$

where h is the formation thickness (m), r is the radial distance(m), r_w is the wellbore radius (m), μ is the fluid viscosity (Pa•s) and s is the Laplace space variable. ω and λ are represented as:

$$\omega = \frac{(\phi CV)_f}{(\phi CV)_f + (\phi CV)_m} \tag{5}$$

$$\lambda = \psi \frac{k_m}{k_f} r_w^{\,2} \tag{6}$$

where C is the total compressibility (1/Pa), V is the ratio of volume of one porous system to bulk volume, k is the permeability (m^2), Ψ is the interporosity shape factor(1/m) and ϕ is the porosity. Subscript m denotes matrix medium.

Time t (s) is related to dimensionless time t_D by

$$t = \frac{S_t r_w^{\,2}}{T_f} \cdot t_D \tag{7}$$

S_t is expressed as:

$$S_t = (\phi CV)_f + (\phi CV)_m \tag{8}$$

Implementation of SA to Interference Test Analysis

Simulated annealing (SA) was first introduced by Metropolis et al.(1953). They used an annealing algorithm to simulate changes in a system of interacting atomics at a fixed temperature. In other words, this basic idea behind SA is an analogy between the way solid cool and anneal. "A solid is annealed by increasing its temperature so that its molecules are highly mobile, followed by slow cooling to force them into the low-energy state of a crystalline lattice" [Dougherty et al., 1991]. Each configuration defined by a set of atomic positions is weighted by Boltzmann probability factor:

$$P_r(E) = \exp\left(\frac{-E}{kT}\right) \tag{9}$$

where E is the energy of the configuration, k is Boltzmann's constant, and T is the temperature.

As for numerical analysis, the probability to accept new configuration can be expressed by

$$P_r(E) = \exp\left[\frac{-(E_{i+1} - E_i)}{kT}\right] \tag{10}$$

where E is the energy. Subscript i denote calculation step. If E_i is larger than E_{i+1}, $P_r(E)$ turned to be over 1. In that case we assume $P_r(E)$ equal to be 1.

Analyzing procedure using SA for estimating four unknown parameters is summarized as follows:

1. Assign initial values to four parameters; λ,ω,T_f and St.
2. Calculate pressure value (ΔP_{cal}) at an observation well by using Equation (1).
3. Calculate the sum of squares between observed pressure value (ΔP_{obs}) and calculated one (ΔP_{cal}). Energy E is then defined:

$$E = \sum_{j=1}^{n} \left(\Delta P_{obs,j} - \Delta P_{cal,j}\right)^2 \tag{11}$$

where n is the total number of pressure measurement, and subscript j denotes the time step.

4. Improve values of four parameters by

$$\log x_{i+1} = \log x_i + ran \cdot \log(dx) \cdot T_i / T_{START} \tag{12}$$

where x represents unknown parameters. Subscript i denotes the temperature step. *ran* is random number between –1 and 1. *dx* is the difference between the possible highest and lowest values of parameters, x is taken logarithm as these parameters may vary in a wide range. T_{START} is equal to be 100°C.

5. Compare the energy between the two steps, E_i and E_{i+1}. If $E_i \geq E_{i+1}$, x_{i+1} displaces x_i. When $E_i < E_{i+1}$, the displacement is treated probabilistically in order to avoid to settle at a local minimum. For this purpose, $P_r(E_{i+1})$ is compared to a newly made random number in a range between 0 and –1, and the displacement is performed when $P_r(E_{i+1})$ is equal to or larger than this random number.

A temperature is lowered for annealing according to a schedule defined by Equation (13).

$$T_i = T_0 \alpha^i \; ; i=0,1,2,3\ldots \tag{13}$$

where α is the coefficient of cooling. Subscript i denote the temperature step. The temperature is decreased down to 0.01°C.

Model Data and its Analysis

We assume an interference test with one production well and one observation well; a distance between the wells is set to be 100m. The well produced continuously at a constant rate of 100m^3/h for fifteen days, then it is stopped producing. Pressure difference at the observation well is calculated using Equation (1). Table1 summarizes parameter values used for making model data, which are denoted as true values. Figure 1 shows pressure versus time at the observation well. Pressure values

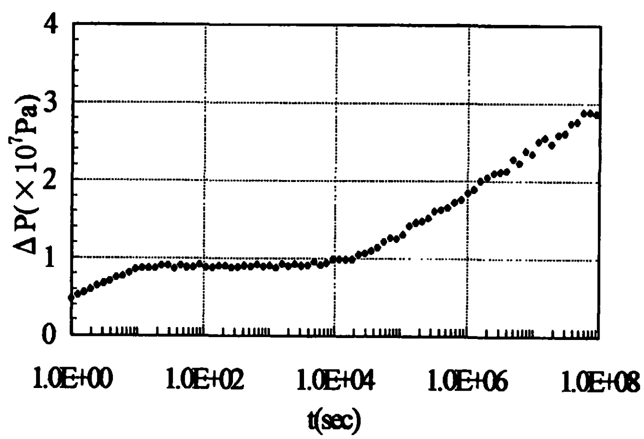

Figure 1. Model data of interference test.

are plotted at equal time intervals on log axis. A noise of 3% at each measurement is added to the pressure values. The curve shows a typical feature of pressure response for a fractured reservoir; a set of linearly increasing pressures separated by constant pressure values.

Table 1. True value and initial value of parameters. Highest value and lowest value of parameters are also shown.

parameter	set value	initial value	upper limit	lower limit
λ(-)	3.16×10^{-8}	1.00×10^{-3}	1.00×10^{-5}	1.00×10^{-8}
ω(-)	3.16×10^{-4}	1.00×10^{-2}	1.00×10^{-2}	1.00×10^{-7}
T_f(m^3/Pa·s)	1.00×10^{-8}	1.00×10^{-5}	1.00×10^{-5}	1.00×10^{-9}
S_t(m/Pa)	1.00×10^{-9}	1.00×10^{-6}	1.00×10^{-6}	1.00×10^{-10}

One of the disadvantages of simulated annealing is that it requires a large computational time. It strongly depends on a magnitude of coefficient of cooling (α). Therefore, we examine its effects on computational time as well as accuracy of estimates by giving six different values of α; 0.95, 0.96, 0.97, 0.98, 0.99 and 0.999. Possible highest values shown in Table 1 are assigned as initial ones to four parameters. Estimated parameters by using different α are shown in Table 2. When a=0.999 is used, all four parameters are in good agreement with the true values. On the other hand the agreement of estimated parameters with other values of a are not as good as the results for α=0.999. Computational time using α=0.999 is longer than those for other values of a, and it is proportional to the magnitude of α. When α=0.999 is used, computational time requires three times longer than that for α=0.95. It is however within an acceptable computational time. Thus, we confirmed that simulated annealing using α=0.999 is of practical use.

Table 2. Estimated parameters for different values of a together with true value.

	λ(-)	ω(-)	T_f(m^3/Pa·s)	S_t(m/Pa)
true values	3.16×10^{-8}	3.16×10^{-4}	1.00×10^{-8}	1.00×10^{-9}
α=0.95	1.77×10^{-5}	1.62×10^{-3}	1.03×10^{-10}	1.78×10^{-6}
α=0.96	1.29×10^{-9}	3.83×10^{-4}	9.36×10^{-8}	2.50×10^{-10}
α=0.97	1.00×10^{-9}	1.00×10^{-4}	2.70×10^{-8}	0.90×10^{-10}
α=0.98	1.63×10^{-8}	3.52×10^{-3}	1.24×10^{-8}	1.63×10^{-10}
α=0.99	1.60×10^{-8}	1.09×10^{-3}	1.19×10^{-8}	2.23×10^{-10}
α=0.999	3.22×10^{-8}	3.17×10^{-4}	1.05×10^{-8}	0.93×10^{-8}

Analysis of Field Data

The Sumikawa geothermal field is located in the Hachimantai volcanic area in northern Honshu, Japan. The area depicted is about 42 square kilometers; the Sumikawa geothermal field lies in the western part of the area (Garg et al.,1991). Two interference tests conducted in 1986 and 1989 are analyzed with the method above. Figure 2 shows well locations at Sumikawa.

Discharge Test in1986

In 1986, a large-scale pressure-interference test was carried out at Sumikawa. Well S-4 was discharged starting on September 2 and was shut in on November 3. Four shut-in observation wells (O-5T, S-3, KY-1 and KY-2) were equipped with down

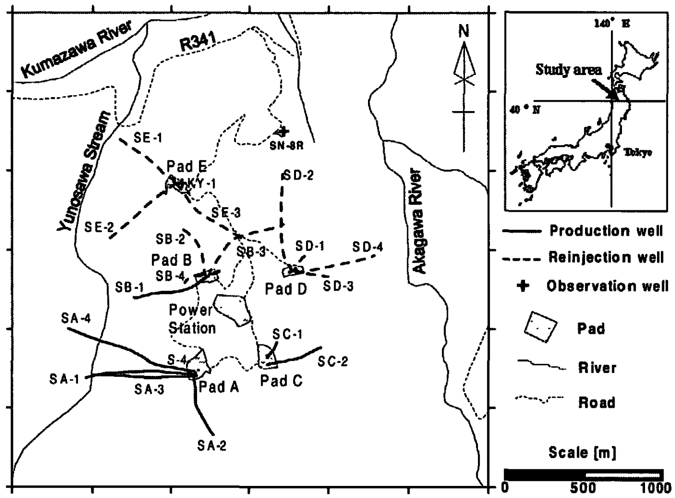

Figure 2. Well location map at the Sumikawa geothermal field (modified from Ariki et al.,2000).

hole pressure gauges of capillary-tube type. No signals attributable to Well S-4 discharge were recorded in Wells O-5T, KY-2 or S-3, but a clear and immediate response was observed in Well KY-1, located 1.1km north of Well S-4 (Pritchett et al., 1989).

We analyze the pressure data measured at Well KY-1 to Well S-4 discharge. Figure 3 shows flow rate history of Well S-4 and pressure measurement at Well KY-1. Four parameters (λ,ω,T_f and S_t) are estimated simultaneously by using simulated annealing. Initial values of parameters are given such that they are equal to the highest values shown in Table 1. Figure 4 shows a comparison between measured pressure and calculated pressure using estimated parameters shown in Table 3. An agreement between two kinds of pressure is relatively good.

Maki et al. (1988) analyzed the data by using the line-source solution for a radial symmetric porous reservoir. They estimated kh to be 2.4 darcy-m where fluid viscosity of 1×10^{-4} (Pa·s) was used. Estimated kh in this study is 2.37 darcy-m, and which is identical to their results. This is because that the test was carried out for a long period so that pressure response as a single porosity system prevails.

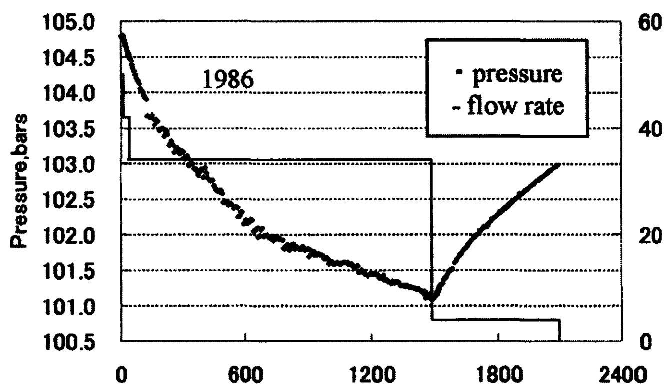

Figure 3. Pressure measurement in Well KY-1 due to discharge of Well S-4 and its flow rate in 1986 (Garg et al.,1991).

Table 3. Estimated value of parameters.

parameter	estimated value
λ(−)	2.63×10^{-7}
ω(−)	4.89×10^{-5}
T_f(m^3/Pa·s)	2.37×10^{-8}
S_t(m/Pa)	2.02×10^{-8}

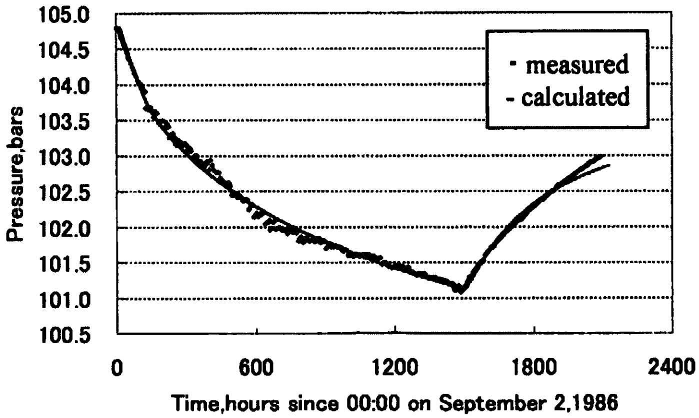

Figure 4. Comparison of calculated pressure response of Well KY-1 with measurements.

Injection Test in 1989

Cold water was intermittently injected into Well S-4 between 16 May and 19 May 1989. Pressure measured down hole in Well KY-1 responded quickly to a change in injection rate of Well S-4. Injection was repeated at short time intervals as shown in Figure 5. Thus, a feature of fractured reservoir in terms of pressure response could be well realized in pressure measurement at Well KY-1. Estimated parameters with the method above are summarized in Table 4. Figure 6 shows a comparison between measured pressure and calculated one using the estimated parameters. A relatively good agreement between measured and calculated pressures is obtained. Discrepancies between two kinds of pressure increase with time in the latter part of measurement. This may be due to influences of other active wells since Well S-4 stopped injecting at time about 70 hours.

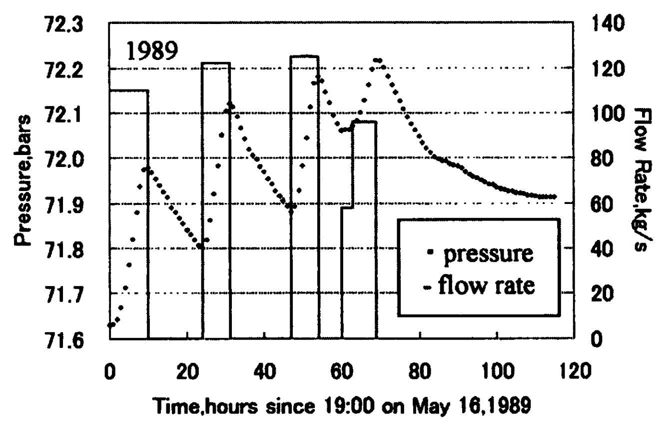

Figure 5. Pressure response of Well KY-1 due to cold water injection into Well S-4 and its flow rate in 1989 (modified from Garg et al.,1991).

Table 4. Estimated value of parameters.

parameter	estimated value
λ(−)	2.27×10^{-7}
ω(−)	1.14×10^{-5}
T_f(m^3/Pa·s)	1.81×10^{-7}
S_t(m/Pa)	7.93×10^{-9}

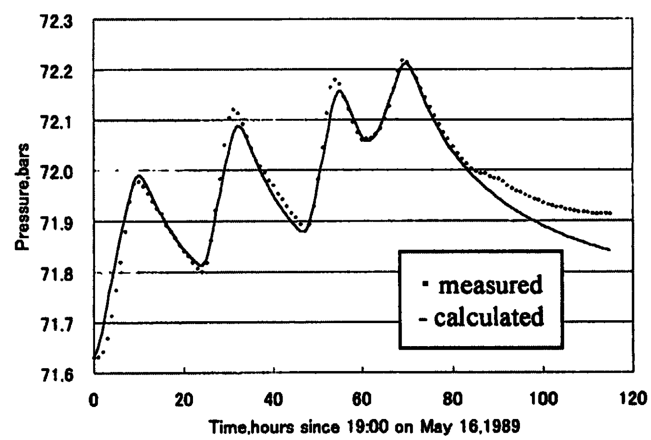

Figure 6. Comparison of calculated pressure response of Well KY-1 with measurements due to cold water injection into Well S-4.

Estimated kh value of 18.1 darcy-m is 7.6 times larger than that for the result of the test in 1986. Large kh value must indicate fracture permeability. Thus, differences of estimated kh values suggest that injecting water into Well S-4 at short time intervals provide information of fracture system in terms of kh value. On the other hand, estimated value of ϕCh is one third of the results for the data in 1986.

Conclusions

A method for analyzing for interference well test data by using simulated annealing is developed.

Model data of interference well test are analyzed for four kinds of reservoir parameters to examine the effects of cooling coefficient (α) on estimates of the parameters and computational time. Good estimates and acceptable computational time are achieved when α=0.999 is given.

Interference well test data at Sumikawa, Japan, are also analyzed. Relatively high transmissivity for a fractured system is estimated when cold water injected at a short time intervals. Simulated pressure using estimated parameter values provides a good agreement with the measured pressure.

References

Arellano, V.M., Iglesias, E.R., Arellano, J. and Perez, M.R. (1990) Automatic interference-test analysis in personal computers, GRC Trans., 14, 2, pp.1271-1278.

Ariki, K., Kato, H., Ueda, A. and Bamba, M. (2000) Characteristic and management of the Sumikawa geothermal reservoir, northeastern Japan, Geothermics, 29, pp.171-189.

Da Prat, G. (1990) Well test analysis for fractured reservoir evaluation, Elsevier, 210pp.

David, E.D. and Robert, A.M. (1991) Optimal Groundwater Management 1. Simulated annealing, Water Resources Reserch, 27, 10, pp.2493-2508.

Garg, S.K., Pritchett, J.W., Ariki, K. and Kawano, Y. (1991) Pressure-interference testing of the Sumikawa geothermal field, Proc. Sixteenth Workshop on Geothermal Reservoir Engineering, Stanford University, pp.221-229.

Itoi, R., Fukuda, M. Jinno, K. and Gotoh, H. (1993) Interference test analysis as the Takigami geothermal field, Japan, Proc. Eighteenth Workshop on Geothermal Reservoir Engineering, Stanford University, pp.267-272.

Metropolis, N., Rosenbluth, A.W., Rosenbluth, M.N., Teller, A.H. and Teller, E. (1953) Equation of state calculations by fast computing machines, J. Chem. Phys., 21, 6, pp.1087-1092.

Maki, H., Prithett, J.W. and Garg, S.K (1988) Interpretation of a pressure interference test of the Sumikawa geothermal field, Proc. International symposium on geothermal energy 1988: Exploration and development of geothermal resources, Kumamoto/Beppu, Japan, pp.557-560.

Warren, J.E. and Root, P.J. (1963) The behavior of naturally fractured reservoirs, Soc. Pet. Eng. J., 3, pp.245-25.