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The Contradictions in Continua Modelling of Flow and Heat Transfer in Fractured and/or Porous Reservoirs

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ABSTRACT

The classical continuum models of flow and heat transfer in fractured and/or porous media are discussed. In simplest examples the internal contradictions of these models are shown for the case of steady process. It is stressed that classical linear exchange terms can give errors even in fully steady conditions, not only in transient processes as it was considered previously. The reasons for such contradictions are analyzed. The ways of contradictions' removal in process of models' formulation through spatial averaging method are discussed.

Introduction

Continuum models of transfer in heterogeneous media are widely used as well as structural models (Pashkevich, 1996). For continuum models the closure problem for determination of interphase flows still remains. The objective of the work is demonstration of internal contradiction of classical models of transfer in fractured and porous media, which use quasi-steady linear presentation of exchange flows. The examples of steady heat transfer and flow in porous (fractured) and fractured porous reservoirs are presented.

After continuum conception the natural heterogeneous rock-fluid medium is substituted for conditional interpenetrating media. Every point of medium is described by two sets of macro-scale physical values and properties, corresponded to each phase. The macro-scale values are an average of micro-scale values. Usually the macro values and macro equations are obtained using a spatial (more often volume) averaging method. The classical continuum model of convective heat transfer in porous (fractured) media is formulated as follows:

$$(1-\phi)C_r\partial t_r/\partial\tau - (1-\phi)\lambda_r^*\nabla^2 t_r - q_t = 0$$
$$\phi C_f\partial t_f/\partial\tau + C_f \mathbf{v}\,\nabla t_f - \phi\lambda_f^*\nabla^2 t_f + q_t = 0, \qquad (1)$$

where ϕ is porosity; C_r and C_f are volume specific heats of rock and fluid, t_r and t_f are temperatures averaged over each phase's (rock and fluid) part in representative elementary volume (REV); $\mathbf{v}$ is Darcy's velocity, which is determined in REV, λ^* is effective thermal conductivity, q_t is interphase heat flow.

At first, system (1) was suggested by Rubinstein (for purely conductive process in 1948, and for convective heat transfer in 1972) and were later named "macroscale interpenetrating media" (MIM) model. The heat flow q_t was considered proportional to specific surface rock-fluid contact (a_{rf}) and difference of fluid and rock temperatures:

$$q_t = a_{rf}\,\alpha\,(t_f - t_r), \qquad (2)$$

where α is interphase heat exchange coefficient. At first linear expression for heat flow q_t in form similar to (2) were suggested by Anzelius, 1926 and Schumann, 1929 for one-dimensional convective heat transfer case. Usually linear character of (2) is advocated as quasi-steady approximation near local thermal equilibrium (Nikolaevskii, 1984, Nigmatulin, 1987, Whitaker, 1977). Discussion of problem arising in proving determination of expression for exchange flow (2), as well as its adequacy to natural processes in geothermal reservoirs is presented in Pashkevich, 1996.

The MIM model (equations (1), (2)) is being widely used for modelling of heat transfer in permeable media, including in geothermal reservoirs and systems (Cheng, 1977, Dyadkin, 1989, Tsherban et al., 1986).

Analogous continuum model for processes of flow in fractured porous media was at first formulated by Barenblatt and Zheltov (1960, see also Barenblatt et al., 1960) and by Warren and Root, 1963. Their formulations for slightly compressible fluid under specific conditions were named dual porosity model. The general Barenblatt-Zheltov model (also named dual permeability model) is formulated as follows:

$$
\begin{aligned}
{}_{fr}c_{fr}\partial p_{fr}/\partial\tau - (k_{fr}/\mu)\nabla^2 p_{fr} - q_p = 0 \\
\phi_{ma}c_{ma}\partial p_{ma}/\partial\tau - (k_{ma}/\mu)\nabla^2 p_{ma} + q_p = 0,
\end{aligned} \tag{3}
$$

where exchange flow between fractures and porous matrix q_p was postulated in form:

$$
q_p = (\alpha_p/\mu)\,(p_{ma} - p_{fr}), \tag{4}
$$

p is pressure, ϕ is porosity, c is compressibility, k is permeability, μ is fluid viscosity, subscripts "fr" and "ma" mean values which refer accordingly to fracture and porous matrix media, α_p is dimensionless coefficient. The pressures p_{fr} and p_{ma} in (3) and (4) are pressures averaged over each phase's (fractures and porous matrix) part in REV.

The linear character of matrix-fracture exchange flow q_p in form of (4) is considered quasi-steady approximation, as well as in above heat transfer model.

The dual porosity model remains widely used in reservoir modelling and is currently modified. Modifications of the model mainly are concluded in generalization of matrix-fracture flow q_p to get over the quasi-steady restriction (short recent review can be found in Shook, 1996 and in Suares et al., 1996). Recent example of such attempts is a work of Shook, 1996. From the other hand, Suares et. al., 1996 introduced the triple porosity model (with additional third open fracture's or fault's phase), for accounting of complex spatial structure of reservoir (here the problem of determination of exchange flows remains). At last, the MINC (Multiple INteracting Continua) method of Pruess and Narasimhan, 1985 and its improving (Pruess, 1990) is another (semi-numerical) extension of dual porosity model. The MINC method is considered free from quasi-steady restriction (Shook, 1996). Nevertheless, after Pruess, 1990, for simple case of two continua (fractures and matrix), "the MINC method reduces to the double-porosity approach ".

Below it is demonstrated on the simple examples, that expression of interaction's terms q_t and q_p as differences of phase's temperatures and pressures (equations (2) and (4)) can lead to errors even in full-steady cases. Therefore classical continuum models for flow and heat transfer are restricted by not only quasi-steady assumption, but in a more general sense.

Heat Transfer In Fractured Or Porous Reservoirs

Consider a conditional reservoir consisted of vertical equidistant fractures filled with solid materials with different thermal conductivity λ_r, λ_f and thickness δ. (see Figure. 1).

Let reservoir be in steady state, so that heat flux Q_t is normal to fracture's plane direction x (i.e., in horizontal direction), be constant and heat flux in other directions be zero. Then the local (micro scale) t' temperature distribution for both phases will be linear in direction x. The volume averaged temperatures t_r and t_f will be equal temperatures averaged over horizontal cross section of fractures. Then averaged temperatures' distributions will be linear too and:

$$
\begin{aligned}
&dt_f/dx=\mathrm{const}_f, \quad dt_r/dx=\mathrm{const}_r; \\
&\nabla^2 t_f = d^2t_f/dx^2 = 0,\ \nabla^2 t_r = d^2t_r/dx^2 = 0.
\end{aligned} \tag{5}
$$

At the same time, from the equations (1) and (2) for considered steady state (also v=0) we get:

$$
\begin{aligned}
\nabla^2 t_r = d^2t_r/dx^2 = (a_{rf}\,\alpha/((1-\phi)\lambda_r^*))(t_r - t_f) \neq 0 \\
\nabla^2 t_f = d^2t_f/dx^2 = (a_{rf}\,\alpha/(\phi\lambda_f^*))\,(t_f - t_r) \neq 0,
\end{aligned} \tag{6}
$$

since $t_r \neq t_f$, so far as the heat transfer occurs in direction x. Thus equations (6) give incorrect expression for laplacian of phase temperatures, which turns into correct equations (5) only in case of thermal equilibrium, when $t_r = t_f$.

Therefore in above considered case the classical continuum model of heat transfer in porous (fractured) media (equations (1) and (2)) gives wittingly wrong result for phase temperature's distribution.

Flow In Fractured Porous Reservoirs

Now consider a conditional reservoir consisted of intermitted vertical porous blocks (with thickness δ_{ma} and permeability k_{ma}) and fractures (with apertures δ_{fr} and permeability k_f), as it shown in Figure. 2.

Let reservoir be in steady state, so that flow rate Q_p in horizontal direction, be constant and in other directions be zero. Then the local (micro scale) p' pressure distribution in blocks and in fractures will be linear in direction x. The volume averaged pressures p_{ma} and p_{fr} will be equal pressures averaged over horizontal cross section of blocks and fractures. Then averaged pressures' distributions will be linear too and:

$$
\begin{aligned}
&dp_{fr}/dx=\mathrm{const}_{fr}, \quad dp_{ma}/dx=\mathrm{const}_{ma}; \\
&\nabla^2 p_{fr} = d^2p_{fr}/dx^2 = 0,\ \nabla^2 p_{ma} = d^2p_{ma}/dx^2 = 0.
\end{aligned} \tag{7}
$$

At the same time, from the equations (3) and (4) for considered steady state we get:

$$
\begin{aligned}
\nabla^2 p_{fr} = d^2p_{fr}/dx^2 = (\alpha_p/k_{fr})(p_{fr} - p_{ma}) \neq 0 \\
\nabla^2 p_{ma} = d^2p_{ma}/dx^2 = (\alpha_p/k_{ma})(p_{ma} - p_{fr}) \neq 0,
\end{aligned}
$$

since $p_{fr} \neq p_{ma}$, so far as the flow occurs in direction x. Thus equations (8) give incorrect expression for laplacian of phase pressures, which turns into correct equations (7) only in case of hydrostatic equilibrium, when $p_{fr}=p_{ma}$.

Therefore in above considered case the classical continuum model of flow in fractured porous media (equations (3) and (4)) gives incorrect result for phase pressure's distribution.

The Reason For Contradiction and Possible Ways of its Removal

The reason for the contradictions in discussed examples is concluded in representation of interphase flows q_t and q_p as expressions proportional to the difference of averaged phases' values (temperatures and pressures) in forms of (2) and (4).

In terms of volume averaging method the interphase flows, entranced in equations (1) and (3), represent the surface inte-

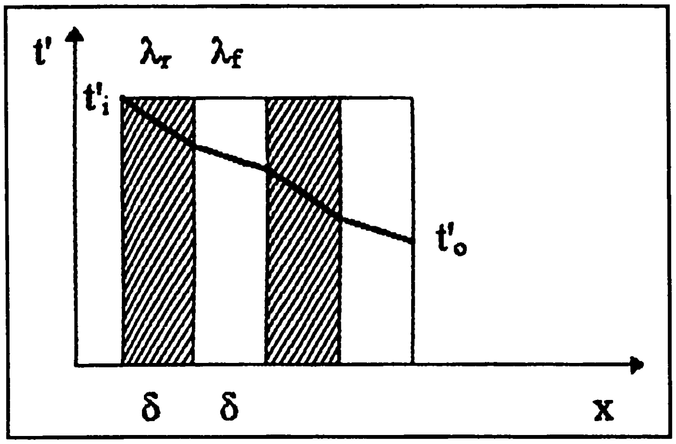

Figure 1. Temperature distribution in system of parallel fractures.

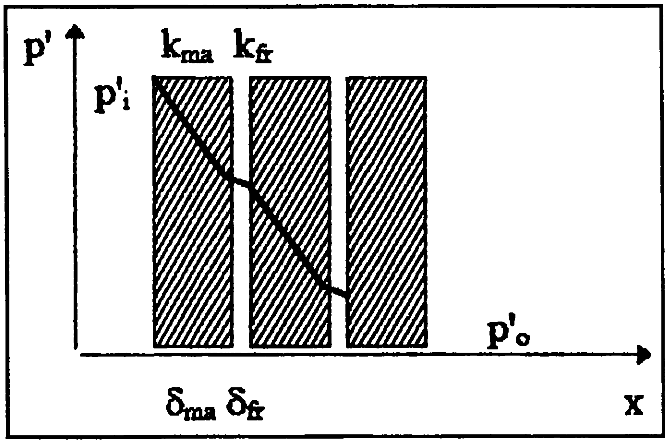

Figure 2. Pressure distribution in system of parallel porous blocks and fractures.

grals of micro-scale interphase flows over surface of interface. That is simple to show on example of heat transfer case. For constant thermal properties of fluid and rock and constant Darcy's velocity, the averaged equations for temperatures are (see, e.g., Whitaker, 1977):

$$\begin{aligned} &(1-\phi)C_r\partial t_r/\partial\tau - (1-\phi)\lambda_r\nabla^2 t_r - \\ &(\lambda_r/V)\nabla\int t_r' \mathbf{n}_{rf}dA - (1/V)\int\lambda_r\nabla t_r' \mathbf{n}_{rf}dA = 0, \\ &\phi C_f\partial t_f/\partial\tau + C_f\mathbf{v}\nabla t_f - \phi\lambda_f\nabla^2 t_f - \\ &(\lambda_f/V)\nabla\int t_f' \mathbf{n}_{fr}dA - (1/V)\int\lambda_f\nabla t_f' \mathbf{n}_{fr}dA = 0, \end{aligned} \tag{9}$$

where $t^{'}$ is microscale temperature, V is averaging volume, the integration goes over surface of rock-fluid interface A in volume V (REV), n_{rf} is outward to phase r unit vector on the surface A.

The first integrals in (9) named thermal tortuosity, are usually included in the effective thermal conductivities (see equations (1)). The last term in equations (9), is the microscale interphase heat flow, integrated over surface of rock-fluid interface in averaging volume, and denoted q_t (see (1)).

As it shown in previous section, the use of standard expression for macroscale interphase heat flow q_t in form of (2) leads to error in simple case of heat transfer in layered system (Figure 1). Nevertheless, in the same case, general equations (9) lead to correct result. Indeed, for the case (see Figure. 1) we have:

$$\nabla\int t_r' n_{rf}dA=0, \quad \nabla\int t_f' n_{fr}dA=0. \tag{10}$$

With the use of corollary of Slattery-Whitaker averaging theorem: $\nabla\varnothing = -(1/V)\int n_{rf}\,dA$, and use of Fourier law: $Q_t = -\lambda_r\nabla t_r$ we can write:

$$\begin{aligned} &(1/V)\int\lambda_r\nabla t_r' n_{rf}dA = Q_t\nabla\phi = 0, \\ &(1/V)\int\lambda_f\nabla t_f' n_{fr}dA = -Q_t\nabla\phi = 0, \end{aligned} \tag{11}$$

since heat flux Q_t is constant and porosity is constant too in the case. Then in the our steady case with v=0, substitution of (10) and (11) in equations (9) gives:

$$\nabla^2 t_r = 0, \nabla^2 t_f = 0. \tag{12}$$

The equations (12) are correct and coincide to equations (5), obtained in the first of the previous sections.

Therefore, the general averaged equations (9) give correct results in considered case, but specific equations (1) and (2), derived from (9), are not. The reason is the expression of interphase heat flow in form of (2). As it above demonstrated, although general heat flux Q_t occurs in the system and is not equal zero, the integrated heat flow over fluid-rock interface surface in the averaging volume $\int\lambda_f\nabla t_f' n_{fr}\,dA$ turns to zero for the considered case. Then the interphase heat flow q_t which associated with this integral, can not be formulated as difference of the fluid and rock temperatures (equations (2)).

The considered example confirms the power and accuracy of the general volume averaging method in formulation of continuum models of transfer in multiphase systems, including in the geothermal reservoirs. The considered contradictions arise from weakness of present closure scheme for interphase flow determination.

The contradiction can be avoided by appropriate choice of form of interphase flow, for example by introduction of third macro-scale phase–temperature, averaged over surface of bulk phase's interface (fluid-rock interfaces). Such approach demands formulation of the additional equation for the surface temperature. It is a subject for further work.

The same illustration and inferences can be made for the process of flow in a fractured porous reservoir, considered in previous section.

Conclusion

The classical continuum models of heat transfer in porous (fractured) media and of flow in fractured porous media (double porosity model) may give incorrect results in fully steady conditions (not only in transient case, as it was well known previously). Therefore in geothermal reservoir modelling such models (approach) can not give adequate description of natural processes in general case. The reasons for internal contradictions of continuum models conclude in linear expression of interphase flow (fluid-rock, fractures-porous matrix) through difference of phase's pressures and temperatures. The contradictions can be avoided by appropriate choice of form of interphase flow, for example by introduction of third macro-scale phase–temperature or pressure, averaged over surface of bulk phase's interface (fluid-rock and fractures-porous matrix interfaces).

The described features of classical continuum models of transfer in permeable medium must be included in design of geothermal reservoir models.

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