RESERVOIR ENGINEERING AND EVALUATION

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INTRODUCTION

An understanding of the physics of fluid flow and heat transport in geothermal systems is important with regard to the classification of and exploration for geothermal resources. This understanding is perhaps even more important in terms of aiding in determining if the resource can be developed economically. Obviously, geothermal systems are very complex with many processes occurring in them. Some of these physical processes include: 1) convective and conductive transport of heat in the fluid and rock matrix, 2) the tranport of water and (or) steam in porous and fractured media, 3) solute transport including chemical reactions that affect the flow characteristics of the medium, and 4) consolidation and land subsidence effects associated with fluid withdrawal. For many field problems, some of these processes have negligible effects and may be omitted, leading to a somewhat simplier description. Even with the simplifications, some rational framework is required in order to keep track of the important processes and how these processes interact.

One approach to attempt this "bookkeeping" of the physical behavior of geothermal systems is to use a mathematical model. Such a model consists of a set of equations that describe the transport processes active within the system and the solution to these equations subject to conditions that prevail at a particular site. This set of equations is general and may be used to consider two major subdivisions of geothermal modeling: 1) modeling the geothermal system under natural conditions in an effort to better understand how it forms and persists within the earth's crust, and 2) modeling the geothermal system during exploitation in order to predict its behavior subject to man-made stresses.

The formation of geothermal systems is thought to be associated with recent volcanism or significant tectonic movements. As a result of these activities, magmatic intrusions may occur at shallow depths in the earth's crust. Meteoric water is recharged to depth where it is heated directly or indirectly by the intruded magma. Density differences caused by the heating drive the fluid buoyantly upward where, hopefully, it can be exploited for power production. To be economically significant, a geothermal resource must have high temperatures and be located at shallow depths within the earth's crust.

Since current technology does not permit economical extraction of heat directly from dry rock, geothermal resources must also contain a fluid (either steam or water) to transfer heat from the geothermal reservoir to the surface. In addition, the reservoir must have sufficient volume, porosity, and permeability to yield adequate flow rates to wells.

One simulates a geothermal system to understand its behavior as well as to estimate the quantity of recoverable energy and the rate at which mass and energy may be extracted. To achieve these objectives, known geological information obtained from both surface techniques and drilling is utilized to determine equation parameters and boundary and initial conditions. During the earlier stages of field development, this information is limited, and the simulation model will undoubtedly be crude. Through the simulation of a variety of producing schemes, however, the energy available and its rate of extraction may be roughly estimated. Reservoir characteristics can be varied to determine what parameters are most critical to reservoir production, and what additional data is required to improve the simulation. If the decision is made to develop the field further, the reservoir model may be used to help answer such engineering questions as optimal well location and spacing, or whether or not to reinject condensate. As more geological information becomes available through continued drilling, the simulation model may be updated to give a more accurate analysis of the geothermal reservoir.

In this discussion, we restrict our attention to hydrothermal systems, that is, geothermal systems containing water. In particular, hydrothermal -convection systems are considered where most of the heat is transferred by circulating fluids rather than by heat conduction. These systems can be further subdivided as either liquid dominated or vapor dominated (White, Muffler, and Truesdell, 1971). In the liquid-dominated type, water is the continuous phase throughout the system that provides the pressure control. Continuity of the liquid phase is evident from reservoir pressures that are near hydrostatic and presence of soluble salts that are not found in significant quantities in low-pressure steam. In the vapor-dominated type, steam is the continuous, pressure-controlling phase, although water is also present.

Hydrothermal Systems

As stated, hydrothermal systems are geothermal systems that contain fluid. Although geothermal fluids contain impurities, many reservoirs are often approximated as pure-water systems. Consider figure 1, a pressure-temperature diagram of the liquid-vapor region for pure water (Whiting and Ramey, 1969). The points A, B, and C represent possible conditions for a hydrothermal system:

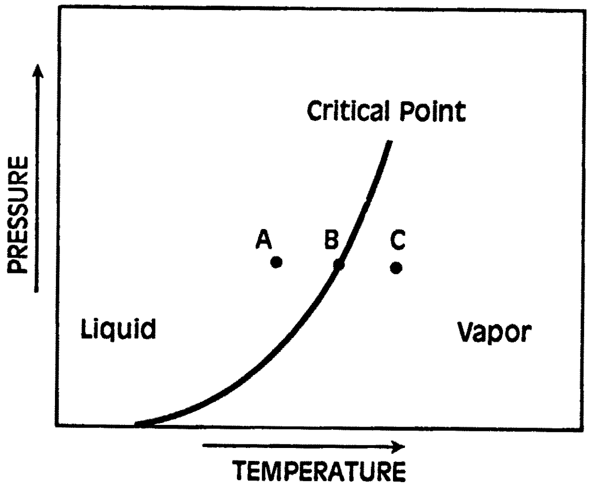

Fig. 1 Generalized pressure-temperature diagram for pure water showing points A, B, and C located in the compressed-liquid, saturated liquid-steam, and superheated steam regions, respectively (after Whiting and Ramey, 1969).

compressed-liquid, saturated liquid-steam, and superheated steam, respectively. These points are shown in figure 2, which is a pressure-enthalpy diagram for pure water (Whiting and Ramey, 1969). The compressed-liquid region is the condition existing in liquid-dominated hydrothermal systems. The vapor-dominated system described by White et al. (1971) is believed to exist mainly in the saturated liquid-steam region. In vapor-dominated systems, it is also probable, especially when influenced by exploitation, that parts of the system contain superheated steam.

At point A (see figure 1) in the compressed-liquid region, the system has one component, water, and one phase, liquid. According to Gibb's phase rule, two independent intensive properties (for example, temperature and pressure) must be specified to determine the thermodynamic state of the system. Once these properties are specified, point A can be located on any other thermodynamic diagram (for example, the pressure-enthalpy diagram). Liquid-dominated reservoirs with these initial conditions yield water to wells.

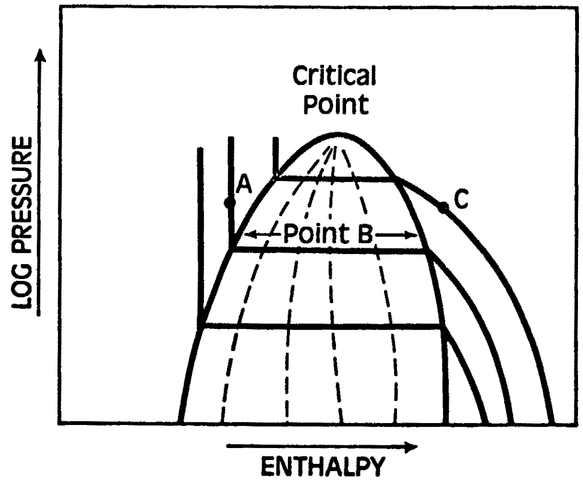

Fig. 2 Generalized pressure-enthalpy diagram for pure water showing points A, B, and C located in the compressed-liquid, saturated liquid-steam, and superheated steam regions, respectively (after Whiting and Ramey, 1969). Bold lines are isotherms and the dashed lines are lines of equal steam quality.

A decrease in pressure in the wellbore, however, causes steam to form, providing a mixture of steam and water at the wellhead. Field data (Grindely, 1965) indicate that production of a geothermal reservoir initially at point A will tend to cause an isothermal depletion, with reservoir pressures decreasing until the vapor-pressure curve is reached.

At the vapor-pressure curve, point B (figure 1), one intensive property determines the thermodynamic state. Although the thermodynamic condition is specified as liquid and vapor in equilibrium, the relative amounts can not be determined unless two independent properties such as pressure and mixture enthalpy are known. Note from figure 2 that temperature and pressure do not determine the relative amounts since temperature and pressure are not independent in the two-phase region.

Once vapor forms in the reservoir, temperatures drop slightly as a result of the heat of vaporization used to form steam. Because the water saturation is high, the relative permeability for the liquid phase dominates, and water continues to be produced from the reservoir. Pressure decreases only slightly in the two-phase region because pressure is maintained by the formation of steam. As production continues, however, water saturations decrease, temperatures decrease, and consequently, pressures also decrease, with more steam being produced from the reservoir.

Point C lies entirely in the vapor (superheated steam) region. This condition probably rarely occurs naturally in a reservoir. As in the compressed-liquid region, two independent intensive properties are required to define the thermodynamic state. In this case, the reservoir would also tend to deplete isothermally.

This description of reservoir behavior has been based on the assumption that the reservoir fluid is pure water and (or) steam. Geothermal fluids, however, contain dissolved solids and noncondensable gases. For example, the total dissolved solids for the Wairakei, New Zealand field is less than 3% by weight, whereas some geothermal fluids in Imperial Valley, California contain as much as 30% dissolved solids by weight. Therefore, the vapor pressure of the fluid in a geothermal reservoir may not necessarily be that represented in the steam tables. For a fixed pressure, the boiling temperature of water will be elevated by the presence of impurities. General field behavior, however, should be qualitatively similar to that outlined for pure water.

THEORETICAL DEVELOPMENT

The physical properties of the fluid and reservoir vary spatially and temporally, and effects of gravity segregation of steam and water require vertical as well as horizontal treatment. Consequently, a general model must be a distributed parameter model with the capability of three-dimensional simulation. To answer important engineering questions, a general model must not only accommodate the transient flow of compressed liquid, steam-water mixture, and superheated steam, but also must allow for phase changes. In this discussion, we neglect solute transport and chemical reactions that may occur in the geothermal system. Furthermore, we assume that the reservoir can be treated as a porous medium.

The general governing equations consist of mass, momentum, and thermal energy balances for each phase present in a geothermal system. Using a set of constitutive relationships, these balance equations may be reduced to two or three nonlinear partial differential equations.

Mass Balance

The mass balances for steam, s, and water, w, may be written as:

$$-\frac{\partial}{\partial x_i}(\bar{v}_s\rho_s) + q_s + d_v = \frac{\partial(\phi S_s\rho_s)}{\partial t} \qquad (1)$$

and

$$-\frac{\partial}{\partial x_i}(\bar{v}_w\rho_w) + q_w - d_w = \frac{\partial(\phi S_w\rho_w)}{\partial t} \qquad (2)$$

where

$\bar{v}$ = phase average velocity $[Lt^{-1}]$
ρ = average density, $[ML^{-3}]$
q = source term, $[ML^{-3}t^{-1}]$
d_v = rate of vaporization, $[ML^{-3}t^{-1}]$
ϕ = porosity, dimensionless
S = saturation, dimensionless, where $S_w + S_s = 1$

Momentum Balance

For velocity, we assume that Darcy's equation for multiphase flow may be used

$$\bar{v}_s = -\frac{\bar{\bar{k}}k_{rs}}{\mu_s}\left(\frac{\partial p_s}{\partial x_j} - \rho_s\bar{g}\right) \qquad (3)$$

and

$$\bar{v}_w = \frac{\bar{\bar{k}}k_{rw}}{\mu_w}\left(\frac{\partial p_w}{\partial x_j} - \rho_w\bar{g}\right) \qquad (4)$$

where

k = local intrinsic permeability tensor $[L^2]$
k_r = relative permeability, dimensionless
μ = dynamic viscosity, $[ML^{-1}t^{-1}]$
p = pressure, $[ML^{-1}t^{-2}]$
$\bar{g}$ = gravitational acceleration, $[Lt^{-2}]$

Energy Balance

The thermal energy equations, neglecting viscous dissipation, for steam, water, and rock (r) are:

$$\frac{\partial(\phi S_s\rho_s U_s)}{\partial t} + \frac{\partial}{\partial x_i}(\rho_s U_s \bar{v}_s) + \frac{\partial\bar{\lambda}_s}{\partial x_i} + p_s\frac{\partial\bar{v}_s}{\partial x_i} + q_s h_s + Q_{ws} + Q_{rs} = 0, \qquad (5)$$

$$\frac{\partial(\phi S_w\rho_w U_w)}{\partial t} + \frac{\partial}{\partial x_i}(\rho_w U_w \bar{v}_w) + \frac{\partial\bar{\lambda}_w}{\partial x_i} + p_w\frac{\partial\bar{v}_w}{\partial x_i} + q_w h_w + Q_{sw} + Q_{rw} = 0, \qquad (6)$$

$$\frac{\partial[(1-\phi)\rho_r U_r]}{\partial t} + \frac{\partial\bar{\lambda}_r}{\partial x_i} + Q_{sr} + Q_{wr} = 0, \qquad (7)$$

in which

U = internal energy per unit mass $[L^2t^{-2}]$
h = enthalpy per unit mass $[L^2t^{-2}]$
Q = interphase heat transfer term $[ML^{-1}t^{-3}]$
$\bar{\lambda}$ = combined conduction - dispersion vector $[Mt^{-3}]$

The first subscript of the Q term refers to the phase from which the energy is obtained, whereas the second subscript refers to the phase receiving the energy.

Constitutive Relationships and Simplifing Assumptions

The equations presented in the previous sections have 23 dependent variables, e.g. densities, relative permeabilities and viscosities. Many of these can be related to particular thermodynamic variables and thus, additional equations are needed expressing them as functions of these variables. To obtain equations (1) - (7), it was necessary to make the following simplifing assumptions: 1) solute transport, chemical reactions, kinetic energy, viscous dissipation, and potential energy can be neglected; and 2) the geothermal reservoir behaves as a porous medium. In order to simplify the equations further and reduce the number of dependent variables, constitutive relationships, (i.e. expressions relating dependent variables), are used. In general, constitutive relationships are determined in the field and used to obtain the most simplified subset of the general equations that describe a particular geothermal reservoir application. For example, if a reservoir is liquid-dominated, with no chance for steam formation, it would be unnecessary to include the steam balance equations. For some applications, such as well-test analysis, isothermal conditions often exist over the period of analysis so that energy balance equations can be neglected.

For general two-phase reservoir applications, the constitutive relationships concern thermodynamics, capillary pressure, relative permeability, viscosity, reservoir consolidation, thermal exchange between phases, and thermal dispersion-conduction.

The thermodynamics of single-component water provide relationships between $S_s, S_w, \rho_s, \rho_w, U_s, U_w, h_s, h_w, p_s, p_w, \rho$ (mixture density of steam and water), h (mixture enthalpy of steam and water), and T (temperature). The final governing equations are usually reduced to two or three that are posed in terms of two or three thermodynamic variables. Models based on pressure-mixture enthalpy, pressure-mixture internal energy, mixture density-mixture internal energy, and pressure-temperature-saturation have been suggested by various authors.

Capillary pressure is the difference between steam pressure and water pressure, defined as

$$p_c = p_s - p_w \tag{8}$$

It is a property of the porous medium, but, in general, is also dependent upon the volume saturation of liquid and the temperature. Capillary pressure has the effect of apparently lowering the vapor pressure of water. Ramey et al. (1973) point out that the reason for this lowering is that vapor pressure data found in steam tables (Meyer, et al., 1968; Keenan, et al., 1969) are based on flat steam-water interfaces, whereas the interface in posous media is curved. The amount the vapor-pressure curve is lowered in a geothermal reservoir is not completely understood. The work of Calhoun, et al. (1949) on consolidated rock does show a lowering with decreased fluid saturation. Cady (1969) and Bilhartz (1971), however, indicate no significant vapor pressure lowering in experiments using unconsolidated sands. An important difference in these results is that the experiments of Calhoun and others were made at a temperature of 36°C, while those of Cady and Bilhartz were done over a temperature range from 121°C to 240°C. Further work on the importance of capillary pressure in geothermal reservoirs is required. For many applications capillary pressure is assumed to be negligible.

Relative permeability is a property of the porous medium, and is also dependent on the volume saturation of liquid. It can be thought of as the ratio of a phase permeability to the total permeability and ranges in values from 0.0 to 1.0. Relative permeability is commonly treated as a function of saturation alone and relationships similar to those in Brooks and Corey (1964) are used. Ramey, et al. (1974), point out that relative permeability can also be a function of temperature. Using unconsolidated sand and working with oil and water, Poston, et al. (1970), observed that for increased temperatures, the relative permeability curves shift to the left on the saturation axis.

Viscosity is dependent on both the pressure and temperature, but the pressure dependence is small and generally neglected.

Coupled equations for consolidation, fluid flow, and heat transport in geothermal reservoirs may be derived and solved, but consolidation is not the process of concern in many studies. A simple approximation of consolidation that incorporates its main effect on fluid flow may be used. This approximation relates the porosity to pressure by,

$$\phi = \phi_i + \beta\,(p - p_i) \tag{9}$$

where ϕ_i and p_i are the initial porosity and pressure, respectively, and β is the intergranular, vertical compressibility coefficient.

The movement of steam and water through porous media is sufficiently slow, and surface areas of all phases are sufficiently large, that it is reasonable to assume that local thermal equilibrium among phases is achieved instantaneously. This common assumption permits the energy equations for rock, steam, and water to be combined and the medium conduction-dispersion term to be expressed as a function of a single temperature. In most developments the lumped conduction-dispersion term is defined by a Fourier-type equation,

$$\bar{\lambda}_s + \bar{\lambda}_w + \bar{\lambda}_r = \bar{\bar{K}}_m \nabla T \tag{10}$$

where T is temperature and, in general, the medium conduction-dispersion $\bar{\bar{K}}_m$ is a tensor quantity. Regarding the thermal dispersion tensor it should be noted that Mercer (1973) separates the medium thermal dispersion tensor into three parts: conduction in the solid phase, diffusion in the liquid phase, and a velocity-dependent dispersion in the liquid phase. This last part is an attempt to take into account the heat transport related to the mixing of different temperature waters as they flow through the pore spaces. Furthermore, Somerton, et al. (1974), point out that the thermal conductivity of the medium is a function of temperature, porosity, and water saturation.

Initial and Boundary Conditions

In order to obtain a solution to the governing equations for geothermal reservoirs, hydrodynamic and thermal boundary and initial conditions must be specified. These conditions can be deduced from a thorough examination of hydrologic and geologic data. There are generally three types of hydrodynamic boundary conditions in geothermal reservoirs: 1) constant pressure boundary conditions, 2) no-flow boundary conditions along impermeable surfaces, and 3) specified flux. The corresponding thermal boundary conditions are 1) constant temperature, 2) no heat flow-insulated boundary, and 3) specified heat flux. When production or injection wells exist in the reservoir, additional boundary conditions specifying either the pressures or the mass flow rates for the sources or sinks are required. Initial conditions, specifying thermodynamic variables throughout the reservoir at the beginning of the simulation, determine whether the reservoir is liquid- or vapor-dominated. In addition, the boundary conditions determine the flow conditions. Depending on the hydrodynamic and thermal boundary conditions, both free (or natural) and forced convection may occur in geothermal systems. Free convection generally dominates flow conditions in geothermal reservoirs before exploitation, while forced convection dominates during exploitation when externally imposed pressure gradients are generated due to production and/or reinjection of fluids.

Equation Parameters

In addition to boundary and initial conditions, many of the equation parameters, which are also field parameters (for example, permeability), need to be determined. Some of these are determined by performing laboratory tests on core samples. Thermal conductivities and porosities may be obtained in this fashion. Permeabilities, however, are generally determined in the field using *in situ* techniques, such as well testing. This involves analyzing pressure data using simplified subsets of the general governing equations. This is often referred to as an inverse problem, since permeabilities are "backed out" from pressure build-up or drawdown data.

Such analyses have been performed on several liquid-dominated reservoirs (Narasimhan, 1977) with limited success. These tests have established that well-testing, based on techniques developed in the fields of petroleum engineering and hydrogeology, is a valuable tool in estimating *in situ* parameters and in deciphering the geometry of geothermal reservoirs.

The application of transient pressure analysis methods to vapor-dominated reservoirs has generally been done using methods developed for noncondensable gas reservoirs. Moench and Atkinson (1977) examine this problem and conclude that the presence of a vaporizing liquid will not complicate evaluation of the reservoir permeability-thickness product from drawdown data when the usual methods of gas reservoir engineering are applied. Simulated pressure buildup data, on the other hand, show characteristics that are markedly different from that expected for noncondensable gas.

CONCLUDING REMARKS

Numerous models based on the concepts presented in this paper have been applied to geothermal systems under both natural conditions and exploitation. A complete review of free convection models applied to natural systems is included in Witherspoon, et al. (1975). A review of reservoir models used to examine exploitation effects is presented by Faust and Mercer (1979). From these reports, it seems obvious that our mathematical tools, both analytical and numerical, are sufficiently powerful to solve the most difficult problems. Unfortunately, the conceptual basis of these mathematical models appears to need reinforcement. The geologic materials in which heat transport processes occur are not easily idealized. Constitutive relationships for thermal dispersion and relative permeability, and the description of flow and transport in fractured rock are areas that need further study. Finally, additional field verification of the complicated mathematical models are needed.

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