A DEPLETION MODEL FOR THE GABBRO ZONE
(Northern Part of Larderello Field)

by

William E. Brigham, Stanford University
Guiseppi Neri, ENEL-Larderello, Italy

Introduction

Task DEA-3,19 of the ENEL/DOE joint agreement is directed toward the development of simple reservoir models which will match past performance data and can be used to predict future production rates and ultimate reserves. To this end, in 1979 the authors studied the pressure and production data available from the Gabbro Zone - a small producing interval north of the main producing area of the Larderello Field. Production began in this field in 1961. A new type of lumped parameter model was developed to match these data. This report summarizes the results of this modeling effort, including projections into the future.

RESERVOIR PRESSURE - PRODUCTION DATA

Four wells provide most of the production from the Gabbro Zone; Wells G-1, G-3, G-6 and G-9. Three additional wells produce minor volumes; Wells G-7, SD-2 and N-155. The detailed monthly production data from these wells were summed and are shown by six month intervals in Table 1. In this table the six month average production rate data are also listed, as well as the average pressures in the Gabbro Zone producing interval.

The reservoir pressure data in the Gabbro Zone deserves special comment. The paper by Celati et al[1] mentions that the original reservoir pressures found at Gabbro indicated a pressure trend toward the main Larderello producing zone. A study was made of the original pressures found in Wells G-1, G-3, G-4, G-6, G-7, G-8, G-9, SD-2, SD-4, SV-9 and N-115. These initial pressures were extrapolated back in time to an equivalent starting date, and it became clear that an original pressure trend existed in the reservoir. The pressure decreased in the south-southeast direction.

TABLE 1

Detailed Production Data, Gabbro Zone

Date	Average Flow Rate ($10^3 T/M_o$)	Cumulative Production (10^6 Tons)	Average Pressure (Kgm/cm^2)
1960	0.0	.0	30.0
1961	41.2	0.49	
1962	81.8	1.48	26.6
1963	100.7	2.08	
	154.2	3.01	
1964	198.2	4.20	
	232.2	5.59	24.85
1965	244.8	7.06	
	243.0	8.52	22.28
1966	247.3	10.00	
	228.5	11.37	21.27
1967	216.7	12.67	
	221.7	14.00	20.03
1968	271.7	15.63	
	261.7	17.20	18.79
1969	263.3	18.78	
	270.0	20.40	18.02
1970	255.0	21.93	
	248.3	23.42	17.45
1971	240.0	24.86	
	238.3	26.29	17.08
1972	231.7	27.68	
	228.3	29.05	16.52
1973	223.3	30.39	
	223.3	31.73	16.07
1974	220.0	33.05	
	225.0	34.40	15.85
1975	213.3	35.68	
	213.3	36.96	15.18
1976	205.0	38.19	
	215.0	39.48	15.28
1977	206.7	40.72	
	218.3	42.03	14.97
1978	197.0	43.21	
	193.0	44.37	

To average the pressure data, it was necessary to take this reservoir pressure trend into account. To this end we mapped the Gabbro Zone initial pressures including well locations and drew initial pressure contours. From this map it appeared that the pressure in Wells G-4, G-9 and SD-2 were very close to the average reservoir pressure, Wells G-1 and G-7 were about 0.6 Kgm/cm^2 above the average, and Well SD-4 was about 3.4 Kgm/cm^2 above the average. Wells G-3 and G-8 were about 1.9 Kgm/cm^2 below the average pressure, and Well 155 was about 3.8 Kgm/cm^2 below the average.

There is a long history of pressure data from Wells G-4, G-8 and SD-4 and the differences in pressure listed above have persisted throughout the producing life of the Gabbro Zone. These three wells supplied most of the pressure data used to determine the average pressures listed in Table 1. In addition there were 8 data points from Well G-7, 3 points from Well G-9, 11 points from Well SD-2 and one data point from most of the remaining wells. Two wells, B-3 and S. Michele, contain a high CO_2 content, thus their indicated pressures are not reliable and were not included in the calculations.

The pressure versus time data from Table 1 are graphed in Fig. 1. Notice that after 1965 the average pressure drops smoothly with production. However, early in the life, from 1960 to 1965, the three data points behave in an erratic manner. It appears there is some error in these data. Also very little data is available during this period. Unfortunately, we could find no logical basis to use to decide which of these three data points are reliable and which are not. As will be seen later, the paucity of data during this early low-production-rate period makes it difficult to decide which of the reservoir model equations we develop later is the most reliable.

RESERVOIR MODELING

The paper by Celati et al[1] clearly shows that the producing interval at Gabbro is brecciated carbonates associated with evaporitic deposits, with a flysh cap rock. Beneath this permeable horizon lies quartzites and phyllites which are somewhat fractured. This sequence of rock types is similar to the geology seen in the main part of the Larderello field.

The volume of steam that has been produced from Gabbro is far greater than could exist in the reservoir as steam alone, thus there must be boiling

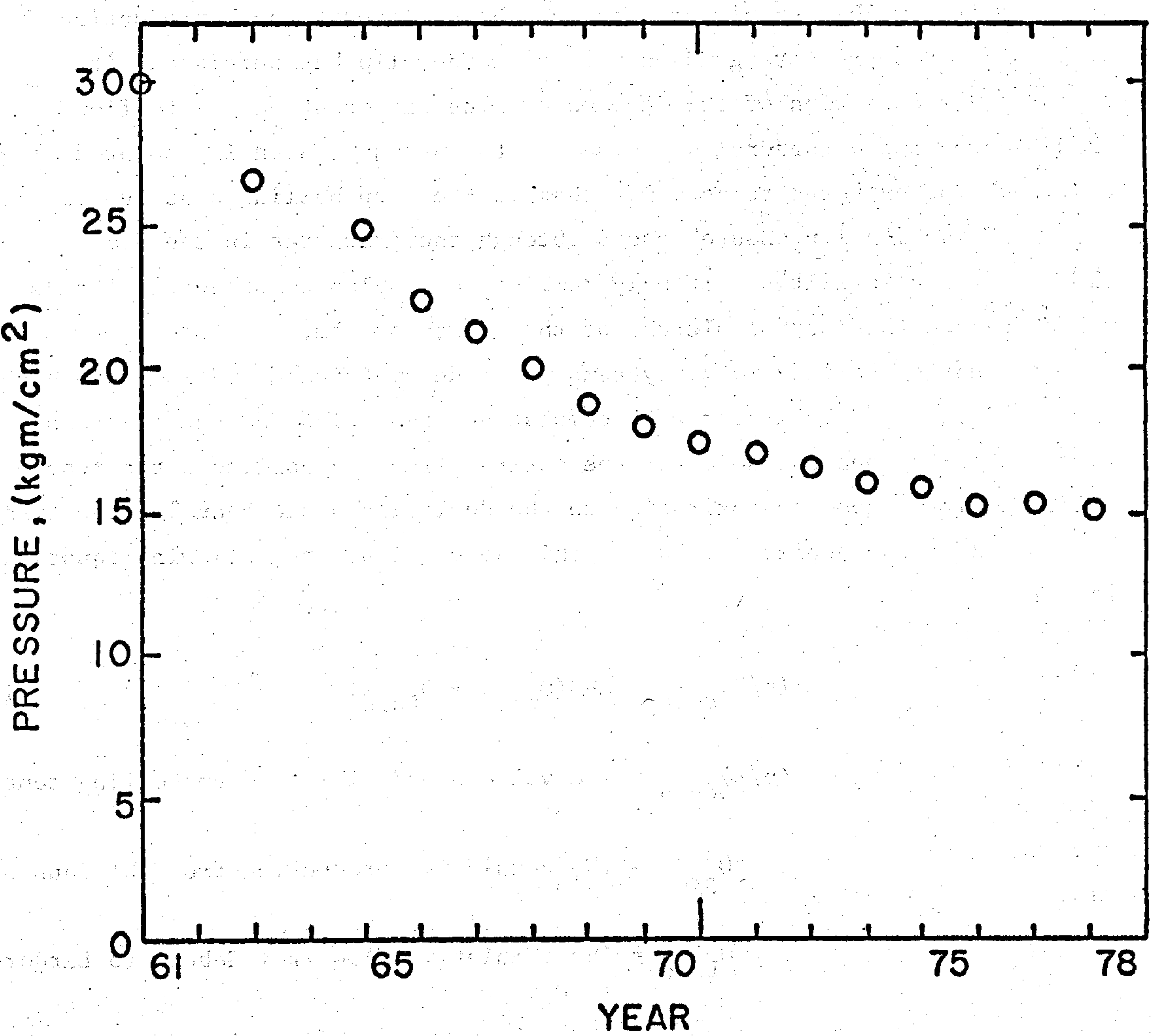

Figure 1. Gabbro zone average reservoir pressure (year end).

water supplying most of the steam to the producing interval. It seems reasonable to assume this water lies somewhere deep within the quartzites and phyllites. This supply of water can be depleted through production. Also, since the pressure gradient toward Larderello has persisted, it appears that depletion of the deep water zone has occurred due to flow from Gabbro toward Larderello. Finally, the pressure seen in the producing zone would be expected to be lower than in the deep boiling zone due to vertical frictional pressure losses through the fractures in the deep quartzites and phyllites. It only remains to develop a mathematical model which matches this verbal picture of the Gabbro system.

A study of boiling water systems was made by Brigham and Morrow[2] which indicated that p/Z is linear with cumulative production for such systems. This linearity does not hold for the entire life of a boiling water reservoir, but it is good approximation to the depletion history during the first 1/3 to 1/2 of the depletion. Using this assumption, the following equation is valid

$$(p/Z)_{deep} = A - B(Q_{prod} + Q_{Lard}) \qquad (1)$$

where

$(p/Z)_{deep}$ = The value of p/Z in the deep boiling zone

Q_{prod} = The cumulative production from the Gabbro Zone

Q_{Lard} = The cumulative flow from Gabbro to Larderello

A and B = Unknown constants, A is the initial p/Z, B is inversely proportional to the volume of the boiling water zone

The flow to Larderello can be treated as steady state flow, as follows:

$$Q_{Lard} = C' \int (p_{Gabbro} - p_{Lard})\, dt \qquad (2)$$

where

p_{Gabbro} = The pressure in the Gabbro zone

p_{Lard} = The pressure in Larderello

C' = An unknown constant which is inversely proportional to the permeability of the flow path between Gabbro and Larderello

Further, since the pressure data are all available in one year intervals, the integral in Equation 2 can be shown as a summation,

$$Q_{Lard} = C'\Sigma(p_{Gabbro} - p_{Lard}) \quad (2a)$$

A study of the main Larderello producing zone shows that its pressure has remained nearly constant at 8 Kgm/cm^2 for the past 15 years. Using this concept, Equation 2a can be combined with Equation 1 to arrive at the following equation:

$$(p/Z)_{deep} = A - B\ Q_{prod} - C\Sigma(p-8) \quad (3)$$

Equation 3 is for the p/Z in the deep boiling horizon. The pressure seen in the producing zone is less than in the deep zone due to frictional pressure losses. This can be expressed as follows:

$$(p/Z)_{top} = (p/Z)_{deep} - \Delta(p/Z)_{flow} \quad (4)$$

where $(p/Z)_{top}$ = the value of p/Z seen in the producing zone. The values associated with Figure 1

$\Delta(p/Z)_{flow}$ = the drop in p/Z due to vertical flow through a deep fractured zone

It only remains to develop an analytic expression for the drop in p/Z due to flow of steam through the fractured zone.

Pressure Drop Due to Linear Flow

To derive an equation for the pressure drop from the deep boiling zone through the fractured zone to the producing horizon, we can envision that the flow geometry is approximately linear. This is transient flow; thus the magnitude of the pressure drop will depend on the terms in the p_d function for linear flow, and the timing of the pressure transient will depend on the terms in the t_d function. Analytic solutions for such problems have been published by Miller[3] and by Nabor and Barham.[4] Nabor and Barham's solutions are summarized in Fig. 2, where their term $F(t_d)$ is the p_d function for linear flow at a constant rate.

The three functions shown in Fig. 2 depend on the outer boundary condition; that is, the boundary condition at the boiling water interface. The $F_1(t_d)$ curve is for a system with a closed outer boundary, the $F_{1/2}(t_d)$ curve is for an infinite system, while the $F_o(t_d)$ curve is for a system with a constant pressure outer boundary. The system that most closely approximates Gabbro is the constant pressure boundary, the $F_o(t_d)$ curve. This is marked more heavily in Fig. 2.

A boiling water system is not an exact constant pressure boundary, for as the system depletes the pressure must decline. However, it is a good approximation to treat this system as constant pressure when the pressure in the boiling zone declines slowly with time - as it must for any reservoir with a reasonably long producing life.

Whenever the flow rate varies, it is necessary to use superposition to calculate the transient pressure drop. If equal time increments are used, the superposition equation is,

$$\frac{kA\Delta p}{\mu L} = q_1 p_d(t_d) + (q_2-q_1)p_d(t_d-\Delta t_d) + (q_3-q_2)p_d(t_d-2\Delta t_d) + (q_4-q_3)p_d(t_d-3\Delta t_d) + (q_{n+1}-q_n)p_d(t_d - n\Delta t_d) \quad (5)$$

Let us study the $F_o(t_d)$ curve of Fig. 2 in detail. A good approximation to this curve is to assume that p_d is proportional to the square root of time until $t_d = 0.785$ and to assume it is a constant equal to 1.00 after $t_d = 0.785$. The maximum error using the approximation is only about 10%.

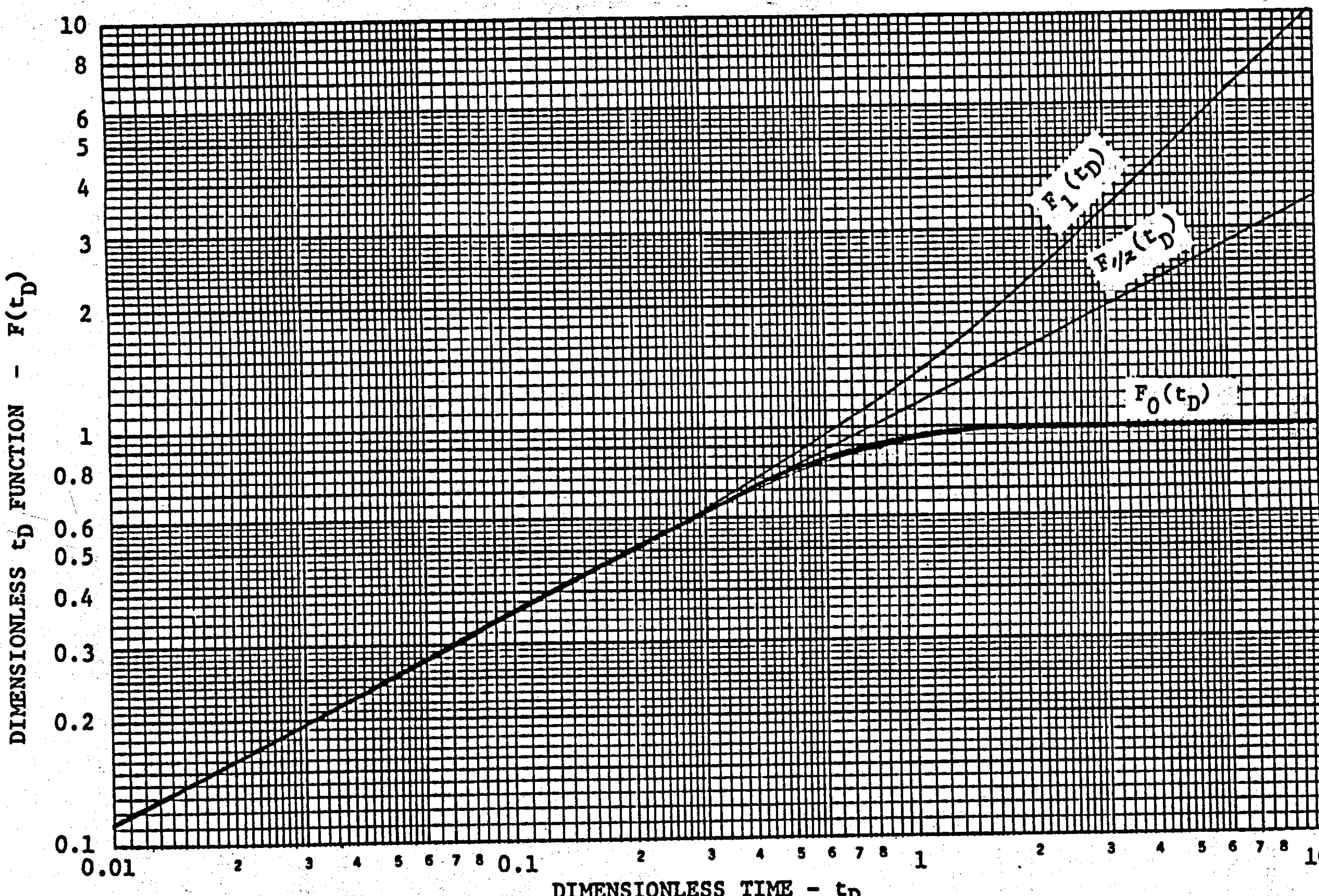

Figure 2. Dimensionless pressure change and efflux functions - linear aquifers.

Ref: Nabor and Barham
Trans AIME (1964) 231, 561

In most real systems we do not know the parameters in t_d well enough to be able to relate real time to t_d; however, we can assume a value for the real time that is equivalent to $t_d = 0.785$ and observe how this affects Eq. 5. Hence forth in this paper we will refer to this time as the "lag time." This phrase was chosen for it is meant to imply the time required to reach effective steady state flow. It is somewhat similar to the common concept of "readjustment time" that is seen in the well test literature; but it is not quite the same, for the readjustment time occurs at $t_d \simeq 1.7$ to 2.0 in Figure 2 where $F_o(t_d)$ is more nearly equal to 1.00.

To determine the effect of the lag time and the above approximations on the superposition equation, let us assume there has been production for six time periods (6Δt) and let us also assume that the lag time is four time periods (4Δt). Using the approximations to the $F_o(t_d)$ curve, Equation 5 becomes:

$$\frac{kA\Delta p}{\mu L} = q_1(1) + (q_2-q_1)(1) + (q_3-q_2)(1) + (q_4-q_3)\sqrt{\frac{3\Delta t}{4\Delta t}} + (q_5-q_4)\sqrt{\frac{2\Delta t}{4\Delta t}} + (q_6-q_5)\sqrt{\frac{\Delta t}{4\Delta t}} \quad (6)$$

Notice the left hand side of Eq. 6 is equal to the flow rate if we had linear steady state flow; thus we can call this term the equivalent steady state flow rate, $q_{eq.}$. The right hand side contains a number of terms that cancel each other, so the equation can be simplified as follows,

$$q_{eq} = q_3\left(1-\sqrt{\frac{3}{4}}\right) + q_4\left(\sqrt{\frac{3}{4}}-\sqrt{\frac{2}{4}}\right) + q_5\left(\sqrt{\frac{2}{4}}-\sqrt{\frac{1}{4}}\right) + q_6\sqrt{\frac{1}{4}} \quad (7)$$

Equation 7 tells us that any flow that occurred at a time further in the past than the lag time has no effect on the pressure drop that is occurring today. Further, Eq. 7 gives us a basis for a general formulation for the equivalent flow rate as a function of the lag time. For example suppose that the lag time is equal to five time periods. The equation for the equivalent steady date flow rate will be,

$$q_{eq} = q_{n-4}\left(1-\sqrt{4/5}\right) + q_{n-3}\left(\sqrt{4/5}-\sqrt{3/5}\right) + q_{n-2}\left(\sqrt{3/5}-\sqrt{2/5}\right)$$
$$+ q_{n-1}\left(\sqrt{2/5}-\sqrt{1/5}\right) + q_n\sqrt{1/5} \qquad (8)$$

The flow rate data from Gabbro were averaged over six month time periods, thus it was logical to use multiples of six months for the unknown lag times. These flow rates are shown in Table II and also the equivalent steady state flow rates are listed for lag times ranging up to 48 months. We also calculated equivalent flow rates for 54 and 60 month lag times, but these are not included in the table for they were found to give unrealistic results, as will be discussed later.

Relating Δ(p/Z) to Flow Rate

Equations 7 and 8 relate the equivalent steady state flow rate to the actual rates; however, Equations 5 and 6 were written for liquid flow rather than steam flow. The correct equation for the equivalent steady state linear flow of steam is

$$q_{eq} = D'' \int_{p_1}^{p_2} \frac{2p\,dp}{\mu Z} = D''\,\Delta m(p) \qquad (9)$$

where

$$m(p) = \int_{p_o}^{p} \frac{2p\,dp}{\mu Z}$$

D'' = an unknown constant inversely proportional to the permeability in the deep fractured zone

TABLE II

Equivalent Flow Rates at Various
Lag Times (10^3T/Mo)

Date	lagtime 0 Mo	lagtime 12 Mo	lagtime 18 Mo	lagtime 24 Mo	lagtime 30 Mo	lagtime 36 Mo	lagtime 42 Mo	lagtime 48 Mo
61	41.2							
	41.2							
62	81.8							
	81.8							
63	100.7							
	154.2							
64	198.2							
	232.2							
65	244.8							
	243.0	243.5	241.4	235.7	227.0	216.0	206.1	198.0
66	247.3							
	228.5	234.0	235.7	236.9	236.4	233.1	227.2	219.0
67	216.7							
	221.7	220.2	221.8	225.2	227.1	228.6	228.9	226.9
68	271.7							
	261.7	264.2	256.7	251.4	249.0	248.8	248.4	248.2
69	263.3							
	270.0	268.0	266.9	267.5	262.7	258.7	256.4	255.8
70	255.0							
	248.3	250.3	253.9	255.1	255.8	257.2	254.6	252.1
71	240.0							
	238.3	238.3	240.5	242.5	245.4	246.9	248.0	249.6
72	231.7							
	228.3	229.3	231.0	232.2	233.9	235.7	238.3	239.9
73	223.3							
	223.3	223.3	224.2	225.2	226.6	227.8	229.3	231.0
74	220.0							
	225.0	223.5	223.5	223.5	224.0	224.6	225.7	226.6
75	213.3							
	213.3	213.3	215.5	216.1	216.8	217.4	218.2	219.1
76	205.0							
	215.0	212.1	212.3	212.4	213.8	214.3	215.0	215.5
77	206.7							
	218.3	214.9	214.9	213.6	213.6	213.5	214.4	214.8

Atkinson and Mannon[5] have shown that m(p) is almost exactly proportional to p^2 for steam reservoirs. Thus Eq. 9 can be simplified to,

$$q_{eq} = D''\Delta(p^2) \tag{10}$$

Note that Eq. 10 relates flow rate to $\Delta(p^2)$, while Eq. 4 requires that the flow rate be related to $\Delta(p/Z)$. There is no theoretical basis whereby these terms can be related, however an empirical power law equation was attempted which is of the following form

$$\Delta(p/Z)_{flow} = D' \frac{\left[\Delta(p^2)\right]^n}{(p/Z)^m_{top}} \tag{11}$$

The applicable pressure and Z factor data are listed in Table 3. To test this empirical equation, pressures ranging from 12 Kgm/cm^2 to 30 Kgm/cm^2 were assumed, with $\Delta(p/Z)$ values ranging from 2.71 to 11.60. These pressure and $\Delta(p/Z)$ values were chosen to cover the range of data seen in the Gabbro history and likely to be seen in the next 15-20 years. These data were fit to Eq. 11 using a lest-square formulation, and the maxiumum error was 2.5%. Most of the errors were less than 1.0%. The values found for the constants were: n = 0.911 and m = 0.519.

It is now possible to write a single equation which combines the concepts discussed above. We can combine Equations 3, 4, 10 and 11 to get,

$$(p/Z)_{top} = A - BQ_{prod} - C\,\Sigma(p-8) - \frac{D(q_{eq})^{0.911}}{\left[(p/Z)_{top}\right]^{0.519}} \tag{12}$$

This is the equation used to match the history of the Gabbro zone and also to extrapolate the producing rate to the future.

Some comments on Eq. 12 seem appropriate at this time. To the best of our knowledge, this is the first time such a depletion material balance model has been used in a geothermal reservoir such that transient flow within the reservoir is coupled with a lumped parameter material balance. In oil reservoir with water influx, a somewhat similar concept has occasionally been used[6] but with a different equation form. Also, as far as we know, this is the first time anyone has shown that $\Delta(p/Z)$ can be simply and accurately related to $\Delta m(p)$. The beauty of Eq. 12 is that it can

TABLE III

Values of Pressure and Gas Deviation Factor, Z, for the Gabbro Zone
(T = ^{o}C)

p, Kgm/cm^2	Z	p/Z, Kgm/cm^2
10.0	0.96150	10.40
12.0	0.95360	12.58
14.0	0.94560	14.81
16.0	0.93690	17.08
18.0	0.92800	19.40
20.0	0.91930	21.76
22.0	0.90974	24.18
24.0	0.90002	26.67
26.0	0.88996	29.21
28.0	0.87935	31.84
30.0	0.86836	34.55

be solved using standard least-squares equations and a programmable calculator, or alternatively a standard regression analysis package can be used on a digital computer.

HISTORY MATCHING

As mentioned earlier there was some doubt about the validity of the first three pressure data points in the pressure/production history. For history matching we assumed the first data point at the end of 1960 was valid, and ignored the two data points at the end of 1962 and 1964. All other data were included in the history matches.

Since we did not know what lag times would be valid for this reservoir, seven lag times were assumed ranging from 18 months to 54 months. The 18 month and 54 month fits of Eq. 12 to the data were quite close; however, the least squares constants were not positive as they must be to be physically realistic. Thus these results are not reported herein. The results from the other five lag time assumptions are shown in Tables IV and V.

In Table IV are listed the pressures calculated from the least-square equation fits for all five lag times. At the bottom of each column is shown the standard deviation of each of the equations. It is clear from this table that any one of the time lag assumptions fits the data quite closely. The standard deviations range from 0.37 to 0.21 Kgm/cm^2. There is a trend toward a lower standard deviation as a longer lag time is assumed, but the reader is cautioned not to place too much emphasis on this trend. When a lag time of 54 months was assumed, the standard deviation was even smaller; but, as mentioned before, one of the constants in the resulting equation was of the wrong sign which is physically unrealistic.

TABLE IV

Calculated Values for p/Z in Gabbro
Compared to Measured Values
(Various Assumed Lag Times)

Date	Measured p/Z Kgm/cm^2	Calculated p/Z for various lag times				
		t_{lag} 24 Mo	t_{lag} 30 Mo	t_{lag} 36 Mo	t_{lag} 42 Mo	t_{lag} 48 Mo
1960	34.55	34.67	34.65	34.62	34.58	34.57
1965	24.55	23.95	24.08	24.29	24.46	24.53
1966	23.40	22.86	22.86	22.90	22.97	23.06
1967	21.76	22.07	22.11	22.08	22.00	21.93
1968	20.40	20.61	20.61	20.55	20.48	20.42
1969	19.40	19.44	19.41	19.41	19.39	19.38
1970	18.80	18.91	18.79	18.66	18.67	18.74
1971	18.40	18.45	18.35	18.25	18.18	18.13
1972	17.70	17.96	17.92	17.86	17.77	17.73
1973	17.20	17.45	17.44	17.43	17.40	17.35
1974	16.95	16.88	16.91	16.92	16.92	16.92
1975	16.20	16.40	16.43	16.46	16.48	16.49
1976	16.30	15.98	16.02	16.07	16.13	16.15
1977	15.90	15.39	15.44	15.51	15.57	15.63
Standard deviation Σ, kgm/cm^2		0.37	0.34	0.29	0.24	0.21

TABLE V

Best Fit Equations to Gabbro Data
Using Various Lag Times

General Equation Form

$$(p/Z)_{top} = A - BQ_{prod} - C\,\Sigma(p-8) - \frac{D(q_{eq})^{0.911}}{\left[(p/Z)_{top}\right]^{0.519}}$$

lag time (Months)	Constants in Best Fit Equations A	B	C	D
24	36.174	0.01027	0.06823	0.16866
30	35.772	0.05872	0.05098	0.21025
36	35.484	0.08818	0.03949	0.23994
42	35.398	0.08888	0.03697	0.25109
48	35.565	0.05825	0.04535	0.23662

Probably the only valid conclusion that can be reached from these results is that any of the five equations and lag times fits the data quite satisfactorily, and we have no valid method of picking one over another.

Table V shows the equations that were generated from the least-squares fits. A study of these equations constants shows that a broad range of reservoir parameters could be used to fit the Gabbro data. For example compare the constants for the 24 month lag time equation with the constants for the 42 month equation. The 24 month equation has the highest initial pressure (A = 36.174), the largest liquid reservoir volume (B = 0.01027), the greatest pressure loss due to flow toward Larderello (C = 0.06823) and the smallest frictional pressure drop (D = 0.16866). For the 42 month equation all these constants are at the opposite extreme; the lowest initial pressure (35.398), the smallest liquid reservoir volume (B = 0.08888), the smallest loss due to flow toward Larderello (C = 0.03697) and the greatest frictional pressure drop (D = 0.25109). These differences are quite significant; for example, the relative reservoir volumes between these two cases are 8.65/1, and the relative flow towards Larderello for these two cases are 16/1. If the results for the 24 month lag time are not considered we find that the ranges of parameters narrows considerably. The maximum range of reservoir size is from the 42 month to the 48 month equation and it is 1.53/1; the maximum range of loss to Larderello is from the 30 month to the 42 month equation, and it is 2.09/1.

In brief, these results tell us that we can match the past data very well indeed using Eq. 12; however, because a number of lag times match the data almost equally well, we do not have an accurate picture of the size of the system, the amount of loss toward Larderello nor the frictional pressure drop.

Fortunately this problem may not be as serious as we might first expect. Often when we achieve a good history match, we find that the projections into the future are realistic even if the specific parameters used to make that match are not. We will test this hypothesis in the next section.

FUTURE PRODUCING RATE

Before projecting the producing rate into the future it was necessary to predict the reservoir flow rate as a function of the average reservoir pressure. In general for gas flow from a reservoir it is possible to calculate the flow rate based on a version of the Forchheimer equation, as follows

$$\bar{p}^2 - p_{tf}^2 = aq + bq^2 \tag{13}$$

where $\bar{p}$ = the average producing zone pressure, Kgm/cm^2

p_{tf} = the flowing wellhead pressure, Kgm/cm^2

q = the producing rate, 10^3T/Mo

a & b = unknown constants

Equation 13 is commonly used for gas flow. The constant, a, expresses the Darcy resistance to flow in the reservoir. The constant, b, expresses the sum of non Darcy effect in the reservoir plus the flowing friction within the well from the bottom to the wellhead.

The historical production rate data at Gabbro were tested against this equation. Individual well data were studied as well as overall reservoir flow data. We found that the past reservoir rate could be matched with a maximum error of 13% and a standard deviation of 5.7%. The best fit equation was,

$$\bar{p}^2 - p_{tf}^2 = 0.00436q^2 \tag{14}$$

The Darcy term in Eq. 13, the constant, a, was found to be negligible.

The producing rates and pressures were projected in the future assuming the producing wellhead pressure would remain constant at 6 Kgm/cm^2 absolute. This projection requires a trial and error calculation each year, for the flow rate and the reservoir pressure are interdependent in Equations 12 and 14. We always found rapid convergence to the answers in 2 to 4 interactions.

The flow rates were projected for 18 years through the year 1995 using all five lag time equations. These projections are shown in Table VI where both production rate and cumulative production projections are shown.

A study of this table shows that all the projections are quite similar even though the parameters in the equations differed markedly. The greatest differences are between the 24 month and the 30 month projections. But even they differ by only 6% in flow rate by 1995, and only differ by 3% in cumulative production at that time.

There is a sound reason to consider the 24 month lag time projections invalid, for the actual production rate from Gabbro at the end of 1978 was 193×10^3T/Mo while the 24 month equation projection was 210.9×10^3T/Mo. Thus all the later projected rates may be too high for this case.

TABLE VI

Projections of Gabbro Production to the Future
Various Lag Times

	$t_{lag} = 24$ END YR		$t_{lag} = 30$ END YR		$t_{lag} = 36$ END YR		$t_{lag} = 42$ END YR		$t_{lag} = 48$ END YR	
Year	q	Q	q	Q	q	Q	q	Q	q	Q
1978	210.9	44.52	195.7	44.43	196.9	44.44	198.2	44.44	199.2	44.45
1979	204.1	47.01	191.1	46.75	192.0	46.77	193.0	46.79	194.1	46.81
1980	198.2	49.42	186.8	49.02	187.6	49.05	188.4	49.08	189.4	49.11
1981	193.2	51.77	181.7	51.23	182.7	51.27	184.3	51.32	185.3	51.36
1982	188.2	54.06	176.9	53.38	178.0	53.43	179.7	53.50	180.9	53.56
1983	183.4	56.29	172.4	55.48	173.4	55.54	175.2	55.63	176.6	55.70
1984	178.9	58.46	168.1	57.52	169.2	57.60	171.0	57.71	172.6	57.80
1985	174.5	60.58	164.0	59.51	165.0	59.60	166.9	59.73	168.7	59.84
1986	170.2	62.65	160.0	61.45	161.0	61.56	163.0	61.71	164.9	61.85
1987	166.2	64.67	156.2	63.35	157.1	63.47	159.3	63.65	161.3	63.80
1988	162.3	66.64	152.5	65.20	153.4	65.33	155.6	65.54	157.9	65.72
1989	158.6	68.57	149.0	67.01	149.9	67.15	152.1	67.38	154.5	67.59
1990	155.0	70.45	145.7	68.78	146.5	68.93	148.7	69.19	151.3	69.43
1991	151.5	72.29	142.5	70.51	143.1	70.67	145.5	70.95	148.2	71.22
1992	148.2	74.08	139.4	72.20	140.0	72.36	142.3	72.68	145.2	72.98
1993	145.0	75.84	136.4	73.86	136.9	74.03	139.3	74.37	142.4	74.71
1994	141.9	77.57	133.6	75.48	134.0	75.65	136.4	76.02	139.6	76.40
1995	139.0	79.25	130.8	77.06	131.1	77.24	133.5	77.64	136.9	78.06

If only the remaining four projections are considered, the lowest is for a lag time of 30 months and the highest is for a lag time of 48 months. In 1995 these differ by only 5% in producing rate and only by 1.3% in cumulative production. Thus our earlier speculation, that the various equations will give similar projections into the future, appears valid. Thus there is oonsiderable reason to assume these projections will be realistic.

The history matches and projections to the year 1995 are shown for three cases in Figures 3, 4 and 5. Figure 3 is for a lag time of 30 months. This is the case which predicts the greatest decline during this period. Figure 5 is for a lag time of 48 months; it predicts the least decline. Figure 4 is for a lag time of 42 months. This is the case which predicts the greatest depletion due to production, the smallest depletion due to flow toward Larderello and the greatest drop in pressure due to transient vertical flow. In spite of the large differences in the parameters used in these equations, the future projections from all three are very similar.

These figures also indicate the times when the four major wells started to produce. Note in particular, that when Well G-9 started producing in 1967, the average reservoir pressure declined more rapidly for a period of time and gradually tended to level off. This can be seen both in the reservoir data and in the equation curve fits. This behavior is a direct result of the transient linear flow assumed in the model.

It would have been possible to extrapolate these production rates further into the future using Equations 12 and 14. Instead we chose to study the possibility of using decline curves for extrapolation. The Fetkovich[7] decline curves were tried, but we met with only limited success. The problem was that we found it difficult to differentiate between the various hyperbolic decline constants due to the scatter in the data at early time.

Another approach was tried, which, as far as we know, has not been used before. We will discuss this new method in the following paragraphs. The general decline equation has the following form

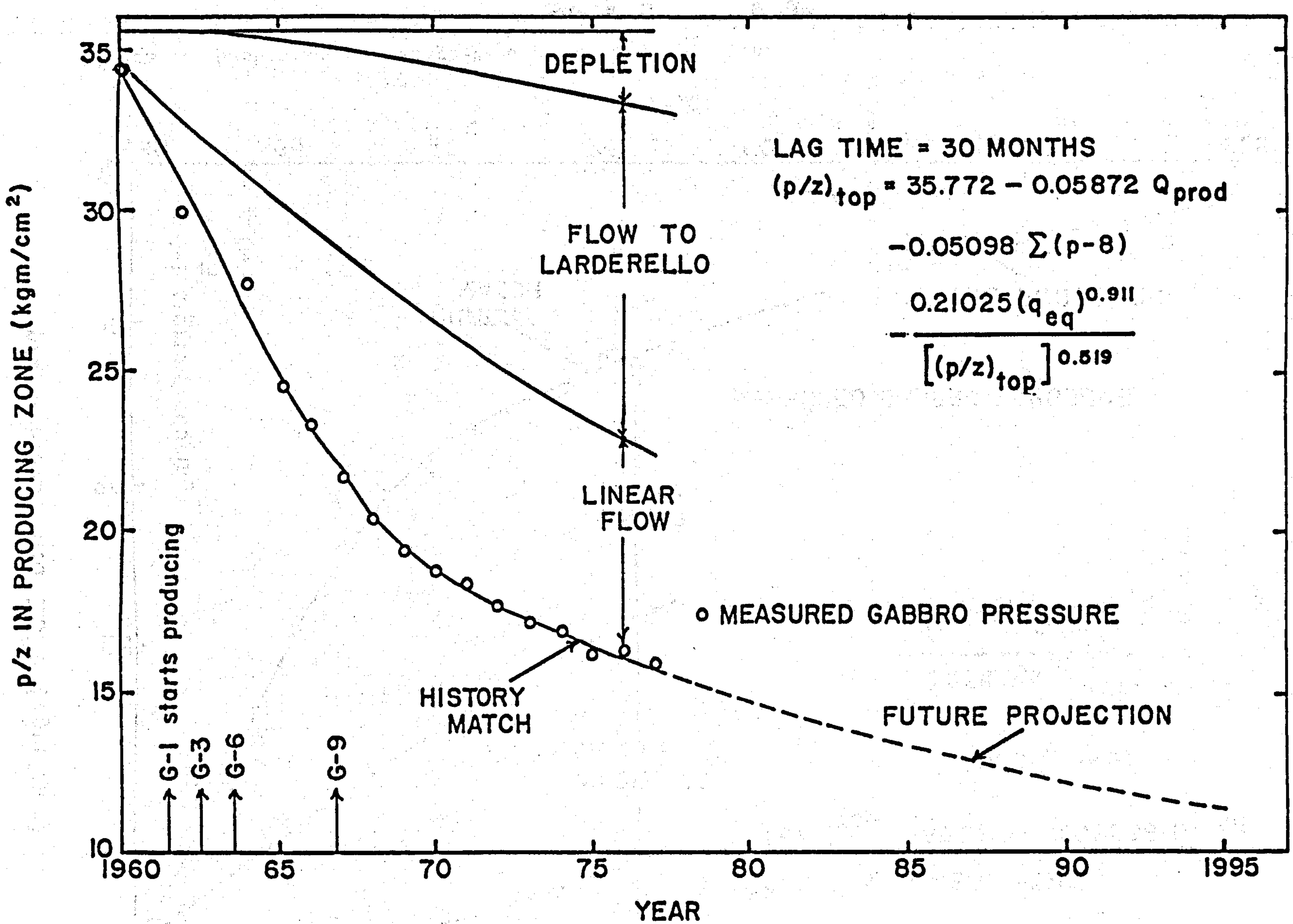

Figure 3. Gabbro zone pressure-production history match.

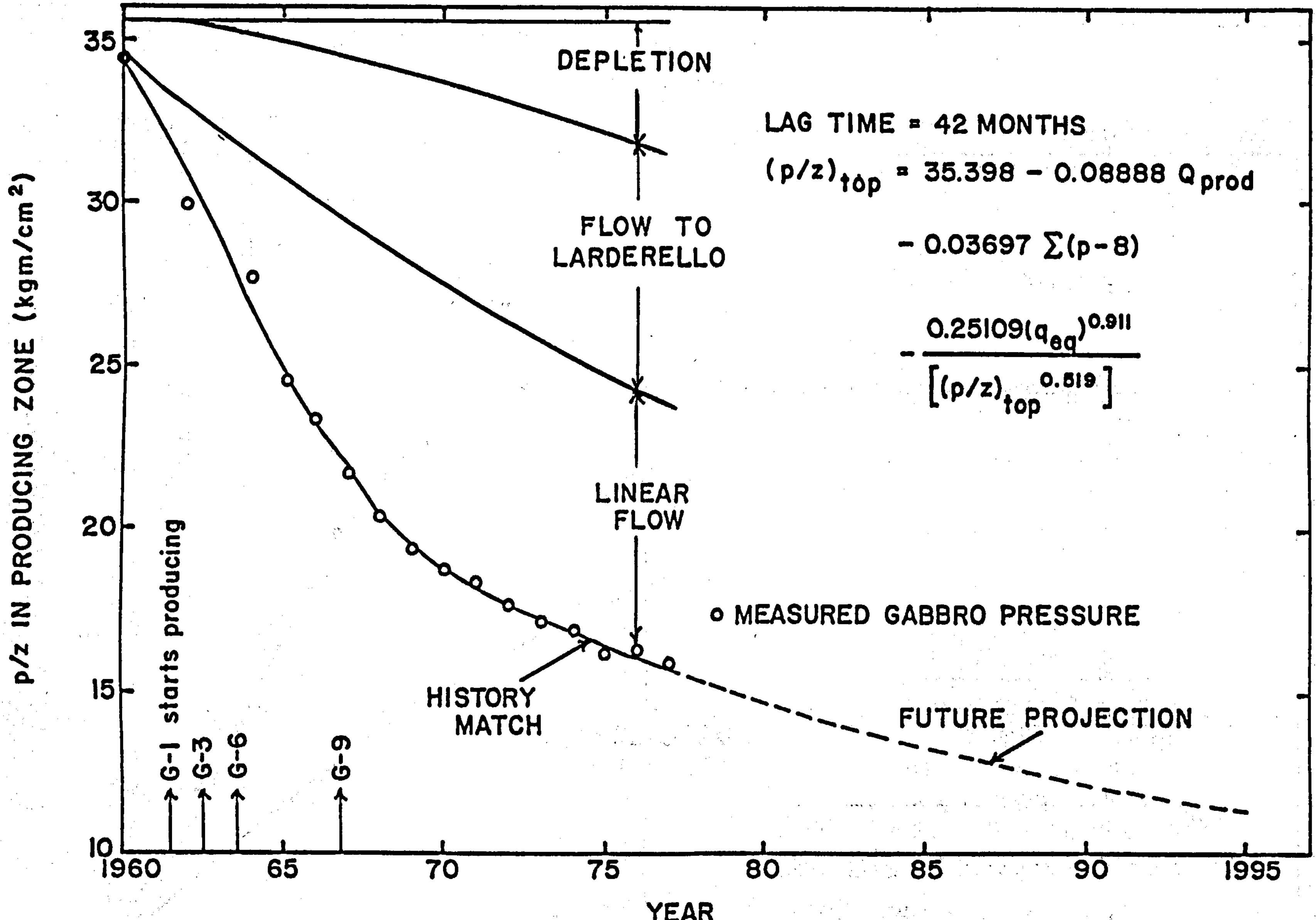

Figure 4. Gabbro zone pressure-production history match.

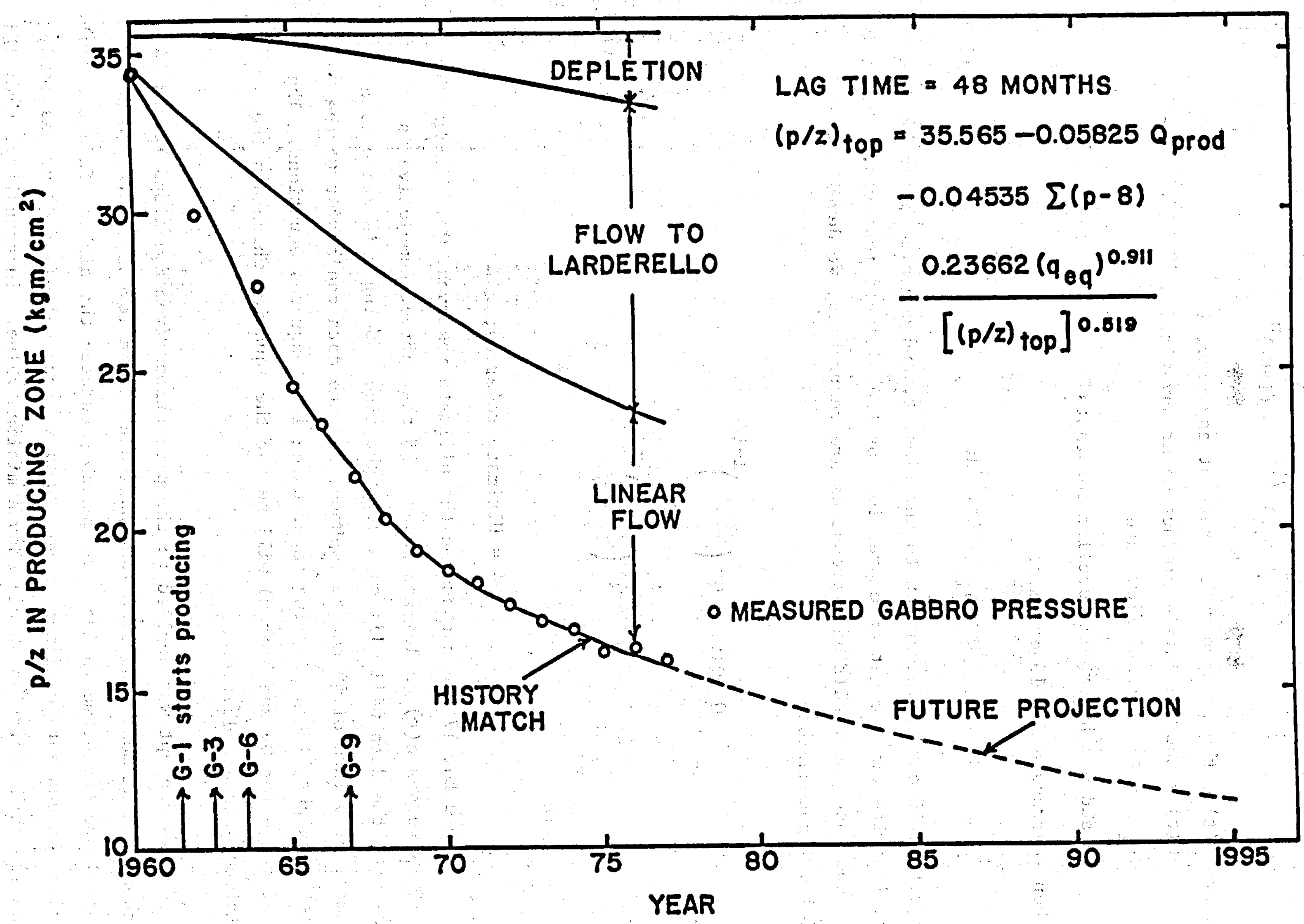

Figure 5. Gabbro zone pressure-production history match.

$$q(t) = \frac{dQ(t)}{dt} = \frac{q_i}{(1+bD_i t)^{1/b}} = \frac{q_i}{(1+Ct)^n} \tag{15}$$

where $q(t)$ = the reservoir flow rate at any time, t

$Q(t)$ = the cumulative production at any time, t

q_i = the initial flow rate

c and n = hyperbolic decline constants

when rearranged Eq. 15 becomes,

$$dQ = q_i \ (1+Ct)^{-n} dt \tag{16}$$

Equation 16 can be integrated to any time, t, and can also be integrated to infinite time to get an expression for the ultimate reserves. When this is done and the resulting expressions are rearranged, we develop the following equation,

$$Q^*-Q(t) = Q^* \left(\frac{q(t)}{q_i}\right)^{1-1/n}$$

$$= Q^* \left(\frac{q(t)}{q_i}\right)^{1-b} \tag{17}$$

where

Q^* = Ultimate reserves, 10^6 Tons

To use Eq. 17 one can assume the ultimate reserves, Q*, then graph Q*-Q(t) versus q(t) on log-log paper. If the correct cumulative is assumed, the data will lie on a straight line. Using this concept, a number of values of Q* can be assumed, and the data can be fit to the best straight line on log-log paper using least-squares equations. The best value for Q* and b(or n) will be for the equation which has the minimum standard deviation.

This concept was applied to the projections from Equations 12 and 14 assuming a 30 month lag time (which predicted the lowest future producing rates) and the 48 month lag time (which predicted the highest future rates). Using the concepts outlined above, when various values of Q* are assumed, the standard deviation should show a minimum value. In actuality, this did not occur; the greater the value of Q* assumed, the smaller was the standard deviation. When a detailed study was made of the data it became clear that this strange behavior was caused by the scatter in the early producing rate data. Figure 6 shows a graph of the producing rate versus cumulative production data which shows this scatter. It is also clear from this figure that the projections into the future, using Equations 12 and 14, decline quite smoothly.

A compromise technique was developed. Best least-squares fits of the data and projections were calculated using Equation 17, and then the standard deviations were calculated based only on the projections into the future. When this technique was used, a range of minimum error was found, but this minimum was very shallow, thus the answer was not exact. For example, the optimum result using the 30 month lag time equation gave an ultimate reserves value, Q*, ranging from 250 to 300 million tons. The optimum using the 48 month lag time equation gave a value of Q* ranging from 350 to 400 million tons.

Such an inexact result is reasonable when one looks in detail at the future projections of producing rate. All the equations predict a very long producing history with a continuing gradual decline in rate. The exact nature of that decline cannot be forecast accurately. It is interesting that this projected behavior is quite similar to that seen in Larderello, which is in a much later stage of depletion than Gabbro.

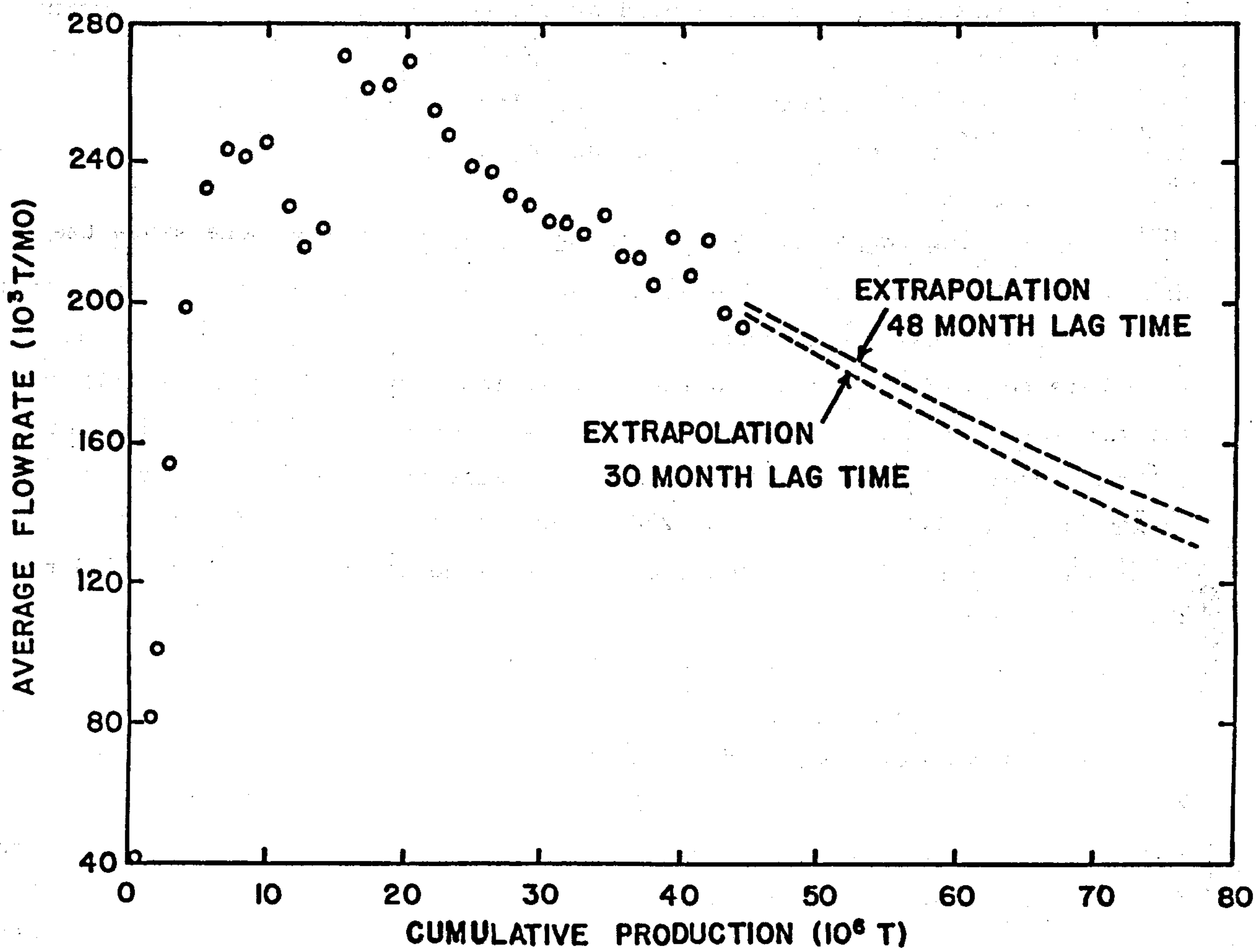

Figure 6. Gabbro zone decline curve.

ANOTHER RESERVOIR MODEL

To be complete in this report we should discuss an attempt to match the data with a slightly different model of the Gabbro system. As formulated in Eq. (12), the model assumes that the depletion from the deep zone due to flow toward Larderello is due to liquid flow (since we are using $\Sigma\Delta p$).

Further, the model assumes the flow is only within the deep zone since the frictional pressure drop term to the shallow zone only includes the actual Gabbro production.

It would be equally logical to assume that the loss to Larderello is steam which first must rise to the shallow zone and then move on the Larderello. If such a model is assumed then the flow to Larderello will be proportional to $\Sigma(p^2-64)$ rather than $\Sigma(p-8)$. In addition we would need a frictional loss term which will include an equivalent Δp^2 term which has been adjusted for the lag time. Such a model can be formulated as follows:

$$(p/Z)_{top} = A - B\,Q_{prod} - C\,\Sigma(p^2-64) - \frac{D(q_{eq})^{0.911}}{(p/Z)_{top}^{0.519}} - \frac{E(p^2-64)^{0.911}}{(p/Z)_{top}^{0.519}} \qquad (19)$$

Equation 19 was tested against the data using a lag time of 36 months. As might be expected, due to the additional constant, the resulting equation fit the data very well indeed. However, the resulting constants were completely unrealistic, sometimes giving negative values when they must be positive from physical considerations. Therefore, further work on this model was abandoned.

CONCLUSIONS

The reservoir pressure data in the Gabbro Zone clearly show a pressure trend toward Larderello which has persisted throughout the productive life of this reservoir. During the past ten years the number of producing wells in the Gabbro Zone has remained constant, and during this time both the producing rate and the Gabbro Zone pressure have declined continuously. Thus there is strong evidence that the Gabbro Zone is exhibiting depletion.

A reasonable hypothesis of the Gabbro Zone production dynamics is to assume that deep within the reservoir is a zone of boiling water which is supplying steam to the upper producing interval in which the wells are completed. Depletion of this deep zone can occur due to production from the Gabbro Zone and also due to flow from the Gabbro reservoir toward the main part of the Larderello Field. Further, since the deep zone seems to be connected to the producing zone by a system of relatively tight fractures, an additional transient pressure drop will occur in the producing zone due to frictional losses as the steam flows vertically.

The above concepts have been successfully incorporated into a reservoir model of the Gabbro Zone production history. This is a lumped parameter model which includes transient flow. The boiling water zone depletion is assumed to fit linearly with p/Z. A steady state flow formulation is assumed for the dynamics of flow from Gabbro to Larderello. The transient linear vertical flow is calculated using a lag time concept to change the transient flow terms to an equivalent steady state flow rate. Various lag times were studied and included in this part of the model is a new empirical equation which accurately relates $\Sigma\Delta p^2$ to $\Delta(p/Z)$.

The resulting predictive equation is linear and thus least-squares equations could be used to match the model to the reservoir data. Several combinations of reservoir size, lag time, linear flow pressure drop, and flow rates to Larderello were found to fit the data equally well. Thus, there is a uniqueness problem associated with these fits of the data. Lag times ranging from 30 to 48 months appear to fit the data equally well. The equation for a 24 month lag time predicted a somewhat higher producing rate in 1978 than actually occurred. Lag times less than 24 months and greater than 48 months were also attempted. The resulting equations fit the data quite well; but some of the constants were negative, which is physically unrealistic.

A study of the extrapolations of these various equations into the future showed that the projections to the year 1995 differ by only 6% in that year. Thus, although the reservoir parameters in the equations varied widely, the projected producing rates were quite similar.

Further projections of these flow rates were made using a revised formulation of the standard hyperbolic decline equations. These projections predicted a long producing life with a gradually declining rate. This is similar to the history seen in Larderello. The calculations project an ultimate reserve ranging from 250 to 400 million tons of steam. This rather large range of predictions is caused by the uncertainty of the decline predictions in the next century.

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