Dynamic Response Of Fluid Inside A Penny Shaped Crack

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ABSTRACT

In order to discuss the method for estimating the geometric characteristics of geothermal reservoir cracks, a theoretical study is performed on the dynamic response of the fluid inside a reservoir crack in a rock mass subjected to a dynamic excitation due to propagation of an elastic wave. As representative models of reservoir cracks, a penny shaped crack and a two-dimensional crack which are connected to a borehole are considered. It is found that the resonance frequency of the fluid motion is dependent on the crack size, the fluid's viscosity and the permeability of the formation. The intensity of the resonance is dependent on the fluid's viscosity when the size, the aperture and the permeability are fixed. It is also found that, at a value of the fluid's viscosity, the resonance of fluid pressure becomes strongest. The optimum value of the fluid's viscosity is found to be almost perfectly determined by the permeability of the formation. Furthermore, it is revealed that, if the fluid's viscosity is fixed to be the optimum value, the resonance frequency is almost independent of the permeability and aperture, but is dependent on the size of crack. Inversely speaking, this implies that the size of the reservoir crack can be estimated from the resonance frequency, if the fluid with the above mentioned optimum value of viscosity is employed for hydraulic fracturing.

Introduction

In geothermal heat extraction using artificial reservoir cracks such as HDR (Duchane, 1991)and HWR (Takahashi and Hashida, 1992), one of the key technologies is the characterization of reservoir cracks which are created by hydraulic fracturing; it is highly desirable to estimate the basic geometrical characteristics, i.e. size, aperture and degree of contract between two crack surfaces, as well as mechanical characteristics, i.e. interfacial stiffness induced by the contact and flow transmissivity of the reservoir crack itself. For these purposes, the so-called AE/MS methods are most promising. Thus, the investigation of dynamic response of a fluid-filled crack is crucial. So far, the dynamic response of a fluid-filled crack has been studied to clarify the source mechanism of volcanic earthquakes, the seismic source of which is considered to be a crack filled with molten magma (Chouet, 1986, 1988; Chouet and Julian, 1985; Ferrazzini and Aki, 1987; Honda and Yomogida, 1993). With regard to the advanced geothermal energy extraction, Ferrazzini et al. (1990), by using a three-dimensional fluid-filled crack model, successfully attempted to estimate the size of a reservoir crack created by hydraulic fracturing during the US HDR project at Fenton Hill, New Mexico. Hayashi and Sato (1992) studied the guided wave in a liquid layer sandwiched between two elastic semi-infinite half spaces which were partially in contact to each other across the layer, following the approach employed by Ferrazzini and Aki (1987). Nagano et al. (1995) measured the guided waves in the reservoir crack in the Higashi-hachimantai HDR model field, where the reservoir crack was connected to two boreholes.

They set an airgun at one of the intersection points between the crack and the boreholes and set a triaxial seismic dector at the other point of intersection. Hayashi et al. (1995) also studied the dynamic response of a two-dimensional very thin crack which was filled with water and found that strong standing waves with wave lengths being equal to L, $2L/3$,$L/2$ ...existed, where L was the length of the two-dimensional crack. Recently, Dvorkin et al. (1992) indicated that the following three factors had significant influence on the dynamic response. The three factors are (1) the spatial variation of a crack aperture, (2) filtration from the crack into the surrounding formation and (3) filtration inside the crack. They studied the dynamic response of fluid in a two-dimensional crack embedded in a rigid body, emphasizing the effects of the three factors.

In the present paper, we study the dynamic response of a fluid inside a three-dimensional crack in a rigid medium. The fluid is subjected to normal harmonic oscillations of the crack surfaces that are permeable and allow the fluid to filtrate into the surrounding formations. As a representative three-dimensional model of reservoir cracks, we employ a penny shaped crack which is connected to a borehole at its center. The basic differential equation for the pressure of the fluid is derived from the Navier-Stokes equation and the equation of continuity, following the approach employed by Dvorkin et al. (1990). The basic differential equation is solved by using the so-called shooting method with the aid of the Runge-Kutta-Gill integration scheme. Then, we discuss the basic characteristics of the dynamic response of the fluid, emphasizing the effects of main factors, such as fluid viscosity, permeability and porosity of the surrounding formation, the crack size and crack aperture and so on, and examine the feasibility of estimating the crack geometry by using the characteristic of the dynamic response. Finally, we also discuss the case of a two-dimensional crack for comparison. The basic characteristics of the dynamic response of the case of a two-dimensional crack were studied in detail by Dvorkin et. Al. (1992). In the present paper, we discuss the case of a two-dimensional crack from the view point of the estimation of the reservoir crack characteristics.

Governing Equation

We examine the dynamic response of the compressible viscous fluid inside a penny shaped crack embedded in a formation. A borehole is crossing the crack perpendicularly at the center of the crack. Crack walls are permeable, allowing the fluid to filtrate into the surrounding formation. The fluid flow is induced by normal harmonic oscillations of the crack walls. Let us introduce a cylindrical coordinate system (r, θ, z) at the center of the penny shaped crack (Figure 1). The information is assumed to be rigid. In the following, the governing equation for the three-dimensional axisymmetric problem is derived, following Dvorkin et al (1990) who treated the two-dimensional problem.

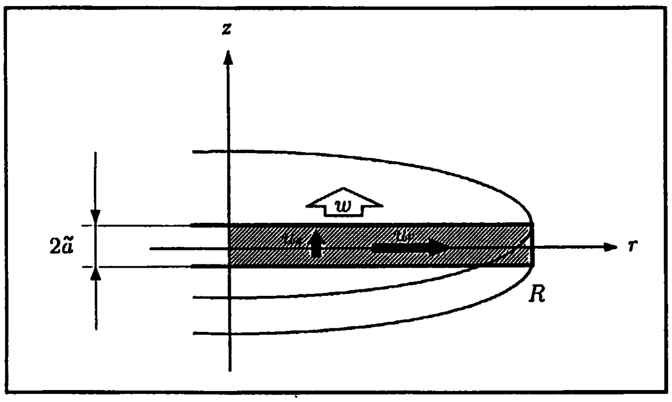

Figure 1. Viscous compressible fluid is a penny shaped crack: u_r and u_z are fluid velocity components and w is fluid velocity of filtration through the walls.

The movement of fluid is axisymmetric. Let us denote the fluid velocity components in the r and z-directions as u_r and u_z, respectively. Furthermore, w is a fluid velocity of fluid filtration into the surrounding formation. We assume that the crack is symmetric with respect to the z-plane. The aperture of the crack $2\tilde{a}$ is a function of the spatial coordinate r and time t similarly to Dvorkin et al. (1990):

$$\tilde{a}(r,t) = a(r) + a_0(r)\varepsilon \exp(i\omega t), \tag{1}$$

where $a(r)$ is zero-frequency aperture of the crack, ω in the angular frequency of oscillation of the walls, and the product $a_o(r)\varepsilon$ gives the amplitude of this oscillation. The amplitude is small compared to the aperture of the crack:

$$a_0(r)\varepsilon \ll a(r). \tag{2}$$

The compressibility of the fluid in the crack can be described by a linear relation

$$dP = c_0{}^2 d\rho, \tag{3}$$

where P is the pressure of the fluid, p is its density, and c_0 is the fluid acoustic velocity. Density variation dp in Equation 3 is much smaller than its reference "undisturbed" value p_o:

$$d\rho \ll \rho_0. \tag{4}$$

We assume that the pressure is constant across the crack and therefore the density of the fluid independent of the z-coordinate.

The equation of mass conservation in the crack is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r) + \frac{\partial}{\partial z}(\rho u_z) = 0. \tag{5}$$

The integration of Equation 5 in the z-direction from 0 to $\tilde{a}$ gives

$$\tilde{a}\frac{\partial \rho}{\partial t} + \rho u_z|_0^{\tilde{a}} + \int_0^{\tilde{a}} \frac{1}{r}\frac{\partial}{\partial r}(r\rho u_r)\,dz = 0. \tag{6}$$

Due to the symmetry of the fluid flow relative to the z-plane,

$$u_z|_{z=0} = 0. \tag{7}$$

At the upper wall of the crack

$$u_z|_{z=\tilde{a}} = w|_{z=\tilde{a}} + \frac{\partial \tilde{a}}{\partial t}. \tag{8}$$

This filtration is modeled as a one-dimensional process in the z- direction. The one-dimensional filtration equation is solved by assuming that the crack is embedded in an infinite porous space. The boundary conditions are the condition of pressure continuity at the crack walls and the condition that fluid pressure in the formation vanishes as $z \to \infty$. The details of this solution are given in Dvorkin et al. (1990). The resulting formula relates w to P is given by

$$w|_{z=\tilde{a}} = (1+i)\frac{k_0}{\mu}\sqrt{\frac{\omega}{2\kappa}}\,P|_{z=\tilde{a}}, \tag{9}$$

where $M = p_0 k_0 c_0^{\,2} / (\varnothing_0\, \mu)$ is hydraulic diffusivity coefficient, μ is fluid's viscosity, k_0 is permeability of the surrounding formation, and $\varnothing_0$ is its porosity.

Assuming that all functions are harmonically time dependent:

$$\left.\begin{aligned} P(r,t) &= P_0(r)e^{i\omega t}, \\ w(r,t) &= w_0(r)e^{i\omega t}, \\ u_r(r,z,t) &= u_{r0}(r,z)e^{i\omega t}, \\ u_z(r,z,t) &= u_{z0}(r,z)e^{i\omega t}. \end{aligned}\right\} \tag{10}$$

Substituting Equations (1) and (9) into Equation (8), we find

$$u_z|_{z=\tilde{a}} = \left[(1+i)\frac{k_0}{\mu}\sqrt{\frac{\omega}{2\kappa}}P_0 + i\omega a_0\varepsilon\right]e^{i\omega t}. \tag{11}$$

We derive the equation of motion from the Navier-Stokes equation defined in the cylindrical coordinate system. Let us assume that the zero-frequency aperture a is almost constant regardless of r, then the flow is approximately radial, and we can neglect $u_r\,(\partial u_r/\partial r)$ and $\partial^2 u_r/\partial r^2$. Hence, the equation of motion becomes

$$\frac{\partial u_r}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial r} + \frac{\mu}{\rho}\left(\frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2}\right). \tag{12}$$

At any station r, with local pressure $\partial P/\partial r$, the solution of Equation (12) for (approximately) parallel-plate radial flow is given by (Mavko and Nur, 1979)

$$u_r = -\frac{1}{\mu n^2}\frac{\partial P}{\partial r}\left[1 - \frac{\cosh(nz)}{\cosh(n\tilde{a})}\right], \tag{13}$$

where

$$n = \sqrt{\frac{i\omega\rho}{\mu} + \frac{1}{r^2}}. \tag{14}$$

Substituting Equation (13) into Equation (6), and using Equations (7), (8), (9) and (11), we arrive at the following differential equation for the function $P_0(r)$:

$$\begin{aligned} &\frac{\rho_0}{\mu n^2}\left[a - \frac{1}{n}\tanh(na)\right]\frac{d^2P_0}{dr^2} \\ &+ \frac{\rho_0}{\mu n^2}\left[\frac{1}{r}\left(1+\frac{3}{n^2r^2}\right)\left\{a - \frac{1}{n}\tanh(na)\right\} - \frac{a}{n^2r^3}\tanh^2(na)\right]\frac{dP_0}{dr} \\ &- \left[i\omega\frac{a}{c_0^2} + (1+i)\frac{k_0\rho_0}{\mu}\sqrt{\frac{\omega}{2\kappa}}\right]P_0 = i\omega\rho_0 a_0\varepsilon \end{aligned} \tag{15}$$

Here, we have set $\tilde{a} \cong a$ in view of Equation (2).

Dynamic Response of Fluid

We numerically solve Equation (15) by using the so-called shooting method with the aid of the Runge-Kutta-Gill integration scheme under the following boundary conditions:

$$P_0 = 0 \quad \text{at} \quad r = R_0, \tag{16}$$

$$\frac{dP_0}{dr} = 0 \quad \text{at} \quad r = R. \tag{17}$$

where R_0 is the radius of the borehole and R is the radius of the crack. The pressure averaged over the area of the crack is given by

$$P_{av} = \frac{1}{S}\iint_S P_0(r)\, r\, dr\, d\theta, \tag{18}$$

where S is the area of the crack. In the numerical calculation, the radius of borehole R_0 is set to be 10cm. Regarding the crack geometry, the radius of the crack R is set to be 10m, 20m and 30m and the aperture of crack $2a$ is set to be 2mm, 5mm and 10mm. In the following, we discuss the dependency of the averaged pressure amplitude on the permeability of the formation k_0, the viscosity of fluid μ and the porosity of the formation $\varnothing_0$.

Figure 2 is an example of the plots of the normalized average pressure P_N, defined by $|P_{av}|/(\rho c_0^2 a_0 \varepsilon/a$, versus frequency $f = \omega/2\pi$, demonstrating the effect of k_0 on the resonance frequency and P_N, where R =10m, $2a$ =10mm, μ =1cP and $\varnothing_0$=3%. Figure 3 shows the variation of P_N with respect to the viscosity and frequency for the case of R =10m, $2a$ =10mm, where the permeability is fixed to be k_0 = 0.09mdarcy and the porosity of the formation if fixed to be $\varnothing_0$=3%. It is seen from Figure 3 that the resonance of fluid pressure becomes strongest at a value of the viscosity. In the following this optimum value of the viscosity is denoted as μ_M. Figures 4(a) and (b) shows also the variation of P_N with respect to viscosity and frequency for the cases of R =20m, $2a$ =10mm and R=10m, $2a$ =5mm, where k_0 =0.09darcy and $\varnothing_0$=3%. By comparing Figure 4 with Figure 3, we can conclude that the optimum viscosity μ_M is almost independent of

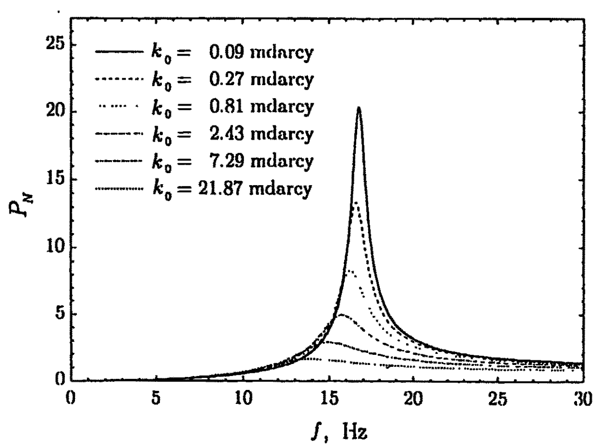

Figure 2. An example of the variation of the normalized average pressure P_N with respect to the excitation frequency f (R =10m, 2a =10mm, μ =1cP).

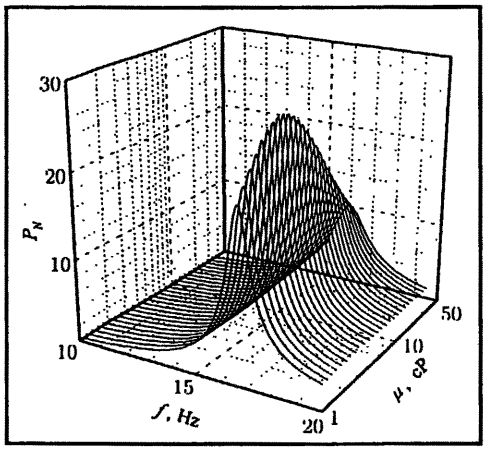

Figure 3. Variation of the normalized average pressure P_N with respect to the fluid's viscosity μ and frequency f (k_0 =0.09mdarcy, $ø_0$ =3%, R =10m, 2a =10mm).

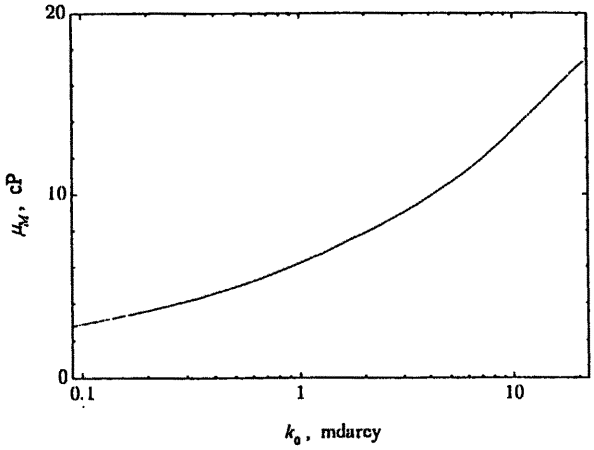

Figure 5. Variation of the optimum viscosity of fluid μ_M with respect to the permeability of the formation k_0. This relationship is independent of R, a, and $ø_0$.

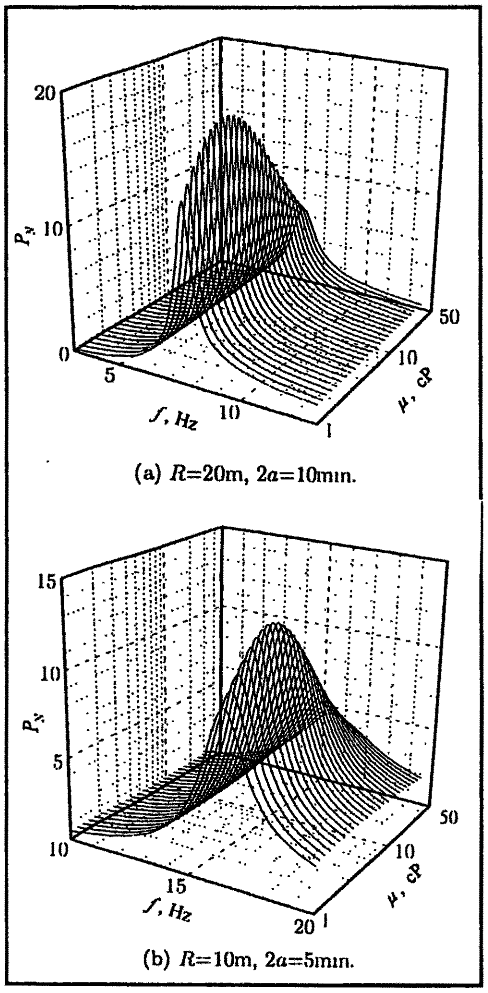

Figure 4. Variation of the normalized average pressure P_N with respect to the fluid's viscosity μ and frequency f (k_0 = 0.09mdarcy, $ø_0$ =3%).

the radius of the crack R and the aperture of the crack a. This means that μ_M is almost perfectly determined by the permeability of the formation k_0, if $ø_0$ is fixed. The relationship between μ_M and k_0 is given in Figure 5. This relationship is independent of the crack geometry, i.e. R and a, as discussed just above. Thus, regardless of R and a, we can expect to get the strongest AE/MS signal during hydraulic fracturing by selecting the viscosity of fluid to be μ_M which is determined by giving the permeability of the formation k_0 as shown in Figure 5. We have checked the effect of the porosity of the formation $ø_0$ and have found that only the amplitude of P_N is dependent on the porosity, although the details are not presented for the sake of brevity. Thus the discussions stated above hold regardless of $ø_0$. In the following, $ø_0$ is fixed to be 3%.

Practically, it is frequently observed that the aperture of the crack mouth at the borehole wall is fairly large due to the collapse of the borehole wall. We have examined the effect of the collapse by calculating with various shapes of the crack opening in the very vicinity of the crack mouth. Resonance frequency of the fluid pressure and intensity of the resonance pressure change an insignificant amount. Thus, we can neglect the effect of the collapse.

Now, let us consider the effects of a, R, k_0 and μ on the resonance frequency f_r. By comparing Figures 3 and 4(b), it is seen that f_r is almost independent of a. But, f_r is dependent on R as can be seen by comparing Figures 3 and 4(a). And f_r is also dependent on μ as can be seen from Figures 3 and 4. It is readily understood f_r is dependent on k_0 as shown in Figure 2. Thus, the effect of the factors R, k_0 and μ on f_r are very complex. However, if we fix μ to be μ_M, the situation becomes much clearer. Figure 6 shows the variation of f_r with respect to k_0 when μ is fixed to be μ_M. As can be seen from Figure 6, f_r is independent of k_0 and determined by R only, if μ is fixed to be μ . Figure 7, reproduced from Figure 6, shows the relationship between R and f_r. Thus, we can estimate R from f_r. Furthermore, we can construct the plots of P_{NR}, defined by $|P_{av}|/(\rho c_0^2 a_0 \varepsilon/R)$, versus k_0 for various values of a. An example of such plots is shown in

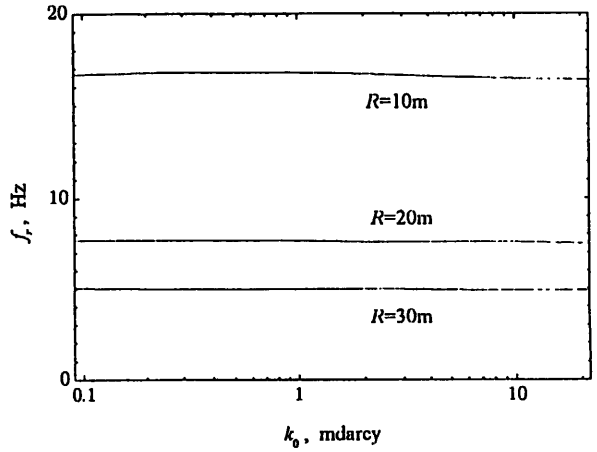

Figure 6. Variation of the resonance frequency fr with respect to the permeability of the formation k_0 ($\mu = \mu_M$). This relationship is independent of a.

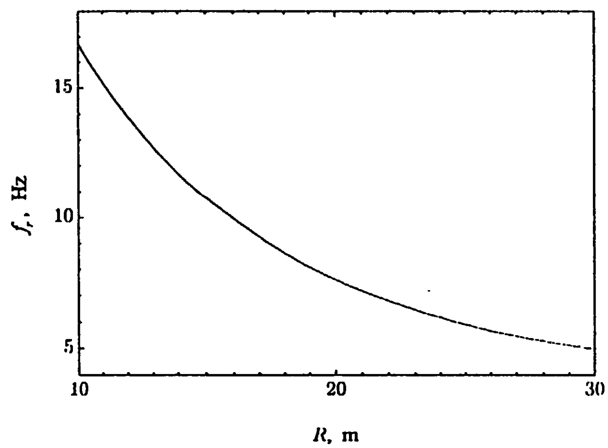

Figure 7. Variation of the resonance frequency f_r with respect to the radius of the crack R at the optimum viscosity of fluid $\mu = \mu_M$.

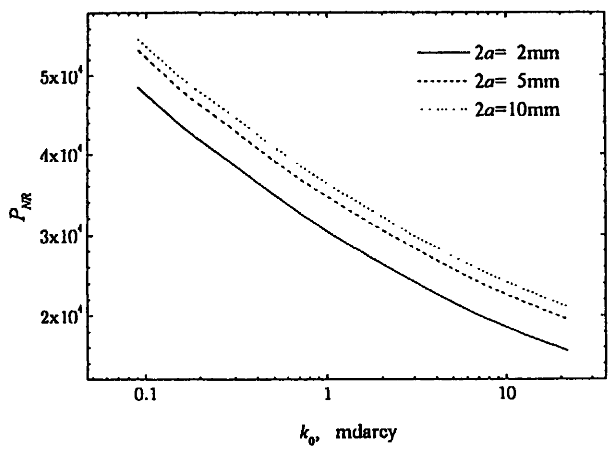

Figure 8. Variation of the maximum of the normalized average pressure P_{NR} ($\equiv |P_{av}|/(\rho c_0^2 a_0 \varepsilon/R)$ with respect to the permeability of the formation ($\mu = \mu_M$, $R = 10\text{m}, \phi_0 = 3\%$).

Figure 8 where R is fixed to be 10m. If R is determined as discussed above, then it is not impossible to estimate the aperture of the crack a from the intensity of the resonance of the averaged pressure P_{NR} by using the relationship between the P_{NR} and k_0 for the determined value of R, such as Figure 8. However, there remain problems of how to measure the amplitude of the excitation input and how to measure the averaged pressure in the crack.

Two-Dimensional Cracks

The governing equation for the case of a two-dimensional crack is given as follows (Dvorkin et al. (1990):

$$\left[1 - \frac{\tanh\left(a\sqrt{i\omega\rho_0/\mu}\right)}{a\sqrt{i\omega\rho_0/\mu}}\right]\frac{d^2P_0}{dx^2}$$
$$+\frac{\partial a}{a\partial x}\tanh^2\left(a\sqrt{i\omega\rho_0/\mu}\right)\frac{dP_0}{dx}$$
$$+\left[\frac{\omega^2}{c_0{}^2} + (1-i)\frac{\omega\rho_0 k_0}{a\mu}\sqrt{\frac{\omega}{2\kappa}}\right]P_0$$
$$= -\omega^2\rho_0\frac{a_0\varepsilon}{a} \quad (19)$$

where x is the distance from the center of the crack measured along the crack. We have analyzed the case of the two-dimensional crack by using this equation similarly to the case of a three-dimensional penny shaped crack.

The results are similar to those of a three-dimensional penny shaped crack. Figure 9 shows the variation of the optimum viscosity μ_M with respect to the permeability of the formation k_0. This relationship between μ_M and k_0 is independent of the length of crack L, the aperture of the crack a and the porosity of formation ϕ_0. There are two lines in the region where the permeability of the formation k_0 is larger than about 5mdarcy.

These two lines correspond to the two peaks of the normalized average pressure P_N. These two values of the two peaks are almost equal to each other and, furthermore, P_N in the region between the two lines is almost equal to the value of P_N at the two peaks, although the value of P_N outside the two lines is smaller than that in the region between the two lines. Thus, for k_0 larger than about 5mdarcy, any value of the fluid's viscosity μ between the two lines can be regarded as μ_M.

Figure 10 shows an example of the plots of the resonance frequency of the fluid motion f_r versus k_0 in the case that μ is fixed to be the value on the upper line in Figure 9, where the length of the crack L =10, 20 and 30m, 2 a = 10mm and =3%. Figure 10 shows that if we chose the fluid with the appropriate value of the viscosity, we can estimate the length of the crack L from f_r, similar to the case of a three-dimensional penny shaped crack. Furthermore, if is determined as discussed above, then it is possible to estimate the aperture of the crack L from the relationship between the intensity of the resonance of the normalized average pressure P_{NL}, defined by $|P_{av}|/\rho c_0^2 a_0\, \varepsilon/L)$, and k_0. Figure 11 is an example of the relationship, when the fluid's viscosity μ is fixed to be the appropriate value selected by using Figure 9, so as to compatible with k_0. However, there remain problems of how to measure the amplitude of the excitation input and how to measure the averaged pressure in the crack.

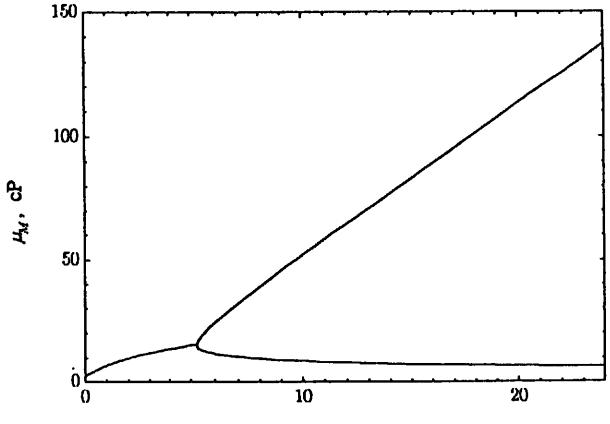

Figure 9. Variation of the optimum viscosity of fluid μ_M with respect to the permeability of the formation k_0 for a two-dimensional crack. The value of the fluid's viscosity in the region between the two lines can be regarded as optimum viscosity.

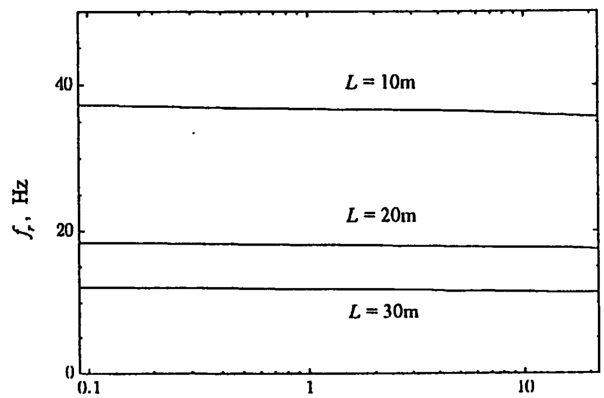

Figure 10. Variation of the resonance frequency f_r with respect to the permeability of the formation k_0 for a two-dimensional crack ($\mu = \mu_M$).

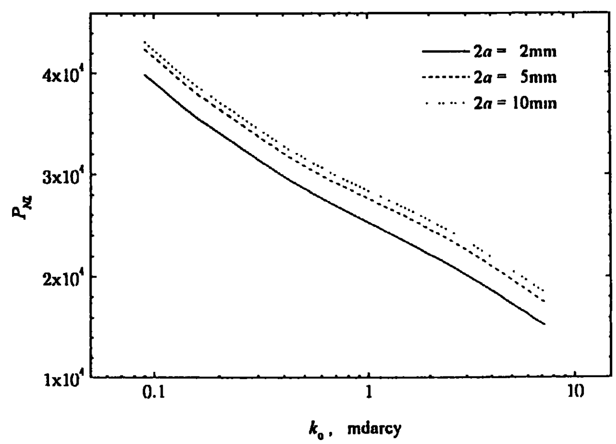

Figure 11. Variation of the maximum of the normalized average pressure P_{NL} ($\equiv |P_{av}|/\rho c_0^2 a_0 \varepsilon/L$)) with respect to the permeability of the formation k_0 for a two-dimensional crack ($\mu = \mu_M$, L =10m, $\varnothing_0$ = 3%).

Conclusions

The dynamic response of the compressible viscous fluid inside a three-dimensional penny shaped crack embedded in a rigid body was examined assuming that the flow was axisymmetric in the penny shaped crack. The case of a two-dimensional crack was discussed for comparison.

In both of the two cases, it was found that there exists an optimum value of viscosity of fluid that maximizes the intensity of the resonance of the fluid pressure. The optimum value of fluid viscosity is almost perfectly determined by the permeability of the formation. This means that we can expect to get the strongest AE/MS signal during hydraulic fracturing by selecting the value of fluid viscosity to be the optimum value which is determined by the permeability of the formation.

Furthermore, when the fluid viscosity is fixed to be optimum, the resonance frequency of the fluid pressure is almost determined by the radius (length) of the crack. So we can estimate the radius (length) of the crack from the resonance frequency of the fluid pressure. And if the radius (length) of the crack is determined, it might be possible to estimate the aperture of the crack by using the relationship between the normalized average pressure and the permeability of the formation.

Acknowledgments

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