6831

Melting from Below in a Binary Eutectic System: Numerical Experiments on Magma Body Formation

*C. M. Oldenburg and F. J. Spera**

It has been estimated that 20 km^3 of basaltic magma is generated in the earth's crust each year, mostly along mid-ocean ridge spreading centers (Crisp, 1984). Associated with melting and partial melting is the accumulation of large volumes of magma into magma bodies. One scenario for melting and the formation of magma bodies involves the heating from below of lower crustal mafic gabbro by hot, refractory, partially molten rock generated in the mantle. We have performed numerical experiments on the melting from below of a silicate solid in the diopside-anorthite system to understand more fully the process of magma body formation. In particular, we are interested in (1) the possibility of forming compositional heterogeneity in the magma body during the time of formation of the body (*ab initio*) and (2) the convective dynamics of this strongly coupled flow system. We present in this report a numerical experiment and a discussion of the strong coupling that arises in this magma-generation problem. Although the primary aim of the study was to understand magma bodies, this work is relevant to current efforts aimed at melting contaminated regions of the ground—a method known as *in situ* vitrification (Jacobs et al., 1992).

MODEL FOR PHASE CHANGE AND CONVECTION

In order to investigate a wide range of magmatic convection problems involving phase change (solidification, melting, or both), we have developed a two-dimensional continuum numerical model for phase change and convection of silicate melt (Oldenburg and Spera, 1991,1992a). This model is an extension of the continuum model developed for phase change in metallurgical systems (Bennon and Incropera, 1987). In the continuum model, one set of equations is valid over the entire domain, regardless of whether the local region is entirely solid, entirely liquid, or a mushy mixture of both. In the experiments discussed here, we use the hybrid model (Oldenburg and Spera, 1992a) for the dependence of the form of the momentum equations on the fraction solid (*fs*) present in mushy regions. The model equations include expressions for conservation of mass, momentum, energy, and species for a binary eutectic system. These equations are solved by a control-volume finite difference method along with supplementary relations that relate enthalpy to temperature and *fs*. The iterative solution method handles the nonlinear couplings involved in phase change and convection. The model, its numerical solution, example calculations, and comparison with laboratory experiments have been presented previously (Oldenburg and Spera, 1990, 1991, 1992a,b).

The model system for the melting experiments is a two-dimensional rectangular domain with bulk composition 80% diopside and 20% anorthite (Di80). This composition models the behavior of mafic gabbro in the lower crust, with generation of basaltic magma upon melting. The phase diagram is shown in Figure 1. The initial temperature (T_0) of the solid body is uniform and 5°C below the eutectic temperature. The bottom of the body (T_b) is held 5°C above the liquidus temperature for Di80. Because our focus is on the bottom-heating scenario, the sidewalls were held insulated for simplicity.

The thermophysical properties of the system are given in Table 1. Because of the practical limits of the computational scheme arising from finite time and space discretizations, the domain is limited to regions of very small size relative to magma bodies. In particular, the domain studied here is 0.20 m in height (L). Nevertheless, the processes of melting and convection and the coupling between these processes are accounted for in the model. We expect the same strong coupling to occur in natural systems despite the different length and time scales.

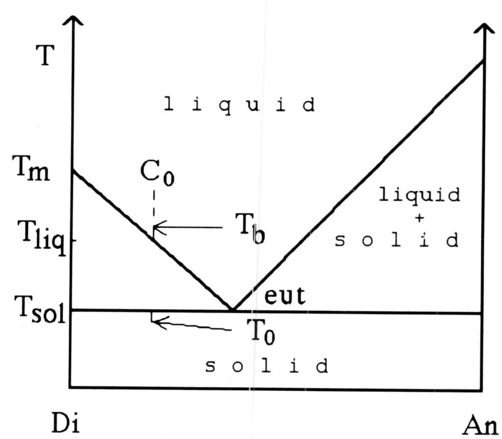

Figure 1. Phase diagram for the pure binary eutectic system diopside-anorthite (Di-An). Initial composition C_0 is Di80. The liquidus temperature (T_{liq}) is shown along with the solidus temperature (T_{sol}), the initial temperature, T_0, and the bottom temperature, T_b. [XBL 936-885]

* Department of Geological Sciences and Institute for Crustal Studies, University of California at Santa Barbara.

Table 1. Thermophysical parameters of Di80 melting problem.

Property	Name	Units	Value
T_{liq}	Liquidus temperature	K	1606
T_{sol}	Solidus temperature	K	1548
T_{bottom}	Temperature at bottom	K	1611
T_{top}	Temperature at top	K	1543
T_0	Initial temperature	K	1543
c_p	Heat capacity	$J\ kg^{-1}\ K^{-1}$	1006
ρ	Density	$kg\ m^{-3}$	2660
κ	Thermal diffusivity	$m^2\ s^{-1}$	10^{-6}
D	Chemical diffusivity	$m^2\ s^{-1}$	10^{-8}
μ	Viscosity	$m\ kg^{-1}\ s^{-1}$	3.6
α	Thermal expansivity	K^{-1}	10^{-5}
β	Compositional expansivity	—	10^{-2}
L	Height of the body	m	.20

RESULTS

In general, the behavior of this silicate binary eutectic system upon melting will involve the initial generation of eutectic-rich melt along the hot bottom boundary. As melting continues along with thermal and chemical diffusion, this zone of melt and mushy two-phase mixture will become thicker and the solid lid will become thinner. When the mushy fluid layer is sufficiently thick, it will become gravitationally unstable and begin to convect because of thermal and compositional buoyancy. Convection will transport heat and mass upward, resulting in more melting of the lid above the plume. The hot eutectic-rich plumes generated at the bottom boundary are a manifestation of the strong couplings between the flow field and the thermal and compositional fields.

Results from a numerical experiment are shown in Figures 2 and 3 as plots of velocity of the mixture and isopleths of the relevant fields. Figure 2 shows the *fs* and mixture velocity fields at four different times ($t = 0.05$, 0.10, 0.15, 0.20), where the time has been scaled by the thermal diffusion time. At $t = 0.05$, the layer of mushy mixture extends to about $Y = 0.35$ and convection has just begun in the lower right-hand corner of the body. Convection is weak in this system, as seen by the slight bowing of isopleths, because of the small height of the system. The region of convecting magma is limited to regions where *fs* is less than about 50%, above which the mixture is effectively locked (Arzi, 1978). Even values of *fs* in the range 0 to 0.5 strongly affect the viscosity and prevent vigorous convection. At $t = 0.10$, the convection becomes more vigorous, as seen by the plume that has formed near $X = 0.25$. The solid lid has melted more, and the original convection cell in the lower right-hand corner remains from earlier times. At $t = 0.15$ and $t = 0.20$, these basic features persist, with the convecting layer getting thicker and the solid lid thinner. The melting process proceeds from the bottom upward. Rather than separating into a liquid and a solid region during heating from below, the body becomes mostly a mushy two-phase mixture.

Shown in Figure 3 are the temperature and dimensionless composition fields for the same experiment at $t = 0.20$. The hot upward plumes and the colder downward plumes are shown by the isotherms. This coupling is the fundamental nonlinearity in pure thermal convection. However, in this binary eutectic system, there is coupling between composition and *fs* fields as well as temperature. In particular, the *fs* is lower in the hot upward plumes, and thus the viscosity is lower and the plume is more mobile than adjacent more *fs*-rich regions (see Figure 2). As for composition, the first melt to form is of eutectic composition, enriched in the light component (anorthite) and therefore less dense due to compositional buoyancy. In Figure 3, we show the normalized mixture composition field, C^*, where $C^* = (C - C_{min})/(C_{max} - C_{min})$ and $C_{max} = 0.23$ and $C_{min} = 0.20$. The C^* field shows a compositional heterogeneity associated with the upward plume near $X = 0.25$. Thus the effects of thermal buoyancy, compositional buoyancy, and *fs* are all contributing to the upward flow. The coupling of the temperature, composition, and *fs* fields leads to this preferential flow phenomena for the present melting scenario with bulk composition Di80.

CONCLUSIONS

The first-order observation is that most of the domain becomes a mushy solid-liquid region. This arises because temperature gradients tend to be rather gentle because of the thermal diffusivity of rock, and the system tends toward intermediate temperatures where both solid and liquid phases are stable. This is in fact what is observed in nature; natural magmas are mixtures of liquid and one or more solid phases. We also observe the *ab initio* formation of compositional heterogeneity from a homogeneous initial condition.

The other important observation is the strong coupling between the velocity, temperature, composition, and *fs* fields and the effect of this coupling on the convective dynamics. In particular, one sees in Figure 3 the strength of the plumes being reinforced by thermal buoyancy, compositional buoyancy, and the small *fs*. It is this sort of nonlinear dynamics that may be responsible for producing much of the natural volcanic activity we observe at the earth's surface. Even in a small and simplified system such as we have modeled, some fundamental nonlinear processes are observed.

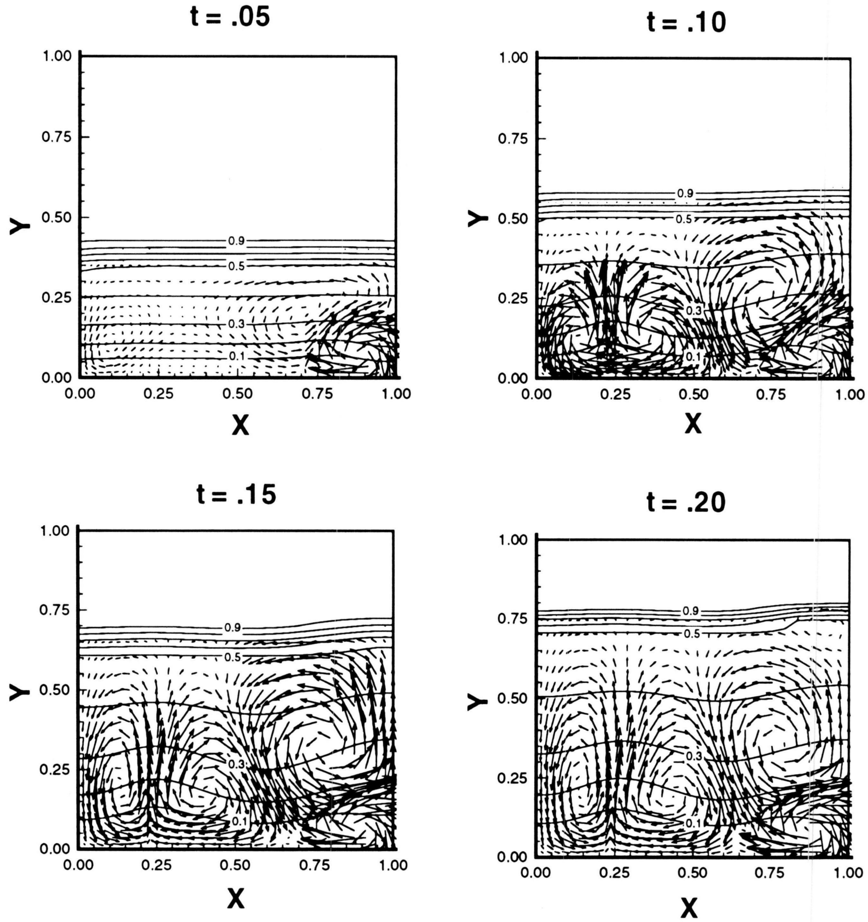

Figure 2. Evolution of the *fs* field. Plot shows vectors of mixture velocity and isopleths of *fs* at $t = 0.05$, 0.10, 0.15, and 0.20. Note the overall decrease in thickness of the solid lid with time and the increase in thickness of the melted mushy layer. Convection becomes more vigorous as the mushy layer thickens. [XBL 936-886]

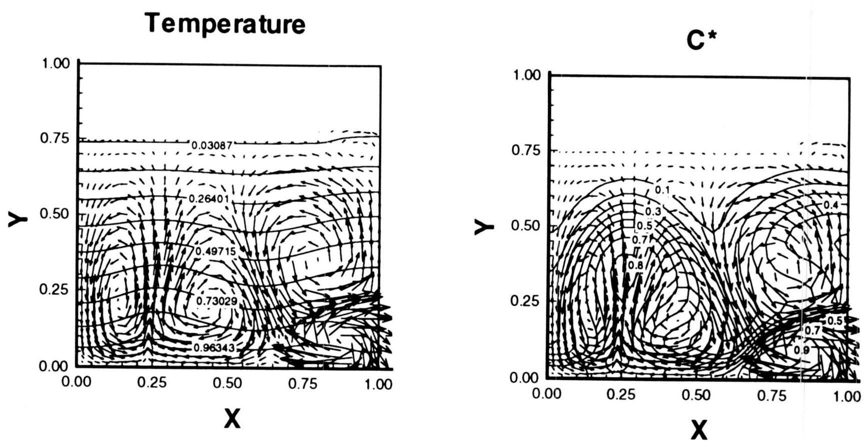

Figure 3. Temperature and normalized composition (C^*) at $t = 0.20$. The isotherms show the plume is hotter than adjacent mushy fluid. The C^* field shows the strong compositional anomaly associated with the plume near $X = 0.25$. The lower *fs* associated with the plume has already been shown in Figure 2. These fields show the strong coupling between velocity, temperature, composition and *fs* fields in this melting problem. [XBL 936-887]

REFERENCES

Arzi, A.A., 1978. Critical phenomena in the rheology of partially melted rocks. Tectonophysics, v. 44, p. 173–184.

Bennon, W.D., and Incropera, F.P., 1987. A continuum model for momentum, heat and species transport in binary solid-liquid phase change systems—I. Model formulation. Int. J. Heat & Mass Transfer, v. 10, p. 2161–2170.

Crisp, J.A., 1984. Rates of magma emplacement and volcanic output. J. Volcanol. & Geotherm. Res., v. 20, p. 177–211.

Jacobs, G.K., Dunbar, N.W., Naney, M.T., and Williams, R.T., 1992. Petrologic and geophysical studies of an artificial magma. Eos: Trans. Am. Geophys. Union, v. 73, p. 401, 411–412.

Oldenburg, C.M., and Spera, F.J., 1990. Simulation of phase change and convection in magma bodies. *In* C. Carrigan and T.Y. Chu (eds.), Heat Transfer in Earth Science Studies (ASME-HTD Vol. 149), p. 35–42.

Oldenburg, C.M., and Spera, F.J., 1991. Numerical modeling of solidification and convection in a viscous pure binary eutectic system. Int. J. Heat & Mass Transfer, v. 34, p. 2107–2121.

Oldenburg, C.M., and Spera, F.J., 1992a. Hybrid model for solidification and convection. Num. Heat Transfer B, v. 21, p. 217–229.

Oldenburg, C.M., and Spera, F.J., 1992b. Modeling transport processes in non-linear systems: The example of solidification and convection. *In* D.A. Yuen (ed.), Chaotic Processes in the Geological Sciences. IMA Volumes in Mathematics and Its Applications, v. 41, p. 205–224.

Effects of Capillarity and Vapor Adsorption in the Depletion of Vapor-Dominated Geothermal Reservoirs

6832

*K. Pruess and M. O'Sullivan**

Vapor-dominated geothermal reservoirs in natural (undisturbed) conditions contain water as both vapor and liquid phases. In response to fluid production, the liquid phase will boil, with heat of vaporization supplied by the reservoir rocks. As reservoir temperatures decline, reservoir pressures will decline also. For depletion of "bulk" liquid, the pressure would decline along the saturated vapor pressure curve, whereas for liquid held by capillary and adsorptive forces inside porous media, "vapor pressure lowering (VPL)" will cause an additional decline.

We have examined experimental data on vapor adsorption and capillary pressures in an effort to identify constitutive relationships that would be applicable to the tight matrix rocks of vapor-dominated systems. Numerical simulations have been performed to evaluate the impact of these effects on the depletion of vapor-dominated reservoirs.

CAPILLARY SUCTION, VAPOR ADSORPTION, AND VAPOR PRESSURE LOWERING

Thermodynamic analysis shows that for pure single-component fluids such as water, coexistence of liquid and vapor phases at any given temperature T is possible only for a certain unique pressure, which is termed the saturated vapor pressure, or saturation pressure, $P_{sat}(T)$. The thermodynamic properties of liquid and vapor, and the conditions under which these phases can coexist, are altered inside porous media. Vapor pressure above a liquid held by capillary or adsorptive forces is reduced in comparison with vapor pressure above the flat surface of a bulk liquid. The reduction is expressed in terms of a vapor pressure lowering factor, or relative vapor pressure, β, defined by

$$\beta = P_v \,/\, P_{sat}(T) \ . \qquad (1)$$

The relationship between β and the capillary pressure, or adsorption-induced suction pressure P_{suc}, is given by the Kelvin equation

$$\beta = \exp\left(\frac{M_{H_2O} P_{suc}}{\rho_l R (T + 273.15)}\right) , \qquad (2)$$

where M_{H_2O} is the molecular weight of water, ρ_l is liquid phase density, R is the universal gas constant, and T is the temperature measured in degrees Celsius. β depends chiefly

* Department of Engineering Science, University of Auckland, Auckland, New Zealand.