INTERFERENCE TEST ANALYSIS METHOD USING THE KALMAN FILTERING AND ITS APPLICATION TO THE TAKIGAMI GEOTHERMAL FIELD, JAPAN

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ABSTRACT

A method to analyze multi-well variable-flowrate interference test data using the line source solution based on an infinite reservoir is described. This presented method employs the Kalman filter technique which provides the best estimates of the reservoir parameters, transmissivity(T) and storativity(S), at every moment when the new pressure value becomes available. Measured pressure data of two observation wells from the interference test conducted for nine months at the Takigami geothermal field, Japan, have been analyzed. The results indicate that the poor initial guesses of parameters will not lead to the divergence of calculations if the ratio of the guesses(S/T) is given to be one or less. The parameter estimation behavior seems to reflect the effects of the faulty pressure data caused by malfunctioning of measurement apparatus and by the fluid flow which dose not satisfy the infinite reservoir model.

INTRODUCTION

Interference well tests have an advantage in providing information on the reservoir parameters(transmissivity and storativity) in the area where observation and active wells are located. Furthermore, the presence of a hydrologic boundary can be detected by analyzing the test data. These physical features of the reservoir are of great importance, for example, in constructing a numerical model of the reservoir.

The test periods may last as long as several months, resulting in a large amount of pressure data from observation wells and changes in flowrate of active wells. Therefore, parameter estimation methods by the use of computers will be a powerful tool for reservoir engineers. A conventional computer assisted parameter estimating method uses the line source solution, superposing to take account for multiple active wells and their changes in flowrates with time, and employs a non-linear least square method since the solutions are non-linear with respect to the reservoir parameters(McEdwards and Benson, 1981;Arellano et al., 1990).Itoi et al.(1990) developed an on-line analysis method for interference tests using the extended Kalman filtering, and demonstrated its advantages for the use with micro computers with small memory. These methods, however, have problems in convergence when the calculations are started with poor initial guesses. This deficiency may require tedious work to find good initial guesses in the absence of prior information on the reservoir parameters. Sen(1984) developed a method to estimate aquifer parameters from a pumping well test by employing the Kalman filter technique, and revealed that the method will lead to the convergence in spite of the poor initial guesses of parameters. Thus, we modified Sen's work, and developed a method which can improve the disadvantage derived from starting with poor initial guesses. The parameter estimating method using Kalman filtering(Kalman, 1966) updates the preceding estimates as the new measured data becomes available, and provides the best estimates at the present time. Therefore, when the measurement data are collected from a reservoir in which fluid flow does not satisfy the infinite-reservoir model or include errors caused by malfunctioning of measurement apparatus, these effects may affect on the estimation behavior.

This presented method is applied to analyze the interference test data at the Takigami geothermal field, Japan. The results show that the method is weakly dependent on the choice of the initial guesses.

ANALYSIS METHOD OF INTERFERENCE TEST

Pressure response at an observation well caused by production or injection of fluid from an active well can be expressed by the line source solution on the basis of isotropic, homogeneous, and infinite porous type of reservoir. The solution is further superposed to account for the effect of multiple active wells and their flowrates change with time on the pressure response. The variable flow rates can be processed by dividing these into a series of linear segments of flow(McEdwards and Benson, 1981). Thus, pressure change at an observation well at time t_k of k-th step(k=1,2, •, •) can be expressed as

$$\Delta p(t_k) = \frac{1}{4\pi T}\sum_{n=1}^{N}\left(\sum_{j=1}^{J} C_{n,j}\right) \qquad (1)$$

where $\Delta p(t_k)$ is the pressure change at the observation well at time t_k, T is the transmissivity(=kh/μ). N is the total number of active wells, j is the number of flow segments and J is the total number of flow segments prior to the time t_k.
$C_{n,j}$ is expressed as

$$C_{n,j} = A_{n,j} + B_{n,j}(t_k - \tau_{n,j})(1 + u_{n,j})[Ei(-u_{n,j+1}) - Ei(-u_{n,j})]$$
$$- B_{n,j}[(t_k - \tau_{n,j})exp(-u_{n,j}) - (t_k - \tau_{n,j+1})exp(-u_{n,j+1})]$$

$$u_{n,j}=\frac{Sr_n^2}{4T(t_k-\tau_{n,j})}\quad , \qquad u_{n,j+1}=\frac{Sr_n^2}{4T(t_k-\tau_{n,j+1})}$$

where $A_{n,j}$ is the flow rate of j-th segment of n-th active well at time τ_j, $B_{n,j}$ is the inclination of this flow segment, $\tau_{n,j}$, $\tau_{n,j+1}$ are the time when j-th flow segment of n-th active well begins and ends, respectively. $Ei(-u)$ is the exponential function of u, S is the storativity($=\phi ch$), and r_n is the radial distance between n-th active well and the observation well. We define $u(t_k)$:

$$u(t_k)=\sum_{n=1}^{N}u_{n,J+1}=\frac{S}{T}\sum_{n=1}^{N}\frac{r_n^2}{4(t_k-\tau_{J+1})} \tag{2}$$

Eqs.(1) and (2) constitute the observation equations required to form the Kalman filter for parameter estimation. The observation equation is assumed to be linear to form Kalman filtering whereas the equations above are nonlinear with respect to T and S. Therefore, the equations are linearized by applying a method proposed by Sen(1984) who analyzed a pumping well test to estimate aquifer parameters using Kalman filtering under conditions of constant flowrate from a single pumping well.

To simplify the subsequent notation , we denote the time t_k by putting suffix, k, to the variables. Taking logarithms of both side of the equations, and by rearranging, we obtain

$$log\ \Delta p_k - log\left[\frac{1}{4\pi}\sum_{n=1}^{N}\left(\sum_{j=1}^{J}C_{n,j}\right)\right]=-\,log\ T \tag{5}$$

$$log\ u_k - log\left[\sum_{n=1}^{N}\frac{r_n^2}{4(t_k-\tau_{J+1})}\right]=-\,log\ T+log\ S \tag{6}$$

Writing the left side of the equations as y_1 and y_2 leads to the observation equation in a vector form:

$$\mathbf{y}_k = H_k\mathbf{x}_k+\mathbf{v}_k \tag{7}$$

where $\mathbf{y}_k=[y_1,\ y_2]^{\mathrm{T}}$, $\mathbf{x}_k=[log\ T,\ log\ S]^{\mathrm{T}}$

$[\ ]^{\mathrm{T}}$ indicates transposition. The reservoir parameters, T and S, can be assumed to be constant during the test, and hence the state equation can be written as

$$\mathbf{x}_{k+1}=\mathbf{x}_k+\mathbf{w}_k \tag{8}$$

where $\mathbf{v}_k$ and $\mathbf{w}_k$ are the measurement and state noises assumed to be zero-mean, mutually uncorrelated, white noise of covariance matrices V_k and W_k, respectively. H_k is the observation matrix and is constant:

$$H_k=\begin{bmatrix}-1 & 0\\ -1 & 1\end{bmatrix} \tag{9}$$

On the basis of the system expressed by Eqs.(7) and (8), the algorithm of Kalman filtering provides the best estimates of parameters at every moment when the new pressure data is obtained by following equations. We denote that $\hat{\mathbf{x}}_{k/k}$ and $\hat{\mathbf{x}}_{k/k-1}$ are the optimal estimates of $\mathbf{x}_k$ on the basis of the observation data up to the time t_k and t_{k-1}, respectively.

$$\hat{\mathbf{x}}_{k/k}=\hat{\mathbf{x}}_{k/k-1}+K_k(\mathbf{y}_k-H_k\hat{\mathbf{x}}_{k/k-1}) \tag{10}$$

$$\hat{\mathbf{x}}_{k+1/k}=\hat{\mathbf{x}}_{k/k} \tag{11}$$

K_k is the Kalman gain matrix and expressed as

$$K_k=P_{k/k-1}H_k^{\mathrm{T}}\left[H_kP_{k/k-1}H_k^{\mathrm{T}}+V_k\right]^{-1} \tag{12}$$

Superscript -1 denotes inversion. P is the error covariance matrix associated with the optimal estimate:

$$P_{k/k}=P_{k/k-1}-K_kH_kP_{k/k-1} \tag{13}$$

$$P_{k+1/k}=P_{k/k}+W_k \tag{14}$$

Parameter estimation can be started by giving initial values of $P_{1/0}$ and $\hat{\mathbf{x}}_{1/0}$, and recursive calculations from Eqs.(10) through (14) provide the best estimates at each time step. The components of the observation vector include the parameters to be estimated as indicated in Eqs.(5) and (6). Hence these values are substituted by the prior estimates: $\hat{\mathbf{x}}_{k/k-1}$(Sen, 1984). This treatment may lead to fail finding the good estimates at each time step when poor initial guesses are given. This is because the parameters are updated using the present observation vector as indicated in Eq.(10). To overcome this deficiency, we iterate a whole calculation procedure in which parameters are estimated using the pressure data from the beginning up to the last measurement. Then, the final estimates obtained during this iteration are provided as the initial guesses for the next iteration. The calculations are repeated until the discrepancy between the newly obtained final estimates and the previous ones satisfy a given convergence criterion.

INTERFERENCE TESTS AT TAKIGAMI

The Takigami geothermal field is located in the central part of Kyushu Island, Japan. Idemitsu Geothermal Co. Ltd has been undertaking exploration this area since 1978, and a power plant of 25 MW is planning to be constructed within the next several years. A long term interference test was conducted from April to December in 1987 with three production wells. The separated water was all reinjected back into the formation through five reinjection wells. During the tests, downhole pressures had been measured at six observation wells with the pressure measurement system of capillary tube type.

Fig. 1 shows the location map of wells together with geological features of this area. The reservoir is water-dominated, and is divided into two areas by the Noine fault running NNW to SSE as shown in Fig.1. Figs. 2 and 3 show the flowrate history of production and reinjection wells, respectively. Flowrates of production wells were kept almost constant during the test period except Well TT4 whose flowrates varied in a range of 200 to 270 m^3/h. On the other hand, all reinjection wells changed their flowrate with time. Both production and reinjection wells stopped their operation by the time about 2×10^8 min since the start of the test. Since the presence of highly permeable zone connecting two reinjection wells, TT11 and TT15, was confirmed, their flowrates are summed up as a single flowrate in Fig.3.

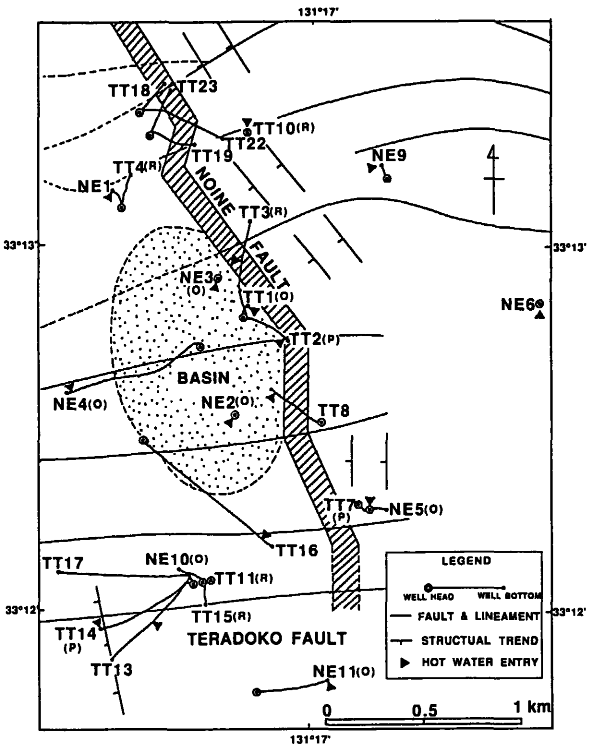

Fig. 1 Location map of wells at the Takigami geothermal field. Signs of (P) indicate the production well, (R) the reinjection well, and (O) the observation well during the interference tests(Hayashi et al., 1988).

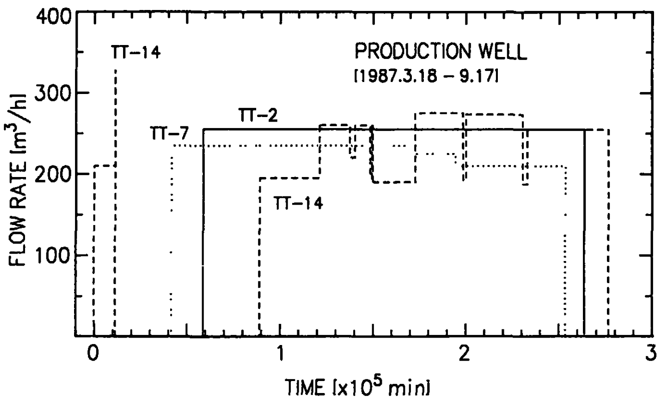

Fig.2 Flowrates of production wells.

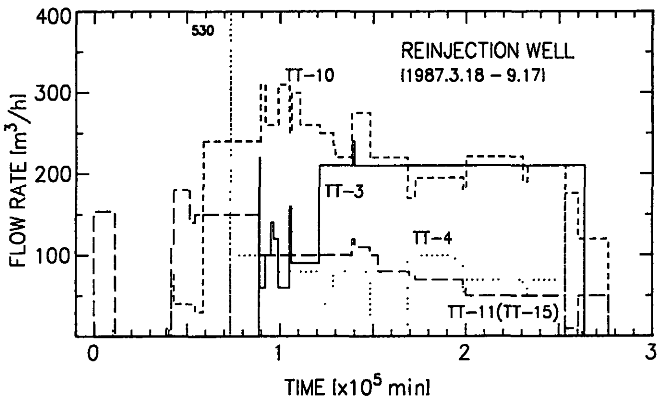

Fig.3 Flowrares of reinjectin wells.

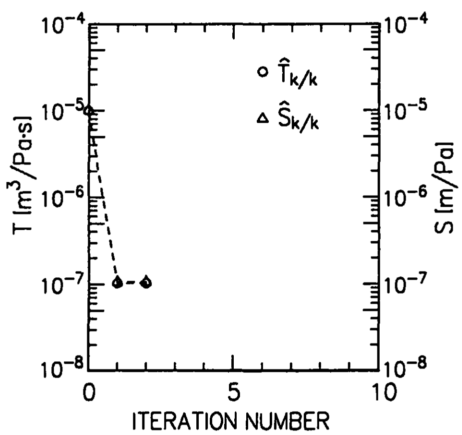

Fig.4 Final estimates of transmissivity(T) and storativity(S) versus the number of iteration of estimation procedure during the analysis of NE4 with an active well TT14.

ANALYSIS OF INTERFERENCE DATA

To demonstrate the use of the method in the preceding section, we have analyzed the down hole pressure data from two observation wells: NE4 and NE11. The pressure data were recorded at an interval of six hours and the total number of data collected for one observation well reached up to 1600 at maximum. The data were, then, processed to extract one from every 10 pressure values and to remove faulty values apparently caused by malfunctioning of the measurement apparatus.

During the course of analysis, initial values of diagonal and nondiagonal components of matrix P are given as 100 and 0, respectively. The matrix elements of covariance of noise vectors are set to be time invariants as 0.01 and 0 for diagonal and nondiagonal parts, respectively.

Well NE4

NE4 is a deviated well and is located in the western most part at Takigami. The number of pressure data after screening is 105 which are used for analysis. Gotoh(1990) reported that the pressure interference between the reinjection and production zones, locating in northern and southern areas respectively, is very little. Therefore, we tried to analyze the pressure data of NE4 for several combinations of active wells: TT2, TT7, TT14, and TT11.

Fig. 4 shows the relation between the final estimates of T and S at each iteration and the number of iteration of estimation procedure when only TT14 is assigned as an active well. Initial guesses of 1×10^{-5} $m^3/Pa\cdot s$ and 1×10^{-5} m/Pa are given for T and S, respectively. The convergence is attained after two iterations. The final estimates obtained during the second iteration are $T=1.04\times10^{-7}$ $m^3/Pa\cdot s$ and $S=1.07\times10^{-7}$ m/Pa.

In order to investigate how good the estimates are at each time step, measured pressure values at time step t_k (p_k) are plotted against pressure values predicted using the estimates at the previous time step: $\hat{x}_{k/k-1}$. Fig. 5 shows the comparison between two kinds of the pressure values during the second iteration procedure. The circle represents the measured pressures and the solid line the predicted ones. The predicted pressure values indicated as $\hat{p}_{k/k-1}$ show a good match with the measured ones except periods around 1×10^5 min. This results imply that the previous estimates are good enough to achieve nearly perfect prediction of the present pressure value, p_k.

Fig.6 shows the change in estimated T and S at each time step during the second iteration. The circle represents the transmissivity and the triangle the storativity. After a slight increase in both estimates at early times, a rapid decrease occurs after the time of 0.4×10^5 min up to 0.9×10^5 min. During this period, TT14 stopped discharging as shown in Fig.2. However, a slight decrease in measured pressure can be recognized in Fig.5. This pressure response may be affected by the start of discharging of other production wells, TT2 and/or TT7. Then, the estimates recovered to the former level and show stable value up to the time 3×10^5 min. They exhibit a slight decrease followed by a small jump as approaching the end of the measurement. The final estimates obtained are $T=1.04\times10^{-7}$ $m^3/Pa\cdot s$ and $S=1.07\times10^{-7}$ m/Pa, and are used to simulate the pressure response of NE4 and the result is plotted in Fig. 7 together with the measured pressure values. A good match is observed. A small discrepancy occurs between the simulated and measured values at later times. This period corresponds to the slight decrease in the estimates of T and S as shown in Fig. 6. As all active wells stopped discharging and reinjecting fluid by about 2.8×10^5 min, this pressure response may be caused by malfunctioning of measurement apparatus.

Next, we analyze the data of NE4 with two active wells, TT2 and TT14, starting with the initial guesses $T=1\times10^{-5}$ $m^3/Pa\cdot s$ and $S=1\times10^{-5}$ m/Pa. Fig. 8 illustrates the estimated T and S versus time. The convergence of parameters is attained after five iterations of estimating procedure. A marked increase in both estimates occur at early times corresponding to the start of discharging from TT2. It can be explained that the previously estimated T and S prior to this jump are small to reproduce the pressure response affected by discharging from TT2. After attaining maximum values, both estimated values seem to decrease followed by stable value up to the time when TT2 stopped discharging. Since an infinite reservoir model is used during this analysis, the effect of the presence of hydrologic boundary can be a cause of a monotonous decrease in estimated T and S. Then, they start decreasing again and quick recovery arises around at the end of the measurement. The final estimates obtained after five iterations are $T=2.10\times10^{-7}$ $m^3/Pa\cdot s$ and $S=1.54\times10^{-7}$ m/Pa. These are 2 and 1.5 times larger than those obtained in the case with TT14. Fig.9 illustrates the comparison between the measured pressure and the simulated pressure using the final estimates of T and S. The fit seems to be fairly

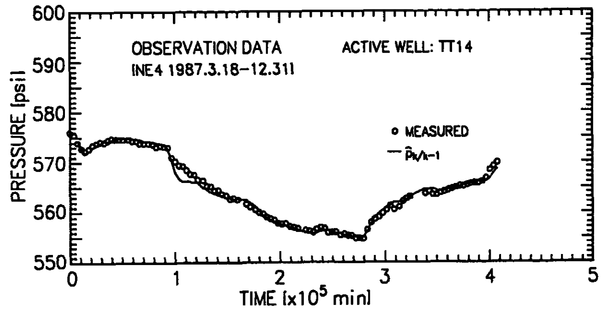

Fig.5 Predicted pressure values are compared with measured ones during the second iteration of parameter estimating calculations.

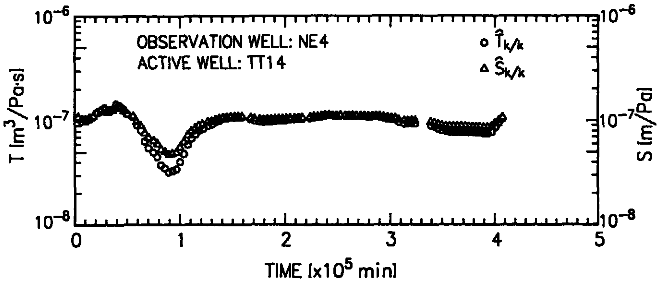

Fig.6 Estimated T and S at each time step versus time .

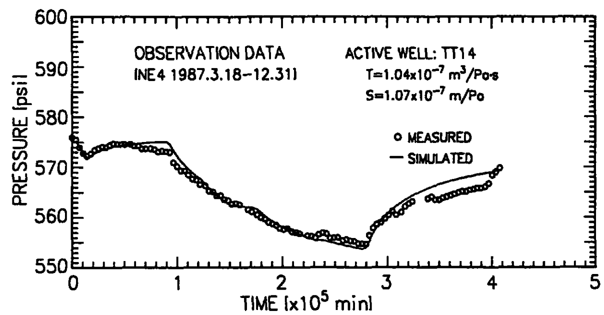

Fig.7 Comparison of simulated pressure values using the final estimates of T and S with measured ones.

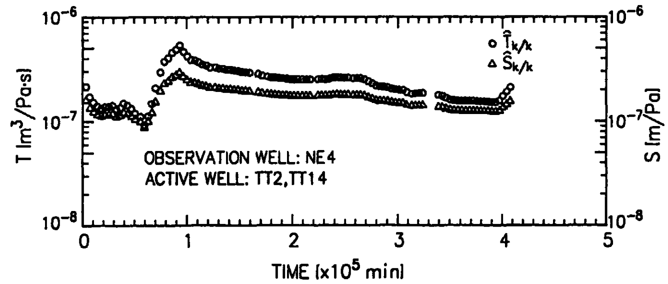

Fig.8 Estimated T and S at each time step versus time during the analysis with active wells, TT2 and TT14.

Table 1 Initial guesses of T and S versus number of iteration needed to attain convergence for the analysis of NE4 with an active well TT14.

Initial guess		Iteration number
T (m^3/Pa·s)	S (m/Pa)	
1x10^{-5}	1x10^{-6}	8
1x10^{-5}	1x10^{-8}	12
1x10^{-8}	1x10^{-8}	2
1x10^{-12}	1x10^{-12}	3

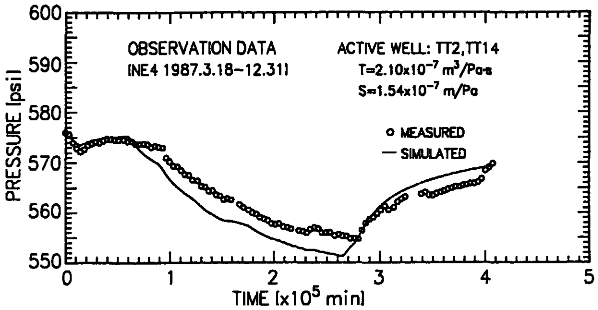

Fig.9 Comparison of simulated pressure values using the final estimates of T and S with measured ones.

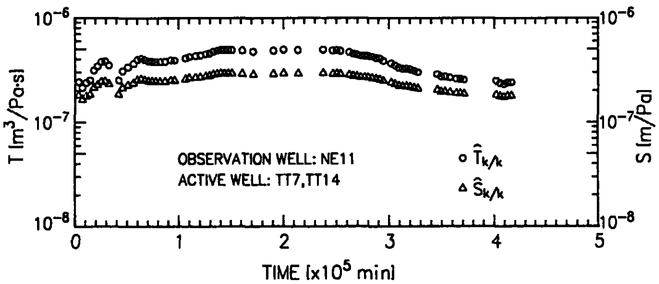

Fig.10 Estimated T and S at each time step versus time during the analysis of NE11 with active wells TT7 and TT14.

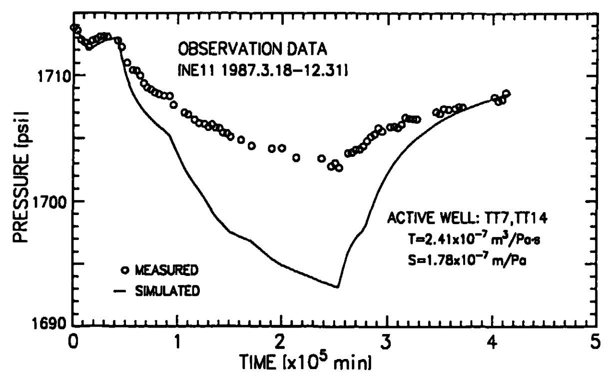

Fig.11 Comparison of simulated pressure values using the final estimates of T and S with measured ones.

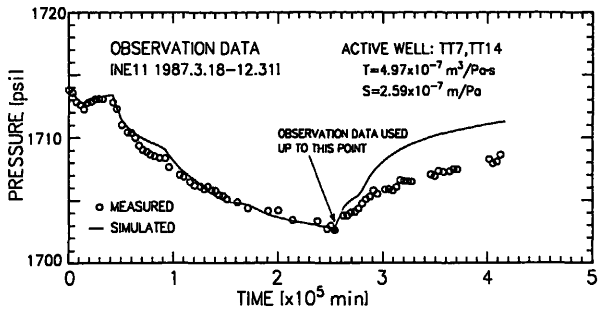

Fig.12 Comparison of simulated pressure values using the final estimates of T and S with measured ones.

good except at the middle times when the simulated values show slightly lower than the measured ones. Further analysis including reinjection well, TT11, exhibits the poorest match between the simulated and the measured pressure values. Hence the effect of injecting fluid into TT11 on the pressure response of NE4 can be judged to be very little or nil.

To examine the effects of initial guesses on the parameter estimation performance, we analyzed the case with TT14 by giving several combinations of T and S. Table 1 summarizes the values of initial guesses and the number of iteration of estimation procedure when the convergence is obtained. The values of final estimates for each case are identical as T=1.04x10^{-7} m^3/Pa·s and S=1.07x10^{-7} m/Pa. Therefore, the present method has rather weak dependence of the initial guesses on the final estimates, although the number of iteration of estimation procedure is different. From our experience, good estimates can be guaranteed as far as the ratio of initial guesses(S/T) being chosen to be one or less.

WELL NE11

NE11 is also a deviated well and locates southern most part of Takigami as shown in Fig.1. Main water loss zone of this well seems to be on southward extension of the Noine fault. A careful examination of the measured pressure data and the flowrate history of active wells seems to support the effect of interference by active wells TT7 and TT14 on the pressure response of NE11. Therefore, we analyzed the pressure data with these two active wells. The number of the pressure data used is 75.

Convergence of calculations is attained after four iterations, and the final estimates obtained are T=2.41x10^{-7} m^3/Pa·s and S=1.78x10^{-7} m/Pa. Fig. 10 illustrates the values of estimates versus time during the fourth iteration. The estimates vary in a fairly small range throughout the measurement period.

The pressure response was simulated using these final estimates, and the result is compared with the measured pressure values in Fig.11. A good match is observed only for short periods at the very early and later times. The simulated pressure departs markedly with time from the measured ones up to 2.5x10^5 min. Then, the discrepancies between two kinds of pressures become small as it approaches the end of the measurement. Since the estimates of T and S during the middle times are higher than the final values as shown in Fig.10, the poor match during the middle times can be attributed to that the values of T and S used for the simulation are slightly low to reproduce this pressure response. A careful examination of the estimation behavior in Fig. 10 suggests that the monotonous decrease in the estimates at later times may be caused by faulty data since the preceding estimates show fairly stable value.

Calculations for parameter estimation are repeated using the pressure data until the time 2.54×10^5 min, indicated by the solid circle in Fig.12. Final estimates obtained after five iterations of calculations are $T=4.97 \times 10^{-7}$ $m^3/Pa{\cdot}s$ and $S=2.59 \times 10^{-7}$ m/Pa. Fig. 12 shows a comparison between the simulated and the measured pressure values versus time. Simulated pressure indicated by the solid line shows a good match to the measured pressure values until the time indicated by the solid circle in the figure. But the simulated values gradually departs from the measured ones with time.

CONCLUSION

We have presented a method for estimating reservoir transmissivity(T) and storativity(S) on the basis of an infinite reservoir model by analyzing interference tests with multiple active wells and variable flowrates. The method employs the Kalman filtering technique and updates the parameters at every moment when the new pressure value from an observation well becomes available. It was demonstrated by analyzing the interference test data at Takigami, Japan, that the poor initial guesses of parameters are improved by iterating the estimating procedure. The method also exhibits that the divergence of calculations can be avoided if the ratio of initial guesses(S/T) is given to be one or less.

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