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Qualitative Theory on the Deep End of Geothermal Systems

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ABSTRACT

A highly active geothermal area liberates about 400 MW of heat and is unlikely to have a collection area much greater than 20 km^2. The conductive supply of heat at this level implies a gradient of 7000°C km^{-1} in the rock that can be maintained for only about 350 yr by transient cooling, and for longer times by physically and geologically unlikely schemes of magma stirring. However, the heat can be supplied easily by a penetrating convective system, where a permeable boundary advances into the hot rock by a process of cooling and cracking. Front advance rates of between 0.2 m yr^{-1} (20 km^2) and 20 m yr^{-1} (0.2 km^2) liberate the required amount of heat and imply system lifetimes between 30 000 and 300 yr based on a penetration depth of 6 km. Prior theory of a one-dimensional convective system derives front advance rates that are of the right general order of magnitude, but somewhat too high (60–200 m yr^{-1}) (Lister, 1974). A qualitative consideration of the three-dimensional case demonstrates that the partially cooled thermal boundary layer has to act as a pressure barrier between the permeable and hot regions. Substantially reduced penetration rates and cracking temperatures are indicated compared to the one-dimensional case, and are likely to be of the right order of magnitude. A simple numerical calculation shows that the probability of finding an active geothermal area along a zone of crustal rifting is about 0.025 per linear km for each cm yr^{-1} of spreading rate.

INTRODUCTION

Geothermal systems are places where considerable volumes of hot water and/or steam make their way toward ground surface. The upwelling may take the form of a thermal plume in rock of fairly homogeneous permeability, such as at Wairakei, New Zealand (Banwell, 1961; Modriniak and Studt, 1959), or the surface manifestation may be the leakage of hot water through a fault zone in largely impermeable rock. In all cases, our knowledge of the overall circulation is limited to borehole data for the uppermost few hundred meters. From what depth the heated water rises is not known, and the mechanism by which it is heated is even more obscure. If a localized swarm of microearthquakes beneath a geothermal area may be taken as a guide, the depth of the system can be about 7 km, at least for one area in Iceland (Ward, 1972). It is clear that the engineering data taken in the exploitation of known geothermal fields cannot provide any information about processes occurring at depths of several km. It is equally clear that some knowledge of how the hot water is generated would be helpful in predicting areas of geothermal promise, and also in estimating the useful lifetimes of both natural and exploited systems. This paper is a form of progress report on fundamental theory, originally developed for application to the problems of heat flow on oceanic ridges (Lister, 1974).

BACKGROUND DISCUSSION

An active geothermal area, such as the one in the Taupo, New Zealand, region, has a natural heat output of about 400 MW and is 5–10 km from its neighbors (Elder, 1965). If the active area were 5 km in diameter, the heat output would be equivalent to a heat flow of 485 μcal cm^{-2} s^{-1} (hfu). In rock of conductivity $k = 0.007$ cal cm^{-1}°C^{-1} s^{-1}, this flux requires a thermal gradient of 7000°C km^{-1} to drive it by conduction. If the hot water were heated by conduction from nearby magma, the separating wall could be only 180 m thick, and the thin wall would have to be present over the whole 5 km diameter area. This is obviously unlikely in steady state, and the transient cooling of a 5 km diameter magma surface is little better as a solution. If a flat rock surface at, say, 1250°C is cooled suddenly to 400°C by the percolating water, the temperature distribution in the rock is given by $T = T_2 + (T_1 - T_2) erf\, 1/2z\, (\kappa t)^{-1/2}$ (Carslaw and Jaeger, 1959), where κ is the thermal diffusivity of the rock (0.009 cm^{-2} s^{-1}). The heat flow through the surface is given by $\kappa (T_1 - T_2)(\pi\kappa t)^{-1/2}$ and decays to 485 hfu in a mere 170 years. Gradual introduction of new magma surface over the 5 km diameter area would extend the high output time by only a factor of 2. Since the thermal areas have been active for considerable periods of time, probably on the order of 10^5 years, the heat output can be maintained only by one of two mechanisms—the continuous introduction of new magma surface next to a permeable formation, or the penetration of the circulating water into the magma or hot rock body itself. Simple conductive heat-exchanger calculations, such as those of Lowell (in press), may explain isolated hot springs in geologically stable areas, but cannot be applied to the highly active systems of interest for power generation.

As far as is known from the history of volcanic eruptions, magma movements occur episodically at intervals of 10^2 to 10^4 yr, and geothermal activity perists for considerable periods after the last evidence of magma movement. This pattern is consistent with the physics of magma injection:

since magma can only flow above the solidus temperature, and rarely appears near the surface significantly above that temperature, injection or eruption must be rapid to avoid blockage of the conduits by cooled material. The most likely mechanism for the heating of the water, and the cooling of the hot rock, is therefore the penetration of the percolating water into the hot body itself. Rock shrinks as it is cooled, and the shrinkage can permit the opening of cracks and the flow of hydrothermal fluids. If the rock changes in temperature from 1250°C to 400°C as the permeable boundary advances, the heat liberated per unit area per unit time is simply the product of the front velocity u, the temperature change ($T_1 - T_2$) and the heat capacity of the rock ρc, 0.77 cal cm^{-3}. Thus, $q = u\rho c(T_1 - T_2) = 2080\ u$ hfu when u is in m yr^{-1}. A front velocity of only 0.23 m yr^{-1} would be needed over a 5 km diameter circle to generate a thermal output of 400 MW, and would take about 3×10^4 years to penetrate to 7 km. Conversely, the output could be generated for a short time by front advance over a 500 m diameter area at 23 m yr^{-1}. Which one is more likely depends on the size of the magma body and the frequency of magma injection: the upwelling plume about 1 km in diameter near the surface at Wairakei (Banwell, 1961) is consistent with sources in this size range. In any case, the concept of the advance of a permeable cooling front removes the difficulty of providing the heat by conduction through rock.

QUALITATIVE THEORY

The One-Dimensional Model

The problem of water penetration into hot rock is complex because it involves the interaction of convection with mechanical shrinkage, rock creep, and rock cracking. None of the individual phenomena are well understood, so that any theory that attempts to combine consideration of all the processes involved into a solution of the whole problem must needs be both highly idealized and tentative. One attempt at this is based on a one-dimensional idealization, where a plane cracking front advances at a characteristic velocity u into the hot rock (Lister, 1974). The mechanical situation is idealized into the stressing of infinite flat layers of rock that remain of constant horizontal dimensions until they crack: thermal shrinkage is exactly balanced by creep. When cracking occurs, the permeability of the rock pattern is a function of the crack opening due to further cooling, and of the crack spacing. This last, the same as the rock column size, is controlled by the advective cooling around the columns. A radial temperature gradient causes hoop tension to develop in the column, and if this is large enough, the column will split. The heat-transport rate through the permeable system above the cracking front is calculated from the experimental results for flat porous cells. Cracking temperatures somewhere between 600°K and 1000°K are plausible on the basis of what (little) is known about rock creep, and the results as presented by Lister (1974) are startling. For a cracking temperature of 800°K, the front velocity is about 60 m yr^{-1}. This is somewhat too high to be reasonable for known geothermal areas, at least on the above estimates, but it is so high that the idealized conditions used in the original calculations (complete horizontal symmetry) cannot exist even on the fastest-spreading mid-ocean ridges.

Modification of the One-Dimensional Model

An indication of what may happen in a more restricted situation can be gleaned from Figure 1, where the convection regime of an initially one-dimensional case is considered in two dimensions. Flow near the lower boundary consists of the main horizontal advection of the convecting fluid, together with a small vertical component of advection due to the advance of the boundary itself into the hot rock. Stabilization of the crack spacing is associated with the latter advection only, and this causes a modification in the results of Lister (1974), where cooling was assumed to be entirely by flow in the cracks a short distance above the cracking front. The modification is not large—for the median cracking temperature of 800°K, u is 200 m yr^{-1} instead of 61, while the crack spacing is 7.6 cm instead of 4.8 cm (Lister, in preparation). More importantly, the cooling ability of the basal boundary layer decreases as it flows horizontally away from the stagnation zone below the downwelling plume. This tends to make the cracking front advance faster where cooling is vigorous than where it is less so, and the planar boundary of the one-dimensional model becomes wavy as depicted in Figure 1.

As soon as the system ceases to be truly one dimensional, the simplicity of the mechanics utilized in previous analysis (Lister, 1974) disappears. The basic pressure relationships can be summarized readily: below the cooled zone, the lithostatic pressure is uniform ("hydrostatic"). Above the cooled zone, in the permeable material full of sub-vertical cracks, the vertical stress is lithostatic, but the horizontal stress is due only to the head of the fluid fill, and is much lower. *Qualitatively*, this produces forces acting in the directions of the arrows in Figure 1: they tend to aid cracking in the zone that is lagging, and reduce cracking in the region that is ahead. Unfortunately for a simple analysis, the cracks that are suppressed and aided are not in the direction of flow in the two-dimensional diagram—they are perpendicular to the paper. This means that the two-dimensional pattern cannot be stabilized at some value of u where the increase in cracking temperature, due to tensile assistance to cracking, compensates for the warming water in the

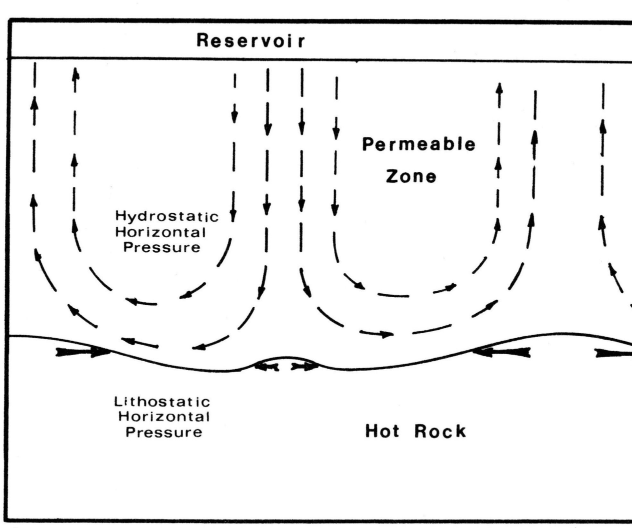

Figure 1. Diagram of perturbations to the one-dimensional model of water penetration induced by cell geometry. Dashed arrows indicate water flow; solid arrows indicate differential pressure forces.

boundary layer and maintains the heat transfer rate. Instead, a breakdown to three-dimensionality is inevitable unless the cell pattern is so unstable that one-dimensionality is maintained statistically. This last is unlikely for steady convection, because upwelling plumes tend to stabilize over the hottest parts of the lower boundary (compare Elder, 1965), and the boundary topography shown in Figure 1 would also tend to stabilize the cell positions where they began.

Toward a Three-Dimensional Model

If the one-dimensional water penetration rate is too fast to exist in nature, and would in any case break down into three-dimensional patterns if allowed to propagate, it may be as well to consider an intrinsically three-dimensional case that could occur in a geothermal region. Suppose a fault or other tectonic event gives cold ground water access to a recent intrusion of limited lateral extent (Fig. 2a). Initial flow is along the newly permeable boundary, but penetration is most rapid near the cold water entrance because $u = \frac{\partial T}{\partial r} \cdot \frac{\kappa}{T_1 - T}$ (Lister, 1974) at some temperature T where the permeability reaches a value comparable to that of the feed system. The thermal gradient is that at the edge of the fluid boundary layer, and is highest when the boundary layer is young and contains mostly cold fluid. If the boundary layer is uniformly two dimensional, the thermal equation is analogous to that applicable to spreading oceanic lithosphere, and the edge gradient is rapidly asymptotic to a $(t)^{-1/2}$ law (Parker and Oldenburg, 1973). Penetration soon becomes more downward than outward (Fig. 2b) as the permeable region continues to expand, and a quasi-steady state may be reached eventually when the permeable region has filled the entire upper cross section of the intrusion (Fig. 2c).

Stress analysis is more complex than for one-dimensional penetration, but simpler than for the breakup of a flat permeable layer. A single permeable region containing subvertical cracks supports the full lithostatic load in the vertical direction, but contains only the pore pressure in the horizontal direction. At a substantial depth below ground surface, this pressure is considerably less than the quasi-hydrostatic pressure in the intrusion, presumed hot enough to be relatively soft if not partially molten. The permeable region must surround itself with a "pressure case" of uncracked rock that is cool enough to creep only slowly under a high stress. The situation is sketched in Figure 3 for the simplified case of a section of vertical cylindrical wall. Stress goes from hydrostatic in the undisturbed material, to highly compressive circumferentially in the "wall," and then tensile just before the rock cracks. Qualitatively, both the cracking temperature and the maximum permissible value of u are substantially reduced compared to the one-dimensional case. The material must become rigid enough to sustain much more than the simple overburden pressure minus pore pressure at a low creep rate: how much more depends on the thickness of the wall and the radius of curvature. Moreover, the wall is subject to continuing creep that slowly compresses the permeable region and continually reduces the temperature at which tension can be generated to propagate the cracks.

It might be thought at first that the permeable region would collapse rapidly because the "wall" region sustaining the external pressure is continually shrinking as it cools.

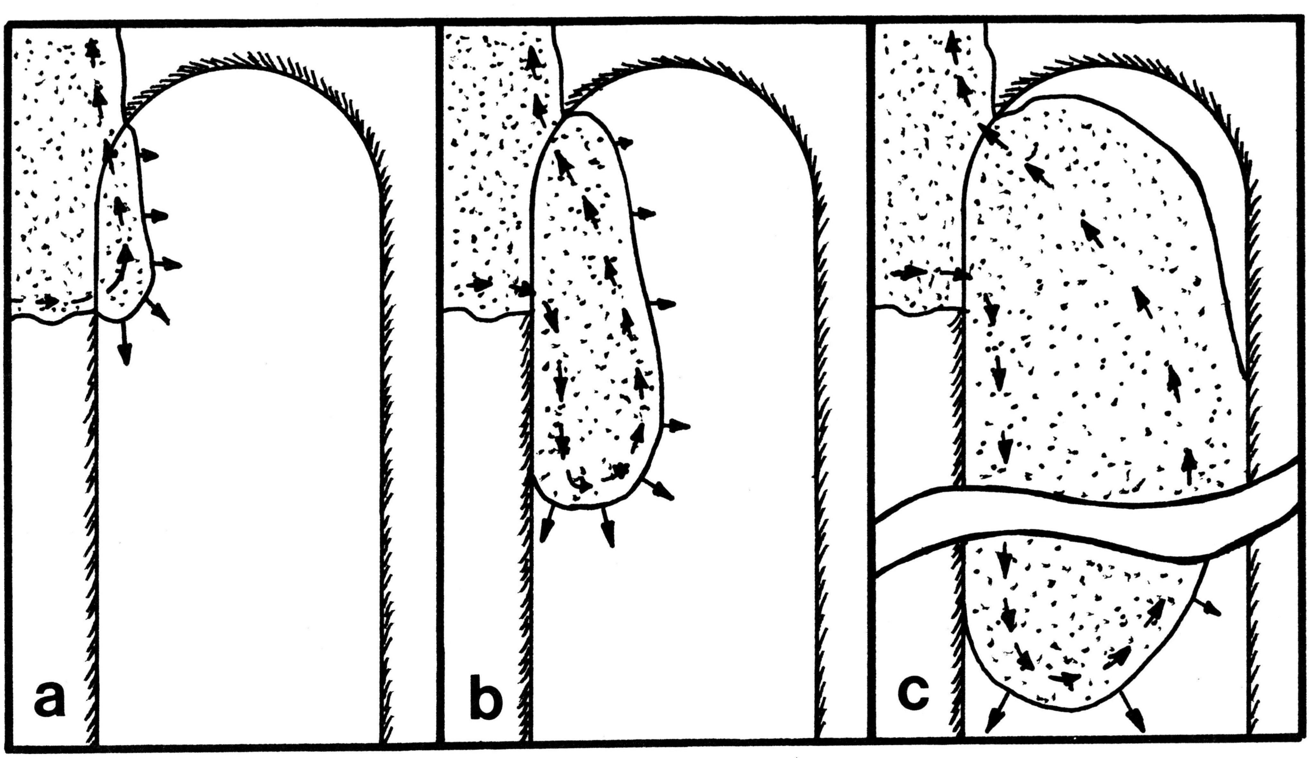

Figure 2. a) Initiation of water penetration into a hot intrusion at a faulted boundary with permeable material. Qualitative variations in the velocity of cracking front advance are indicated by the arrows. b) Actively penetrating phase of the geothermal system not limited by intrusion size or other inhomogeneities. c) Late stage of activity in an intrusion of moderate size where the upper part is fully cooled to the water temperatures.

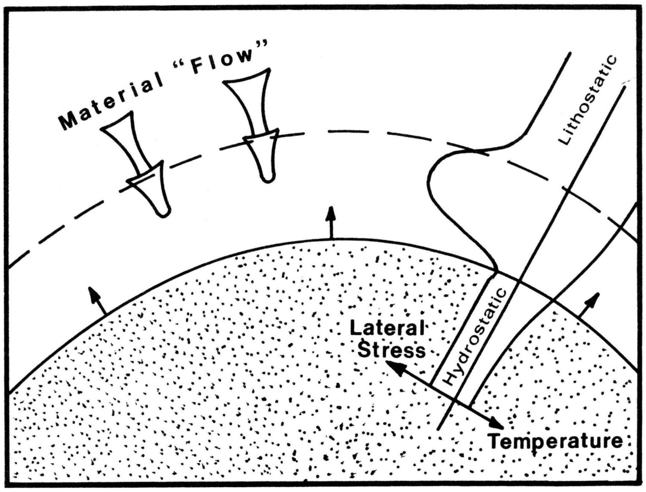

Figure 3. Diagram of the stresses in the boundary region of the permeable zone, idealized as a section of a vertical cylinder. The curved wall of partially cooled rock acts as a barrier for the horizontal pressure difference between the permeable and uncooled regions. Arrows indicate a way of visualizing the growth of the cylinder as the flow of material through the boundary. This does not disturb the static "pressure case" analysis of the boundary region.

However, the shrinkage is balanced exactly by the outward migration of the cooled zone, and this is indicated in Figure 3 by the arrows representing the "flow" of material through the thermal boundary layer. If it were not for the gradual increase in radius of curvature of the cooled zone, the picture in Figure 3 could be considered as static in time—material flows through it but the temperature and stress distribution remain unchanged. Aside from the second-order effect of radius change, the overall permeability loss can be estimated from the stress-temperature distribution considered as static. Rock stiffens rapidly as temperature is reduced, how rapidly depending upon the activation energy of the dominant creep process in the material. Lister (1974) made use of transient creep data for "wet Llerzolite" (Carter and Ave'Lallement, 1970) and estimated cracking temperatures of about 800°K for the one-dimensional model. In fact, the intrusive rock can probably be considered as "dry" by laboratory standards, and, not only is the intrinsic creep of dry rock lower at high temperatures, but the activation energy is much higher: 120 Kcal mole^{-1} rather than the 80 assumed by Lister (1974). Thus the paradox that unrealistic rock properties had to be assumed to produce reasonably low cracking temperatures is removed in the three-dimensional model: the rock must be very stiff in the stressed zone for the permeable system to have a reasonable life.

Water Temperatures and Limits on Penetration

One of the less reasonable results of the one-dimensional model was the relatively low water temperature obtained for a plausible cracking temperature by maximizing the penetration rate of the convective system. Another difficulty was finding a suitable mechanism for the termination of the penetration of the system at a reasonable depth. Both these problems are essentially eliminated by the three-dimensional model. If water access is restricted to a system such as in Figure 2b or c, it grows rapidly at first because the surface area of hot rock is small. As the surface area rises, the temperature of the peripheral upwelling water approaches the cracking temperature of the rock, itself slowly decreasing with time due to the stress/creep history of the system. Meanwhile the active bottom end of the system, always "young" in stress history, continues to propagate at much the same rate as long as cold water can obtain easy access. Thus the temperature of the hot water converges with the cracking temperature associated with a very slowly propagating cylindrical front, unless the size of the intrusive body is less than the natural size of the system (Fig. 2c).

In addition to the primary mechanism of static fatigue advocated by Lister (1974) to terminate water penetration, the three-dimensional "cold finger" model has another mode of disruption. Convection cells prefer to be approximately equidimensional (Lapwood, 1948), and the long thin cell should eventually break down in spite of the strong geometric and wall-temperature stabilization. When this happens, temperatures in the lower part of the system will rise rapidly, greatly reducing the permeability and accelerating the onset of static fatigue. At this point the hot, highly stressed rock will shatter, generating microearthquakes and signalling the death of the geothermal system through the catastrophic drop in permeability (Lister, 1974).

CONCLUSIONS

It has been shown that the sustained high output of major geothermal systems is not consistent with the conductive extraction of heat from a magma body of reasonable size. The output *is* consistent with the penetration of the permeable region into the hot body itself at rates between 0.2 and 20 m yr^{-1}, depending upon the surface area of the advancing front. System lifetimes between 300 and 30 000 years are plausible, based on a maximum penetration to 6 km and the above penetration rates. A one-dimensional model of water penetration into hot rock (Lister, 1974) predicts rates of between 60 and 200 m yr^{-1} for a cracking temperature of 800°K: they are somewhat too high but of the right general order of magnitude. Qualitative consideration of a three-dimensional penetrating convectional system suggests that the front velocities will be reduced considerably into the right order-of-magnitude range.

Tectonic action is required to expose the hot body to fluid in a nearby permeable region, and is most likely to be associated with the active phase of magma injection. Highly active systems are therefore not likely to persist for geologically long times after volcanic activity in a region has ceased. Storage systems capped by impermeable rock may occur in some places, but, per unit volume, they store only about a tenth of the thermodynamically useful heat energy available in an uncooled intrusive body. Unless extraordinarily large, they are unlikely to be of more than local economic significance. Useful levels of power generation can be obtained for extended periods of time only where naturally active systems exist, or an active system can be started artificially. The places where frequent magma emplacement is expected are: subduction zones in an active volcanic phase; continental rift zones; and the major oceanic spreading systems. Known examples of these categories are respectively Italy and Japan; New Zealand and Baikal (USSR); and Iceland. The basin and range province of the western USA might be considered a hybrid between the latter two categories—spreading that is as slow as in a continental rift zone (Baikal or East Africa) but associated with easy crustal extension more like that of an oceanic ridge.

The probability of finding an active geothermal area at a spreading or rifting zone can be related simply to the spreading rate. Taking a median system 1.5 km in diameter and with a 3000-year life, one may be expected at the same place every 110 000 years (allowing for circularity) for a 1 cm yr^{-1} spreading rate. This is a probability of 0.025 per km of rift per cm yr^{-1} of spreading rate (full rate, both sides). Thus, an active geothermal area may be expected every 20 km on the Mid-Atlantic Ridge, every 3 km on the fast-spreading East Pacific Rise (almost continuous activity), and every 100 km or so on a slow-spreading continental rift zone. The probability of finding an area per linear km is of course independent of the areal size and related lifetime of the geothermal area, and dependent only on the mean power level, depth of penetration, and rock supply temperature. It is interesting that the high concentration of active areas along the New Zealand volcanic zone (Elder, 1965) implies an opening rate of 2 cm yr^{-1} or more. This region and the almost comparable but shorter spreading zone in the Salton trough, California, might be considered prime continental areas for sustained high-level power generation.

ACKNOWLEDGEMENT

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