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## ABSTRACT

A Simple method to estimate both flow time of water traveling through a path connecting two wells and the Peclet number from well-to-well tracer tests is presented. For this analysis, only the first tracer arrival time and the peak concentration time of the tracer response at an observation well are used. The method was tested using actual field data, and the resuts show that this method is simple, accurate, and, with only a few limitations, can provide reliable information about water flow patterns and traveling time.

## INTRODUCTION

In the geothermal field, well-to-well tracer tests are of ten carried out to evaluate the interference of reinjection wells with production wells. The traditional method of analyzing the tracer response is to obtain the desired parameters by matching an observed tracer concentration/time curve with a theoretical one by the trial and error method. A simpler method to estimate both the average water traveling time and the Peclet number (water flow velocity times average water traveling distance/coefficient of hydrodynamic dispersion, ux/D) using only the first arrival time and the peak concentration time, is presented in this paper. First of all, some concentration/time behaviors are calculated, and from the results of these calculations, the relationships between the water traveling time and both the first arrival time and the peak concentration time with different Pe values are derived. The applicability of this method was tested by applying it to some tracer tests in the geothermal fields.

## MOVEMENT OF A TRACER IN A SEMI-INFINITE AQUIFER

The partial differentioal equation governing the tracer distribution in a semi-infinite aquifer can be reduced to

$$
\begin{equation*}
\frac{\delta \mathrm{C}}{\delta \mathrm{t}}=\mathrm{D} \frac{\delta^{2} \mathrm{C}}{\delta \mathrm{x}^{2}}-\mathrm{u} \frac{\delta \mathrm{C}}{\delta \mathrm{x}} \tag{1}
\end{equation*}
$$

Solving (1) for the following initial and boundary conditions

| $\mathrm{t}=0$ | $\mathrm{C}=\mathrm{Co}$ | at x |
| :---: | :---: | :---: |
| $\mathrm{t}_{1} \equiv \mathrm{t}>0$ | $\mathrm{C}=\mathrm{C}_{\text {I }}$ | at $\mathrm{x}=0$ |
| $\mathrm{t}>\mathrm{t}_{1}$ | $\mathrm{C}={ }^{\text {Co }}$ | at $\mathrm{x}=0$ |
| t > 0 | $\mathrm{C}=\mathrm{Co}$ | at $\mathrm{x}=\infty$ |

$$
\begin{align*}
& \text { yields } \\
& C(x, t)=C_{B}+\frac{C_{1}-C_{\theta}}{2}\left[\left(\operatorname{erfc}\left(\frac{x-u t}{2 \sqrt{D t}}\right)+\exp \left(\frac{u x}{D}\right)\right.\right. \\
& \left.\quad \times \operatorname{erfc}\left(\frac{x+u t}{2 \sqrt{D t}}\right)\right\}-\left\{\operatorname{erfc}\left(\frac{x-u\left(t-t_{1}\right)}{\left.2 \sqrt{D\left(t-t_{1}\right.}\right)}\right)\right. \\
& \quad+\exp \left(\frac{u x}{D}\right) \times \operatorname{erfc}\left(\frac{x+u\left(t-t_{1}\right)}{\left.\left.\left.2 \sqrt{D\left(t-t_{1}\right)}\right)\right\}\right]} \quad(2)\right. \tag{2}
\end{align*}
$$

and (2) can be rewritten in a form of relative concentration, $\mathrm{C}^{*}$, as a function of $\mathrm{a}, \mathrm{Pe}$, and t ,

$$
\begin{align*}
C^{\cdot} & =\frac{1}{2}\left[\operatorname{erfc}\left(\frac{a-t}{2 \sqrt{(a / P e) t}}\right)+\exp (P e)\right. \\
& x \operatorname{erfc}\left(\frac{a+t}{2 \sqrt{(a / P e) t}}\right)-\left\{\operatorname{erfc}\left(\frac{a-\left(t-t_{1}\right)}{2 \sqrt{(a / P e)\left(t-t_{1}\right)}}\right)\right. \\
& \left.\left.+\exp (\operatorname{Pe}) \operatorname{erfc}\left(\frac{a+\left(t-t_{1}\right)}{2 \sqrt{(a / P e)\left(t-t_{1}\right)}}\right)\right\}\right] \tag{3}
\end{align*}
$$

where

$$
\operatorname{erfc}(y)=\frac{2}{\sqrt{\pi}} \int_{y}^{\infty} \mathrm{e}^{-z 2} \mathrm{dz}
$$

and

$$
C_{1}=C_{0}+\frac{1}{G i v i t_{1}} \times 10^{3}
$$

## CONCEPTUAL MODEL OF TRACER RETURN AND WATER FLOW

To apply (2) to a well-to-well tracer test, a conceptual flow model like the one shown in figure 1 is considered: a tracer solution released from the injection well, RW, returns with a return ratio of $f$, through the path to the production well, PW, mixing water of concentration Co, from other sources. $C_{T}, C$ and $C s$ are tracer concentrations at the released point, at the bottom and at the surface of the production well, respectively, and $x$ is the average water traveling distance through the path between the two wells.

## MODEL CALCULATION OF THE TRACER RESPONSE

To evaluate the effect of a and Pe on the concentration/time behavior, the model calculation of the tracer response is made by changing both a and Pe , where $\mathrm{t}_{1}$ is fixed as 0.2 (see figures 2 and 3). It is clear from these figures that as a and Pe become smaller, the first tracer arrival time occurs earlier, and also the concentration/time curve becomes both sharper and with a higher peak. More importantly, for each combination of different values of a and Pe , we


Figure 1. Conceptual model of tracer return and water flow
obtain only one concentration/time curve combining the first arrival time and the peak concentration time. This makes it possible to estimate a and Pe by using only these two field time data. Figures 4 and 5 show the relationships between $a$ and the first arrival time, $t_{0.01}$ (here chosen as the time of $1 \%$ of the peak concentration for practical reasons), and between a and the pcak concentration time, $t_{m}$. From these figures, it is clear that both $t_{0.01}$ and $t_{\text {m }}$ have nearly linear relations, although the slopes are different. Figure 6 show the relationship between $t_{0.01}$ and $t_{m}$. For this analysis, either figure 6 or both figures 4 and 5 together can be used.


Figure 2. $C$ versus $t$ curves at $P e=20$


Figure 3. $C$ versus $t$ curves at $a=200$

To evaluate the effect of $t_{1}$ on $t_{0.01}$ and $t_{m}$, the latter two are calculated by assigning the following values: $t_{1}=0.2,1.0$ and $3.0 ; a=10$ and 500 ; and $\mathrm{Pe}=5$ and 100 . The calculation, tabulated in table 1 , indicate that, within the arranged range of $t_{1}$ values, $t_{0.01}$ and $t_{m}$ do not vary greatly.

Table 1. Effect of $t_{1}$ on $t_{0.01}$ and $t_{m}$

|  |  | $t_{1}(\mathrm{~h})$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 0.2 | 1.0 | 3.0 |
| $\begin{array}{rr} a & =10 \\ P e & =5 \end{array}$ | $\begin{aligned} & t_{0.01} \\ & t_{\mathrm{m}} \end{aligned}$ | $\begin{aligned} & 1.4 \\ & 5.8 \end{aligned}$ | $\begin{aligned} & 1.6 \\ & 6.2 \end{aligned}$ | ---- |
| $\begin{array}{r} a=10 \\ \mathrm{Pe}=100 \end{array}$ | $\begin{aligned} & t_{0} .01 \\ & t_{\mathrm{m}} \\ & \hline \end{aligned}$ | $\begin{aligned} & 6.4 \\ & 9.8 \end{aligned}$ | $\begin{array}{r} 6.7 \\ 10.2 \end{array}$ | $\begin{array}{r} 7.0 \\ 11.3 \end{array}$ |
| $\begin{array}{r} a=500 \\ \mathrm{Pe}=5 \end{array}$ | $\begin{aligned} & t_{0.01} \\ & t_{m} \\ & \hline \end{aligned}$ | $\begin{array}{r} 65.5 \\ 288.2 \end{array}$ | $\begin{array}{r} 65.9 \\ 283.1 \\ \hline \end{array}$ | $\begin{array}{r} 66.9 \\ 284.3 \\ \hline \end{array}$ |
| $\begin{array}{r} a=500 \\ \mathrm{Pe}=100 \end{array}$ | $\begin{aligned} & t_{0} .01 \\ & t_{\mathrm{m}} \end{aligned}$ | $\begin{aligned} & 317.3 \\ & 484.8 \end{aligned}$ | $\begin{aligned} & 317.7 \\ & 486.6 \end{aligned}$ | $\begin{aligned} & 318.7 \\ & 487.2 \end{aligned}$ |



Figure 4. $\mathrm{t}_{0.01}$ versus a


Figure 5. $\mathrm{t}_{\mathrm{m}}$ versus a


Figure 6. $\mathrm{t}_{\mathrm{m}}$ versus $\mathrm{t}_{0.01}$

## PROCEDURES

The basic procedures to estimate a and Pe are as follows:

1. read a and Pe in figures 4 and 5 at observed $\mathrm{t}_{0.01}$ and $\mathrm{t}_{\mathrm{m}}$
2. plot these values of a versus $P e$ on the same graph and connect them with smooth lines
3. the cross point of the two lines will give the desired values of a and Pe .

To verify the results, the concentration/time curve is simulated by substituting the obtained a and Pe into (2), and it is then compared with the observed data by applying the concentration peak of the simulation curve with that of the data.

If more than one peak are clearly recognizable, more than one path are assumed between the two wells. In such a case, first, the estimation of $\mathrm{a}_{1}$ and $\mathrm{Pe}_{1}$ for the first path is made from the first arrival and the first concentration peak times, $t_{0.01}$ and $t_{m, 1}$. Then, by subtracting the first simulation curve with $a_{1}$ and $\mathrm{Pe}_{1}$ from data, the estimation of $\mathrm{a}_{2}$ and $\mathrm{Pe}_{2}$ for the second path is made from the first arrival and the second peak concentration times, $t_{0.01,2}$ and $t_{m, 2}$, of the residue. The total simulation curve, that is, the sum of the first and the second one, is compared with actual data for the verification of the result.

As described above, in a multiple-peak case, $\mathrm{a}_{1}$ and $\mathrm{Pe}_{1}$ can be estimated only if $\mathrm{t}_{\mathrm{m}, 1}$ of the data is equal to the peak concentration time of the first path (see figure 7). However, they are not always coincident as seen in figure 8, where the $t_{m, 1}$ value of the data is slightly larger than that of the first path, because of the effect of the concentration/time behavior through the second path. If this method is applied to the latter
case, an error in the result of the estimation arises. The accuracy of the estimation will be evaluated by comparing the simulation curve with the observed data.


## APPLICATION

This method was applied to the tracer
response in some well-to-well tracer tests
conducted in geothermal fields. Shown in figures
9 and 10 are comparisons of the data with
simulated concetration/time curves. In these
figures, dots indicate the observed data, and
solid lines the simulation curves, while the 'x'
marks in figure 10 show the residues obtained by
subtracting the first simulation curve from the
data. Both simulated curves are closely compared
with the data: one path in the former case and two
paths in the latter.

SUMMARY
The proposed method to estimate the average water traveling time and the Peclet number by using the first tracer arrival and the peak concentration time, is much simpler than the traditional method, and can be used not only in a one concentration peak cases but also in multiplepeak cases. If, however, the first peak

Fukuda et al
concentration time is affected by a following concentration/time behavior, the application of this method results in an error. The solution for such a case is left to future studies.


Figure 9. Tracer test data (by Ito et al. 1977) and result of analysis


Figure 10. Tracer test data. (by NEDO, 1990) and results of analysis

NOMENCLATURE

```
a : water traveling time (=x/u) [h]
C : observed tracer concentration [mg/1]
CI
    [mg/]
Co: background tracer concentration [mg/1]
C
        production well [mg/1]
D : coefficient of hydrodynamic dispersion [m}\mp@subsup{m}{}{2}/\textrm{h}
G1: flow rate of injected water [kg/h]
Gs: flow rate of produced steam [kg/h]
Gw: flow rate of produces water [ [kg/h]
I : amount of tracer injected [kg]
Pe: Peclet number (=ux/D)
t : time [h]
t
u}\mathrm{ : water flow velocity [m/h]
Vi: specific volume of injected water [ [m}
Vs: specific volume of produced stcam [ [m}
Vw: specific volume of produced water [m}3/\textrm{kg}
x : average water traveling distance [m]
```


## REFERENCE

Ito, J., Kubota, Y. and Kurosawa, M. (1977) Geothermal water flow in Onuma geothermal reservoir (in Japanese), Chinetsu, 4, 3, 139-151.

NEDO(New Energy and Industrial Technology Development Organization) (1990) Report on
'Water Reinjection Mechanism (in Japanese)'

## APPENDIX

The return ratio, $f$, of injected water to the production well is essentially given by

$$
f=\frac{G r}{G i} \quad(A-1)
$$

where Gr is the flow rate of the returning water.
It is also obtained from the tracer return as

$$
\begin{equation*}
f=\frac{G v V_{v}}{G i V i} \frac{C s-C_{0}}{C-C_{0}}=\frac{G V V_{V}}{G i V i} \frac{C_{s^{\prime}}}{C^{\prime}} \tag{A-2}
\end{equation*}
$$

where Cs'=Cs-Co and $C^{\prime}=\mathrm{C}-\mathrm{Co}$
or

$$
f=\frac{G V V_{v}}{G i V_{i}} \frac{C s m^{\circ}}{C_{z^{*}}}=\frac{G V V_{v}}{G_{i} V_{i}} \frac{C_{s m^{\circ}}}{C_{i}^{*} C_{m}^{\circ}} \quad(A-3)
$$

where $\quad C_{I}{ }^{\prime}=C_{I}$-Co and a subscript means a peak value. Because $G w, G i, V w, V i$ and $C^{\prime} I$ can be assumed to be constant, $(A-3)$ is rowritten as

$$
C^{\circ} m=K \operatorname{Csm} \quad(A-4)
$$

where

$$
K=\frac{G\|V\|}{i G i V i C_{i}}
$$

is constant. $K$ can be obtained by comparing peak of calculated relative concentration, with the peak of the observed data, Csm'. using the $k, C^{*}$ can be re-drawn as $C^{\prime}$ or $C$.

