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Economic Factors in Resource Exploration and Exploitation

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ABSTRACT

Both replenishable and nonreplenishable resources are exhaustible, and even finite nonreplaceable resources can have infinite economic lives. The concept of ecological equilibrium, in which total recruitment of new mass is equal to the harvest rate, is relevant to both types of resources. The rate of use of existing stock is the intensive margin, and investment in renewal through exploration and development represents the extensive margin. Both the rate and level of recovery are influenced by the economic motivation of the resource owner to maximize the present value of the resource. Unlike other branches of economics in which current production is pushed to the point where marginal profits are zero, it is shown that the profit-maximizing resource owner will postpone the current production of an additional unit if the present value of the profit which that unit could earn at some future date is larger than the marginal profit which can be earned today. Further results of the analysis are the following:

1. The optimal conservation of the known stock is determined by maximizing, over the set of possible lifetimes and given discount rates, the present value of the resource. This maximization process determines the lifetime of the resource, the optimal reserve to output ratio, and the rate of recovery.
2. The time to begin developing a proven reserve is when the value of the resource in the ground stops rising faster than the discount rate.
3. The time to prospect fields with suspected reserves is when the lease value stops rising faster than the discount rate.

INTRODUCTION

Depletion of natural resources is an issue of continuing importance. This paper is a discussion of economic factors in the optimal depletion of resources. It might at first seem puzzling that there could be any such thing as optimal depletion, but depletion is associated with economic development and nondepletion is associated with monopolization. Furthermore, conservation and depletion are not opposites since conservation carries with it the concept of an optimum rate of depletion.

The opposite of depletion is augmentation. There is increased depletion of resources when current production is increased and when current exploration is decreased. In this case the ratio of reserves to output—the so-called Life Index—falls. There is augmentation of resources when there is (1) decreased current production and consumption; (2) increased exploration; and (3) technological progress which increases efficiency of recovery, permits the substitution of lower-quality for higher-quality deposits, and makes feasible alternative sources of supply. When there is augmentation the Life Index rises.

The main contribution of this paper is the refinement and extension of an economic model which was first suggested by Mason Gaffney in 1967. Inputs to this model are estimates of resource availability and cost functions supplied by geologists and engineers. Outputs of the model are (1) optimal lifetime of resource; (2) optimal annual production; and (3) optimal ratio of reserves to output, the Life Index. First, however, several earlier economic optimization models are briefly discussed.

MAXIMIZATIONS

Maximization of Ultimate Recovery

The volume of oil and gas ultimately recoverable from a given reservoir may have a tendency to decline with increasing rates of production once a threshold level is reached. Figure 1 is a graphical presentation of this kind of situation (Davidson, 1963).

Maximum ultimate recovery according to Figure 1 is obtained by annual production rates which are equal to or less than the maximum efficient rate of production (MER). MER is not, however, an economic concept: "MER is without economic content. Even with zero interest, it would never make sense to maximize ultimate recovery" (Gaffney, 1967). During World War II, for example, many oil fields were operated at rates in excess of MER because the benefits of extra production exceeded the costs of reduced ultimate recovery (Davidson, 1963).

Maximization of Average or Annual Profit

If future profits are not discounted (as might be the case with a zero rate of interest), then profits per unit of resource will be maximized. Figure 2 shows the situation of a firm which can sell as much as it wants at the going price (P). Since average revenue (AR) is constant it is also equal to marginal revenue (MR). Marginal cost (MC) is equal to average cost (AC) at the minimum point of the average cost curve and, beyond that point, MC exceeds AC (Lockner, 1965).

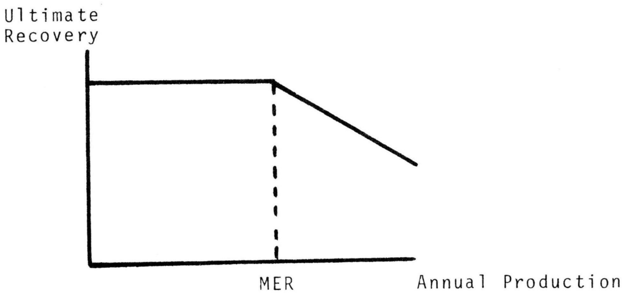

Figure 1. Maximum efficient rate of production: *MER*.

Figure 2 indicates that profit per unit of resource is maximized, and hence total profit per reservoir, when the gap between price per unit and cost per unit is greatest. This occurs when annual production is *A* units.

When only *A* units are produced, however, marginal revenue is still in excess of marginal cost. It pays in terms of *annual* profit to expand production until marginal revenue is no longer greater than marginal cost, that is, until marginal revenue equals marginal cost. This occurs when annual production is increased to a level of *B* units (Fig. 2). Profits per year (current profits) are maximized when annual production is *B* units. There is some sacrifice of total profits over the lifetime of the resource if current profits are maximized. Such a sacrifice of downstream profits is justifiable if profits now are more valuable than profits later as indeed they would be if the rate of discount is greater than zero. The higher the rate of discount, the closer production will be pushed to the point *B* where current profits are maximized.

Present Net Value Per Life of Resource

Future dollars have less value than current dollars because the rate of interest (rate of discount) is greater than zero. If the rate of interest were zero, business firms would be indifferent as to the time distribution of their receipts and, as mentioned earlier, would maximize profit per unit of resource by producing *A* units per year in Figure 2. In this case marginal revenue exceeds marginal cost by an amount X:

$$MR - MC > 0$$

$$MR = MC + X \tag{1}$$

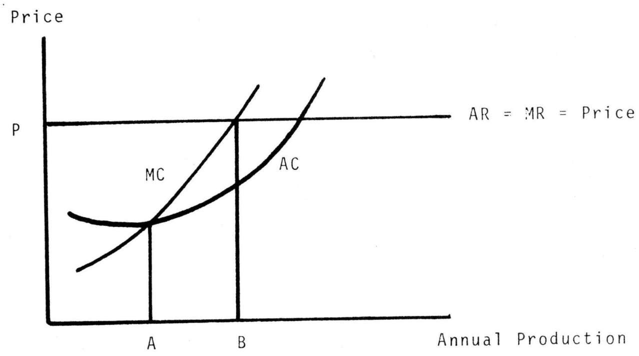

Figure 2. Costs and revenues. Key: *AR*, average revenue; *MR*, marginal revenue; *P*, price; *AC*, average cost; *MC*, marginal cost.

where $X = MR - MC$. The amount X will later be referred to as "marginal user cost," MUC.

If the rate of interest is very high, current profits will be maximized and a higher level of annual production (*B* units in Figure 2) is scheduled because production is carried to the point where marginal profit—marginal revenue less marginal cost—is equal to zero. In this case marginal user cost will also be zero.

The present value of a dollar a year hence is $\$1/(1 + r)$, where r is the rate of interest. If the present value of profits obtainable over the given life of a resource is maximized, the present value of marginal profits next year, $MP_{T+1}/(1 + r)$, must be equal to marginal profits this year, MP_T (Herfindahl, 1967; McDonald 1967):

$$MP_T = \frac{MP_{T+1}}{1 + r} \tag{2}$$

$$MR_T - MC_T = \frac{MP_{T+1}}{1 + r} \tag{3}$$

$$MR_T = MC_T + \frac{MP_{T+1}}{1 + r} \tag{4}$$

$$MR_T = MC_T + MUC_T. \tag{5}$$

If marginal revenue this year merely covers current marginal cost (i.e., $MR_T - MC_T = 0$), then the interest rate must be so high that the term $MP_{T+1}/(1 + r)$ in Equation (4) is virtually zero. In this case maximization of present value is equivalent to maximization of current profit. If the interest rate is zero, on the other hand, then marginal profits this year are equal to undiscounted marginal profits next year (Eq. 2) and maximization of present value is equivalent to maximization of profit per unit of resource.

The discounted value of marginal profits next year, $MP_{T+1}/(1 + r)$, is forfeited if current profits are maximized. This is the justification for referring to the present value of next year's marginal profit as a marginal user cost:

"For any particular producer oil in the ground is a stock. The more used today, *ceteris paribus*, the less will be available tomorrow. Consequently, for production to occur under conditions of [discounted] profit maximization, marginal revenue must not only cover marginal operating and royalty costs, but must also cover the present value of marginal profits given up by producing this week [year] rather than later" (Davidson, 1963).

The present value of future marginal profit will be forfeited if current production is carried to the point *B* in Figure 2 where marginal revenue merely covers marginal cost. Unless interest rates are very high, business firms will have an economic incentive to conserve their resource because of the opportunity cost (marginal user cost) of excessive current production.

If discounted future marginal profits are greater than current marginal profits, the firm will be able to increase the present value of its resource by increasing production in future time periods relative to current levels. This will have the effect of increasing marginal costs in the later period, as production is pressed closer to capacity, until the point is reached where discounted future marginal profit is no longer greater than current marginal profit. When future prices are expected to rise relative to future costs, the firm will withhold current production in favor of increased production in the future. If future interest rates are expected to be lower than current interest rates, the present value of future marginal profit increases and more production will be allocated to future periods.

When current production exceeds the marginal efficient rate (MER), then the present value of future recovery forfeited must be covered by current marginal revenue, that is, marginal user cost also includes the cost of reduced ultimate recovery. If several independent firms are producing from a common reservoir, then current production foregone by one firm is likely to be captured by its rivals. In this case marginal user cost will be negative and each firm is encouraged to overproduce (at levels even higher than *B* units of production in Fig. 2). This is an aberrant situation which arises from the common law concept of the "rule of capture" and will not occur if common reservoir pools are unitized (Davidson, 1963).

The foregoing analysis had been based on two key assumptions:

1. The economic life of the resource is given. This means that the year (*t*) of exhaustion was somehow predetermined and, given this fixed lifetime, the firm decides to allocate production over these years such that no recoverable resource remains in year *t*.
2. There is no need for steadiness of production. Annual production rates have been assumed to change in response to changes in expected prices and costs. Such flexibility in production is realistic in special circumstances such as excess capacity sponsored by cartel arrangements. For example, the number of days' production allowed for wells controlled by prorationing in Texas rose from 97 days in 1962 to 365 days in 1972 (Kahn, 1964).

Large capital investments require a reliable supply of, and demand for, the resource so that steady production can be achieved (Hotelling, 1931). Although rate of use is subject to some short-run control, such control is usually of second-order importance. The basic cost determinant is the capital invested in year zero (Gaffney, 1967). It is also in year zero that the firm must decide on the optimal operating life of its resource. The question that then needs to be answered is as follows: Given a steady annual rate of production and a possible lifetime of resource which can vary between 1 and 100 years, for example, which lifetime is optimal with regard to maximization of present net value?

Table 1. Effect of costs and interest rates on years of life of resource (figures shown in parentheses are present net values).

Cost*	Interest rate .05	 .10	 .15
$500 000	5 $766 000	4 $667 000	3 $594 000
$800 000	6 $713 000	5 $598 000	4 $514 000
$1 000 000	7 $684 000	6 $559 000	5 $470 000
$2 000 000	11 $573 000	9 $418 000	8 $311 000
$5 000 000	20 $373 000	19 $178 000	18 $63 000
$10 000 000	34 $182 000	(< $0)	(< $0)
$15 000 000	55 $66 000	(< $0)	(< $0)
$20 000 000	(< $0)	(< $0)	(< $0)

Note: The physical quantity of the resource was assumed to be 1 000 000 units with a price per unit in each time period of $1.00.

* Cost = present value of the cost of extracting entire resources in one year = *K*.

OPTIMAL LIFETIME OF RESOURCE

Gaffney Model

The following model is an extension of preliminary ideas set forth by Mason Gaffney (1967). Given an estimate of the physical quantity (Q) of a resource whose price is assumed to be $1.00 per unit in every time period, the business firm will determine the economic life (L) of its deposit by choosing that lifetime which maximizes the present value of the resource. Optimal annual production will be Q/L and the present value of revenues, *PVR*, is given by following formula:

$$PVR = \frac{Q}{L}\left(\frac{1}{1+r} + \frac{1}{(1+\;)^2} + \dots + \frac{1}{(1+r)^L}\right) \tag{6}$$

$$= \frac{Q}{L}\,\frac{1-(1+r)^{-L}}{r},$$

where *r* is the rate of interest.

A simplified cost function can be obtained by assuming that doubling of life cuts costs in half because only half as much capacity is required. If the present value of extracting the entire resource in one time period is denoted by *K*, the present value of costs (*PVC*) is given by

$$PVC = K/L \tag{7}$$

The present net value ($PNV = PVR -$ PVC) of the resource is

$$PNV = \frac{Q}{L}\,\frac{1-(1+r)^{-L}}{r} - \frac{K}{L}. \tag{8}$$

Given the cost (*K*) of exhausting the resource in one time period (year) and the rate of interest (*r*), the possible values for lifetime of resource (from $L = 1$ to $L = 100$, for example) are tried in Eq. 8 and that lifetime is chosen which maximizes present net value.

A necessary but not sufficient condition for maximization, over all possible lifetimes, of present net value can be obtained by differentiation. The first derivative is

$$\frac{dPNV}{dL} = \frac{1-(1+r)^{-L}}{r} - L(1+r)^{-L} - \frac{K}{A}, \tag{9}$$

but this expression still contains lifetime (*L*) as a variable so that an analytical solution is not readily available.

Global maximization is easily obtainable, however, once a computer program is written which iterates Eq. 8 over all possible lifetimes. The results of such a program for various values of *K* and *r* are reported in Table 1.

Table 1 indicates that increases in costs of "early extraction"—that is, cost of extracting the entire resource in one year—lengthen the period of exploitation. When the rate of interest is 5% the years of life of resource is (1) 5 years when early extraction cost (*EEC*) is $500 000; (2) 55 years when EEC is $15 000 000; and (3) infinite, due to economic infeasibility of any extraction, when EEC is $20 000 000.

Positive net values in Table 1 mean that the yield—percentage rate of return—exceeds the given rate of interest. When early extraction cost is $10 000 000, for example, the rate of return is greater than 5% (since present net value is greater than zero when a 5% rate of interest is used in the discounting) and is less than 10% (since present net value is less than zero when a 10% rate of interest is used). With an early extraction cost of $10 000 000, the resource will have a lifetime of 34 years if the rate of interest is 5% but will be uneconomic if the rate of interest is 10%. It was asserted that a resource has an infinite life if there is zero production. It is perhaps equally plausible to argue that such a "resource," being uneconomic, has a zero life.

An increase in the rate of interest has several effects:

1. The present net value of resources declines when interest rates rise. Exploration and development activity will thus be discouraged.
2. Some resources which are economic at low interest rates become noneconomic if interest rates rise. When early extraction cost is $10 000 000, production is feasible at a 5% rate of interest but is infeasible if interest rates are 10%.
3. For resources which are economic at both low and high interest rates, the effect of an increase in the rate of interest is a reduction in lifetime of resource. When early extraction cost is $500 000, the years of life of resource is 5 years if the rate of interest is 5% but is only 3 years if the interest rate is 15%.
4. The reserves-to-output ratio, or Life Index, increases with decreases in the rate of interest. When early extraction cost is $800 000, the half-life of the resource is 3 years if the interest rate is 5% and hence the average ratio of reserves to output is 3:1. If the rate of interest is 15%, however, the half-life drops to 2 years and the reserve-output ratio falls to 2:1.

Extensions of the Gaffney Model

It is unnecessarily restrictive to assume that price per unit of resource will be the same in each time period. When price can vary from year to year, the formula for present net value becomes

$$PNV = \frac{Q}{L}\left(\frac{P_1}{1+r} + \frac{P_2}{(1+r)^2} + \dots + \frac{P_L}{(1+r)^L}\right) - \frac{K}{L} \qquad (10)$$

where P_L is price per unit in year L, the year of exhaustion.

Table 2, showing the effects of rising prices, depletion allowances, and profit tax, is based on a resource of 100 units ($Q = 100$) with an early extraction cost (K) of $150, a 20% interest rate (r), and initial price per unit (P_1) of $1.00. When prices are expected to remain constant, present net value is maximized (at $30.40) when lifetime of resource is 6 years and annual production is 16.67 units. If prices are expected to rise 10% a year, annual production drops to 12.5 units, lifetime rises to 8 years, and present net value increases (to $43.90). If prices are expected to fall 10% a year, annual production increases to 20 units, lifetime is shortened to 5 years, and present value drops (to $20.80).

Table 2. Effects of rising prices, depletion allowances, and profits tax.

Cost parameters	Maximized present net value*	Annual production	Lifetime of resource (yrs)
(1) $P_{T+1} = P_T$ (constant price)	$30.4	16.67	6
(2) $P_{T+1} = 1.10P_T$ (rising price)	$43.9	12.5	8
(3) $P_{T+1} = .90P_T$ (falling price)	$20.8	20.0	5
(4) $P_{T+1} = P_T$, plus depletion allowance†	$59.7	20.0	5
(5) $P_{T+1} = 1.10P_T$, plus depletion allowance†	$76.6	16.67	6
(6) $P_{T+1} = .90P_T$, plus depletion allowance†	$47.9	25.0	4
(7) $P_{T+1} = 1.10P_T$, plus 50% profits tax	$21.95	12.5	8
(8) $P_{T+1} = 1.20P_T = (1 + r)P_T$	No finite value‡	0.0	Infinite

*Calculated as $PNV = \frac{Q}{L}\left(\frac{P_1}{1+r} + \frac{P_2}{(1+r)^2} + \dots + \frac{P_L}{(1+r)^L}\right) - \frac{K}{L}$, with $K = 150$, $Q = 100$, $r = .20$, and $P_1 = \$1.00$.

†Based on a depletion allowance which raises revenue per unit by 50%.

‡See Table 3.

Percentage depletion allowances exempt part of business income from taxation and in effect increase after-tax revenue per unit. If a depletion allowance is granted which increases after-tax revenue per unit by 50%, then present net value of the firm and annual production will both increase. Although the lifetime of given resources will be reduced by depletion allowances, the rather dramatic increase in present net value will make submarginal resources economically viable and will encourage both exploration and development. As a result, both reserves and output will increase so that the Life Index—ratio of reserves to output—will tend to remain constant (Peterson, 1975; Peterson and Seo, 1975).

If a 50% profits tax is imposed, present net value will

Table 3. Discounted revenues and costs for resource lifetimes of 1 to 4 years.

Lifetime of resource (yrs.)	Annual production	Discounted cost	Discounted revenue
1	100	150	$100\left(\frac{1.2}{1.2}\right) = 100$
2	50	75	$50\left(\frac{1.2}{1.2} + \frac{1.2(1.21)}{1.44}\right) = 100$
3	33.33	50	33.33(3) = 100
4	25	37.5	24(4) = 100

be cut in half in the rising-price case of Table 2 (to \$21.95), but annual production will remain constant at 12.5 units. The decrease in present net value will discourage exploration and development activity, however, so that the ratio of reserves to output will fall.

Finally, if prices are expected to rise at a rate equal to the rate of interest (20% in Table 2), then there is no finite maximum to present net value. That is, the longer the lifetime of resource, the larger is present net value. This means that annual production will be zero, as long as prices are expected to rise by 20% per year. It is a standard result in resource economics that resources should be withheld from production if it is expected that future net prices will rise as fast as the rate of discount (Gordon, 1967; Vickrey, 1967; Hotelling, 1931; Lockner, 1965). In this case, in terms of Figure 2, production will be less than *A* units per year. Since the expectation of future prices is subjectively determined, there will be variation among firms in their rates of production.

EXPLORATION AND DEVELOPMENT

If the net per unit price, after deduction of costs, is rising 20% or more a year (for example) when rates of discount are 20% or less, the resource is more valuable in the ground, *in situ*, than it is *ex situ* or at the wellhead. This result can be generalized in terms of the following propositions (Gaffney, 1967):

1. The time to begin developing a proved reserve is when the value of the resource in the ground stops rising faster than the discount rate.
2. The time to prospect fields with suspected reserves is when the lease value stops rising faster than the discount rate.

The present net value of holding suspected or proved reserves (*PNVH*) is given by the following formula:

$$\frac{NV_1}{1+r} + \frac{NV_2}{(1+r)^2} + \dots + \frac{NV_H}{(1+r)^H}, \qquad (11)$$

where NV_H is the net value in year H. Various values for H (from $H = 1$ to $H = 100$, for example) are inserted in Eq. 11 and that value of H is chosen which maximizes present net value of holding (*PNVH*).

It is assumed that the holder of reserves desires to maximize, over all possible holdings periods, the present net value of his asset. At the end of this holding period he will either begin prospecting (in the case of suspected reserves) or production (in the case of proved reserves).

If the net value of his asset is rising as fast as the rate of discount r—that is, if $NV_{T+1} = (1 + r)NV_T$—the longer the holding period is the greater will be the present net value of his holding. This means that there is no finite maximum for present net value and hence the holding period is infinite for both suspected and proved reserves. As current production decreases, however, current price will rise relative to future price so that it is unlikely that holding periods will be excessively long. The general proposition, then, is that an expectation of rising net prices leads to longer holding periods.

SUMMARY

Five models of optimal exploitation of resources have been discussed:

1. Maximization of ultimate recovery. This is a physical rather than an economic concept and is based on the idea that excessive current production, especially from oil and gas reservoirs, can lead to a reduction in total recovery. The maximum efficient rate of production (MER) is the highest rate of annual recovery from a reservoir that is allowable lest total recovery be reduced.
2. Maximization of profit per unit of resource. If profit per unit of average profit is maximized, then total undiscounted profit per resource-field or reservoir is also maximized and annual production is carried only to the point where average costs are minimum. This model appears to have economic relevance only in the unlikely event that discount rates are zero.
3. Maximization of profit per year. When production is extended to the point where marginal revenue no longer exceeds marginal cost, then current profits are maximized. This model has relevance only if discount rates are extremely high or if marginal user costs, discussed below, are close to zero.
4. Maximization of discounted profits per life of resource. Production is carried only to the point where marginal revenue covers both marginal cost and marginal user cost. Marginal user cost is the discounted value of future marginal profit which is forfeited by production now instead of production later. The lifetime of the resource is assumed to be determined by outside or exogenous forces rather than by the business firm. It is also assumed that rates of production can vary rather widely from year to year.
5. Maximization of discounted profits per resource. Given an estimate of quantity of resource, a cost function, and the rate of discount, the business firm is assumed to choose the lifetime of his resource so as to maximize present net value. Given the need for steady annual production, the optimal rate of recovery and ratio of reserves to output are obtained as soon as optimal lifetime is determined. The lifetime of resources can vary greatly due to variations in the rate of discount and in the path of future prices.

REFERENCES CITED

Davidson, P. 1963, Public policy problems of the domestic crude oil industry: Am. Econ. Review, v. 53 (March), p. 93.

Gaffney, M., 1967, Editor's conclusions, *in* Gaffney, M., ed., Extractive resources and taxation: Madison, Univ. of Wisconsin Press.

Gordon, R. L., 1967, A reinterpretation of the pure theory of exhaustion: Jour. Political Econ., 75, p. 280.

Herfindahl, O. C., 1967, Depletion and economic theory, *in* Gaffney, M., ed., Extractive resources and taxation: Madison, Univ. of Wisconsin Press, p. 131.

Hotelling, H., 1931, The economics of exhaustible resources: Jour. Political Econ., v. 39 (April), p. 163.

Kahn, A. E., 1964, The depletion allowance in the context of cartelization: Am. Econ. Review, 54 (June), p. 300.

Lockner, A. O., 1965, The economic effect of the severance tax on the decisions of the mining firm: Natural Resources Jour., 4 (January).

McDonald, S. L., 1967, Percentage depletion, expensing of intangibles, and petroleum conservation, *in* Gaffney, M., ed., Extractive resources and taxation: Madison, the Univ. of Wisconsin Press, p. 275.

Peterson, R. E., 1975, Economic factors in the longevity of resources: Hawaii Geothermal Project.

Peterson, R. E., and Seo, K. K., 1975, The Reich case: Economic implications of depletion allowances: Hawaii Geothermal Project.

Vickrey, W., 1967, Economic criteria for optimum rates of depletion, *in* Gaffney, M., ed., Extractive resources and taxation: Madison, Univ. of Wisconsin Press, p. 317.