CHARACTERISTICS OF SHUT-IN CURVES OF TRANSVERSE CRACKS IN HYDRAULIC FRACTURING STRESS MEASUREMENTS

Takatoshi Ito and Kazuo Hayashi

Institute of Fluid Science, Tohoku University, Sendai 980, Japan

ABSTRACT

Characteristics of pressure decay curves after shut-in, i.e., shut-in curves, in hydraulic fracturing stress measurements were studied theoretically and experimentally for the case that a transverse crack is induced perpendicularly to a wellbore axis. Based on the characteristics, a method to evaluate the shut-in pressure, i.e., the tectonic stress normal to the crack plane, was developed. The method utilizes the plot of the inverse of pressure decrease rate vs the pressure after shut-in. The plot can be divided into two segments, where the plot fits to a straight line in each segment. Then, the shut-in pressure can be evaluated as the pressure at the intersection of the two straight lines. The method is successfully applied to the pressure decay curves after shut-in obtained in laboratory experiments.

INTRODUCTION

In most geothermal heat extraction subsurface systems, natural cracks and/or artificially created cracks serve as heat exchange surfaces. The behavior of the cracks is mainly governed by the tectonic stresses. In this sense, establishing the methodology for measuring tectonic stress is essential for designing reservoirs and preserving reservoirs stably during geothermal heat extraction.

Hydraulic fracturing is now widely used for in-situ tectonic stress measurements (Zoback et al., 1977; Haimson, 1978; Pine et al., 1983; Cornet and Valette, 1984; Hayashi et al., 1989). With this technique, an interval of a wellbore is sealed off with a straddle packer system and then pressurized by injection of fluid to induce and extend cracks emanating from the wellbore. Here, two types of cracks can be created by hydraulic fracturing, i.e., the longitudinal type and the transverse type. The former grows in parallel to the wellbore axis; the latter develops across the wellbore along a preexisting plane of weakness or along an interface such as a vein. Several characteristic wellbore pressures related to the tectonic stresses are observed in the downhole fluid pressure - time history during hydraulic fracturing. The tectonic stresses are computed from those pressures by using the relations between the tectonic stresses and the pressures which are derived theoretically in advance. Among those pressures, the so-called shut-in pressure is taken as an indicator of the tectonic stress normal to the crack plane. The presumption is that the shut-in pressure corresponds to the pressure at which the fluid pressure in the crack balances the compressive tectonic stress normal to the crack plane during the pressure decay following shut-in, i.e., the cessation of fluid injection (Kehle, 1964). By examining theoretically on the closure process of a penny shaped crack in an infinite formation, Kehle (1964) suggested that the final stable pressure after shut-in should be taken as the shut-in pressure. However, in most cases, the pressure decreases gradually after shut-in and approaches a pore fluid pressure in the formation, therefore, we cannot find out the final stable pressure representing the tectonic stress. Hence, several methods have been proposed in order to identify the indistinct shut-in pressures from the gradual pressure decay curves (Gronseth and Kry, 1983; McLennan, 1980; Enever and Chopra, 1986; Zoback and Haimson,1982; Aamodt and Kuriyagawa, 1983; Lee and Haimson, 1989). However, there seems to be no deterministic method for evaluating the shut-in pressure so far. The principal reason for the absence of the deterministic method is that the closure process of the crack has not yet fully understood. In this reason, Hayashi et al.(1989, 1991) analyzed recently the crack closure process after shut-in based on linear theory of elasticity and fracture mechanics for the case that the longitudinal cracks are induced by hydraulic fracturing. The results show that the closure process after shut-in consists of three major stages, and the inverse of pressure decrease rate is linear with respect to the pressure in the first and final stages. On the basis of the characteristics, it is shown that the shut-in pressure, i.e., the tectonic stress normal to the crack plane, can be determined as the pressure at the lower end of the first stage.

In the present paper, the functional characteristics of the pressure decay curves after shut-in is clarified, following the analytical procedure of Hayashi and Haimson (1991), for the case that the transverse crack is induced perpendicularly to the wellbore axis by hydraulic fracturing. The validity of the characteristics is confirmed through laboratory hydraulic fracturing experiments. Then, a new method for determining the shut-in pressure is developed on the basis of the characteristics. The method is successfully applied to the pressure decay curves after shut-in obtained in laboratory experiments.

ANALYSIS OF PRESSURE DECAY AFTER SHUT-IN

(a) Basic equations Let us consider the pressure decay

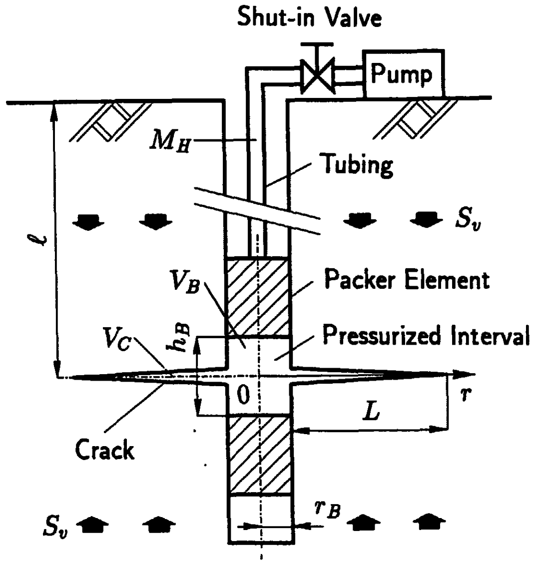

Figure 1. Hydraulic fracturing system and the induced transverse crack.

process after shut-in for the case that a transverse crack is induced by hydraulic fracturing (fig.1). Recently, Hayashi and Haimson (1991) reported that, for the case that a set of two longitudinal cracks are induced, the pressure decay process after shut-in consists of three major stages, i.e., from cessation of crack growth until crack tip closure (stage I), from just after crack tip closure until complete crack closure (stage II), and from just after complete crack closure until the test is stopped (stage III). In the case of a transverse crack, the pressure decay process also consists of these three stages. Let us consider the pressure decay process during stages I and III for the case that a transverse crack is induced.

In stage I, the pressure decrease after shut-in is governed by the following differential equation (Hayashi and Sakurai, 1989) derived from global mass conservation of the fracturing fluid:

$$\frac{dP}{dT} = -\frac{\rho_c Q_\ell}{\frac{d}{dP}(\rho_c V_C + \rho_c V_B + M_H)} \tag{1}$$

where P is the interval pressure, T is the time after the onset of pressurization, ρ_c is the mass density of the injected fluid in the pressurized interval, Q_ℓ is the volumetric fluid loss rate due to permeation into the rock, V_C is the volume of the crack, V_B is the volume of the pressurized interval, and M_H is the fluid mass in the tubing connecting the straddle packer system to the pump (fig.1). The injected fluid density in the pressurized interval is given by

$$\rho_c = \rho_0\{1 + \kappa(P - P_0)\} \tag{2}$$

where ρ_0 is the fluid density at the pressure of the air on earth's surface, P_0, and κ is the fluid compressibility. The volumetric fluid loss rate can be expressed as (Nolte, 1986):

$$Q_\ell = \frac{A_0 C}{\sqrt{T_0}} 2\left(\sqrt{\frac{T}{T_0}} - \sqrt{\frac{T}{T_0} - \frac{A}{A_0}}\right) + 2\pi r_B h_B \frac{C}{\sqrt{T}} \tag{3}$$

where C is the fluid loss coefficient, T_0 is the time at shut-in, r_B is the wellbore radius, h_B is the length of the pressurized interval (fig.1). The areas of fluid permeation from the crack before and after crack tip closure are denoted by A_0 and A, respectively. The volume of the pressurized interval is given by

$$V_B = \pi r_B{}^2 h_B \tag{4}$$

Taking account of the tubing deformation due to the fluid pressure inside and outside the tubing, the fluid mass in the tubing is given by (Hayashi and Haimson, 1991)

$$M_H = \pi \ell r_i{}^2 \rho_0\{1 + \kappa(P - P_w - P_0)\}\left(1 + \frac{PU - P_w V}{E_t}\right) \tag{5}$$

$$U = 2(1+\nu_t)\frac{(1-2\nu_t)r_i{}^2 + r_o^2}{r_o^2 - r_i{}^2} \quad , \quad V = U + \frac{4(1-\nu_t^2)r_o^2}{r_o^2 - r_i{}^2} \tag{6}$$

where ℓ is the length of the tubing, r_i and r_o are the inner and outer radius of the tubing, E_t and ν_t are the Young's modulus and Poisson's ratio of the tubing, P_w is the average pressure outside the tubing and is given by

$$P_w = \rho_0 g \frac{\ell}{2} \tag{7}$$

where g is the acceleration of gravity.

In order to discuss the pressure decay process after shut-in based on eq.(1), we need to obtain the relation between the interval pressure and the crack volume. As a typical transverse crack, here we consider the penny shaped crack which is induced perpendicularly to the wellbore axis (fig.1). The pressure in the crack is equal to the interval pressure, and it is assumed that the wellbore is located at the center of the crack and the size of the wellbore is so small compared with the crack length. Then, the crack volume is approximately given by (Tada et al., 1985)

$$V_C \cong \frac{16}{3E'}(P - S_v)(r_B + L)^3 \tag{8}$$

where $E' = E/(1-\nu^2)$, E and ν are the Young's modulus and Poisson's ratio of the rock, S_v is the compressive tectonic stress normal to the crack plane, and $r_B + L$ is the radius of the crack (fig.1). Furthermore, in this case, eq.(3) can be rewritten as

$$Q_\ell = \frac{C\sqrt{T_0}}{r_B}\frac{r_B^3}{T_0}\left\{a\left(\sqrt{\frac{T}{T_0}} - \sqrt{\frac{T}{T_0} - \frac{A}{A_0}}\right) + \frac{b}{\sqrt{T/T_0}}\right\} \tag{9}$$

$$a = 4\pi\left(\frac{L}{r_B}\right)^2 \quad , \quad b = 2\pi\frac{h_B}{r_B} \tag{10}$$

By using the eqs (4)-(8), we have

$$\frac{d}{dP}(\rho_c V_C) = \frac{16\rho_0}{3E'}(r_B + L)^3\{1 - \kappa(S_v + P_0) + 2\kappa P\} \tag{11}$$

$$\frac{d}{dP}(\rho_c V_B) = \pi \rho_0 r_B^2 \kappa h_B \tag{12}$$

$$\frac{d}{dP}(M_H) = \left[\kappa\left(1 - V\frac{P_w}{E_t}\right) + \frac{U}{E_t}\{1 - \kappa(P_w + P_0)\} + 2\frac{\kappa}{E_t}UP\right]\pi\rho_0 \ell r_i^{\,2} \tag{13}$$

These equations show that the denominator of the right-hand side of eq.(1) can be expressed in the following form:

$$\frac{d}{dP}(\rho_c V_C + \rho_c V_B + M_H) = \alpha + \beta P \tag{14}$$

where α and β are the constants independent of the interval pressure. In general, the constants in eqs (11)-(13) take the values as follows:

$\kappa \sim 0.45 \times 10^{-3}$ MPa^{-1}, $1/E' \sim 0.3 \times 10^{-4}$ MPa^{-1},
$1/E_t \sim 0.5 \times 10^{-5}$ MPa^{-1} (Steel tubing),
$1/E_t \sim 0.5 \times 10^{-2}$ MPa^{-1} (Rubber tubing),
$P_0 = 0.1$ MPa

Taking account of these values, the constants α and β can be written as follows:

$$\alpha = \rho_0\left[\frac{16}{3}(r_B + L)^3\frac{1}{E'} + V_t\left\{\kappa\left(1 + \frac{V_B}{V_t}\right) + \frac{U}{E_t}\right\}\right] \tag{15}$$

$$\beta = \rho_0\left[\frac{32}{3}(r_B + L)^3\frac{\kappa}{E'} + 2UV_t\frac{\kappa}{E_t}\right] \tag{16}$$

where $V_t = \pi r_i^{\,2}\ell$.

After complete crack closure (stage III), the crack volume is zero and hence, fluid leakage from the crack surfaces into the rock is negligibly small, so that eqs (1) and (2) become:

$$\frac{dP}{dT} = -\frac{\rho_c Q_\ell}{\frac{d}{dP}(\rho_c V_B + M_H)} \tag{17}$$

$$Q_\ell = \frac{C\sqrt{T}_0}{R}\frac{r_B{}^3}{T_0}\frac{b}{\sqrt{T/T_0}} \tag{18}$$

Then, through the similar process of formulation for stage I, we finally get

$$\alpha = \rho_0\left[V_t\left\{\kappa\left(1 + \frac{V_B}{V_t}\right) + \frac{U}{E_t}\right\}\right] \tag{19}$$

$$\beta = \rho_0\left[2UV_t\frac{\kappa}{E_t}\right] \tag{20}$$

Here, eqs (19) and (20) correspond to eqs (15) and (16), respectively.

(b) Characterristics of pressure decay after shut-in
Let us introduce the following nondimensional notations:

$$\left.\begin{array}{l} p = P/S_v\ ,\ t = T/T_0\ ,\ k = C\sqrt{T_0}/r_B\ , \\ \phi = \alpha S_v/(r_B{}^3\rho_0)\ ,\ \psi = \beta S_v^2/(r_B{}^3\rho_0)\ , \\ q_\ell = Q_\ell/(kr_B{}^3/T_0) \end{array}\right\} \tag{21}$$

Then, the basic eq. (1) can be rewritten as follows:

$$\frac{dp}{dt} = -k\frac{1 + \kappa(P - P_0)}{\phi + \psi p}q_\ell \tag{22}$$

Noting that $\phi >> \psi p$, and $1 >> \kappa(P - P_0)$ under normal conditions, we can reduce eq.(22) to:

$$\frac{dp}{dt} = -\frac{k}{\phi}q_\ell \tag{23}$$

for stages I and III. Furthermore, taking account that $a >> b$ in general, the nondimensional fluid loss rate is given by

$$q_\ell = \begin{cases} a(\sqrt{t} - \sqrt{t-1}), & \text{(for stage I)} \\ \dfrac{b}{\sqrt{t}}, & \text{(for stage III)} \end{cases} \tag{24}$$

The solution of the differential equation (23) for stage I is:

$$p = p_1 - \frac{2}{3}\frac{k}{\phi}a[\{t^{3/2} - (t-1)^{3/2}\} - \{t_1^{3/2} - (t_1 - 1)^{3/2}\}] \tag{25}$$

where t_1 is the time at the end of the instantaneous crack growth just after shut-in due to equilibration of injected fluid pressure within the crack (Hayashi and Haimson, 1991), and p_1 is the pressure at t_1. Taking account that the first term in square brackets can be approximated in $(\sqrt{t} - \sqrt{t-1} + 1)/2$ (Hayashi and Haimson, 1991), eq.(25) can be rewritten with the aid of eq.(23) in the following form:

$$p \cong \frac{1}{3}\left(\frac{ka}{\phi}\right)^2\frac{dt}{dp} + p_1{}^* \tag{26}$$

where

$$p_1{}^* = p_1 - \frac{1}{3}\frac{k}{\phi}a + \frac{2}{3}\frac{k}{\phi}a\{t_1^{3/2} - (t_1 - 1)^{3/2}\} \tag{27}$$

In eq.(26), k, a, ϕ and $p_1{}^*$ are independent of p and t, so that it can be readily understood that the inverse of the pressure decrease rate is linear with respect to the interval pressure in stage I.

On the other hand, the solution of the differential equation (23) for stage III is:

$$p = p_2 - 2\frac{k}{\phi}b(\sqrt{t} - \sqrt{t_2}) \tag{28}$$

where t_2 is the time at the completion of crack closure and p_2 is the pressure at time t_2. From eqs (23) and (28), we have

$$p = 2\left(\frac{k}{\phi}b\right)^2\frac{dt}{dp} + p_2{}^* \tag{29}$$

where

$$p_2{}^* = p_2 + 2\frac{k}{\phi}b\sqrt{t_2} \tag{30}$$

In eq. (29), k, b, ϕ and $p_2{}^*$ are independent of p and t. Therefore, as same as the case of stage I, the inverse of the pressure decrease rate is linear with respect to the interval pressure after the completion of crack closure.

LABORATORY EXPERIMENTS AND RESULTS

(a) Experimental procedure In the previous section, it is shown that the inverse of the pressure decrease rate is linear with respect to the interval pressure in stages I and III. In order to verify these theoretical results, laboratory hydraulic fracturing experiments were conducted as follows.

As a rock sample, Honkomatsu andesite was used. Specimen size was 0.3 × 0.3 × 0.3 m and the wellbore with diameter of 10 mm was drilled into the specimen. In order to simulate the tectonic stresses, the specimen was loaded under triaxial compressive stresses as shown in fig.2, i.e., the two horizontal compressive stresses S_1 and S_2 ($S_1 > S_2$), and the vertical compressive stress S_v. The triaxial loads were applied through three pairs of flat jacks that filled the spaces between the sides of the specimen and the inner wall of a heavy steel frame (fig.3). A viscous hydraulic oil (Tellus Oil 32) was employed as a fracturing fluid, and hydraulic fracturing was performed through the following procedure of two steps. At the first step, a vertical wellbore was drilled only part way through the specimen leaving the rock itself to form the bottom end. By using a single packer jig to pressurize the wellbore, hydraulic fracturing was performed under vertically unstressed condition, i.e., S_1 = 15 MPa, S_2 = 10 MPa and S_v = 0 MPa. Thus, a transverse crack was induced at the bottom of the wellbore perpendicularly to the wellbore axis (fig.2). At the 2nd step, the wellbore was extended completely through the specimen. In this case, a simple double-packer jig analogous to a straddle packer system was employed and set in a way the interval contained the transverse crack. Then, the hydraulic fracturing was performed and the interval pressure - time history was recorded. The relation between the inverse of the pressure decrease rate and the pressure after shut-in was investigated for this 2nd step.

(b) Results and discussions At first, the experiments were conducted under axisymmetric loading conditions, i.e.,

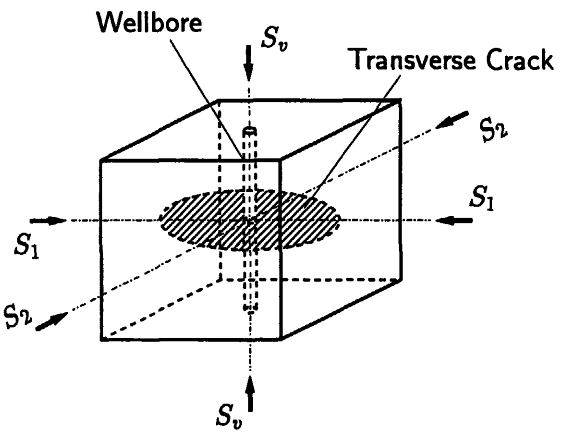

Figure 2. Geometry of the specimen and triaxial compressive loads.

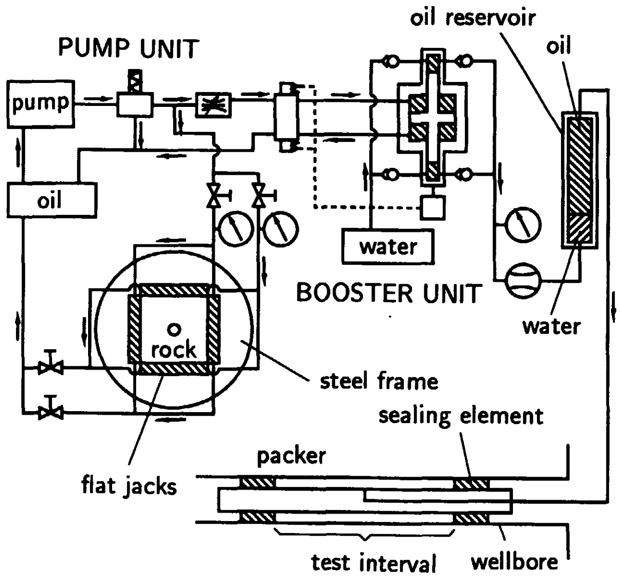

Figure 3. Laboratory hydraulic fracturing system.

$S_1 = S_2 = 20$ MPa and $S_v = 15$ MPa. As described above, the transverse crack was induced perpendicularly to the wellbore axis. Under these conditions, the whole of the crack closes simultaneously when the decreasing pressure in the crack is equal to the compressive tectonic stress normal to the crack plane (see Appendix). Hence, stage II, i.e., from just after crack tip closure until complete crack closure, vanishes and stage III begins at the end of stage I. Figure 4(a) shows an example of results of the experiments. As shown in this figure, the pressure decreases gradually after shut-in and there is no obvious "breaks" or "knees", so that it seems to be impossible to detect the shut-in pressure directly from the pressure decay curve. On the other hand, the plot of dT/dP vs P (fig.4(b)) which is constructed from the same experimental results given in fig.4(a), can be clearly divided into two segments, and the plot in each segment fits to a straight line. Then, the magnitude of the pressure at the intersection of the two lines is very close to the magnitude of S_v. Besides, as can be seen from fig.4(b), the plots of dT/dP vs P is convex. Furthermore, the plots of dT/dP vs P for other all experiments conducted in this study are also convex. In order to investigate the reason why the plot is convex, let us examine the slope of the two straight lines which fit to the plot of dT/dP vs P. To this end, Δ is defined as the ratio of the slope of the segment in the higher pressure region to that in the lower pressure region on the plot of dT/dP vs P. From eqs (26) and (29), the ratio Δ is given by

$$\Delta = \frac{3}{2}\frac{r_B{}^2 h_B^2}{L^4}\left(\frac{\alpha_1}{\alpha_3}\right)^2 \tag{31}$$

where α for stages I and III are denoted by α_1 and α_3, respectively. By using eq.(31), Δ is evaluated for the experiments conducted in this study. Thus, the following parameters are

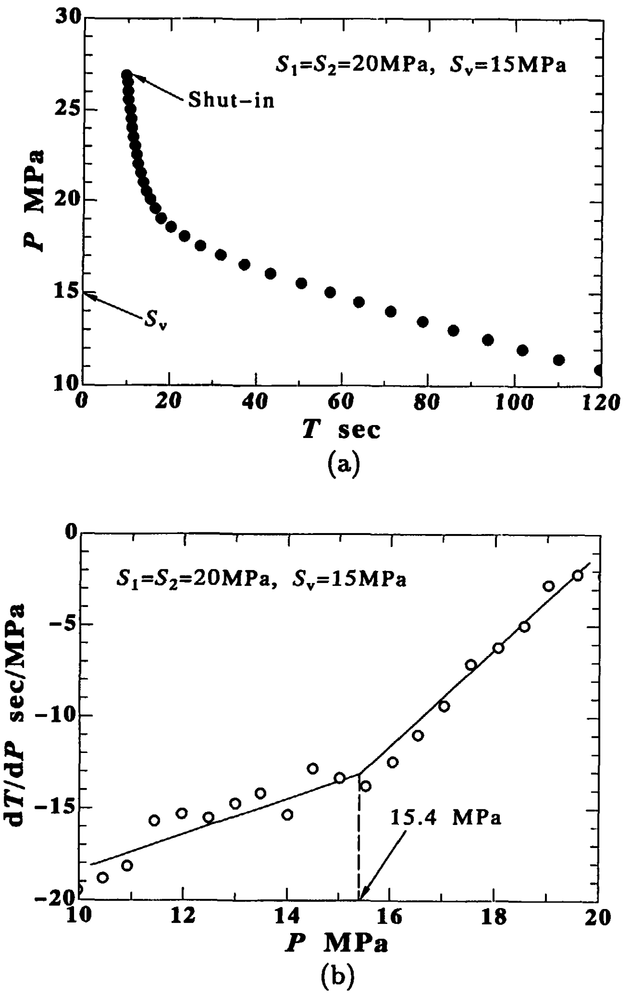

Figure 4. An example of the results of laboratory experiments for the case that $S_1 = S_2$.

used to evaluate Δ:

$$r_B = 5 \times 10^{-3} \text{ m} , \ h_B = 4.3 \times 10^{-2} \text{ m} ,$$
$$L = 1.5 \times 10^{-1} \text{ m} , \ r_i = 10^{-3} \text{ m} , \ r_o = 2 \times 10^{-3} \text{ m} ,$$
$$\ell = 2 \text{ m} , \ E = 3 \times 10^4 \text{ MPa} , \ \nu = 0.18 ,$$
$$E_t = 2 \times 10^5 \text{ MPa} , \ \nu_t = 0.3 , \ \kappa = 0.45 \times 10^{-3} \text{ MPa}^{-1} ,$$
$$\rho_0 = 1000 \text{ kg/m}^3$$

The evaluated result is $\Delta = 2.9 (> 1)$. It means that the plot of dT/dP vs P is convex. Moreover, the evaluated value is close to the ratio which is obtained from the experimental result shown in fig.4(b), i.e., $\Delta = 2.8$. Thus, it can be concluded that the segments in the higher and lower pressure regions in fig.4(b) represent stages I and III, respectively.

Next, in order to investigate the effect of S_2 on the pressure decline after shut-in, the hydraulic fracturing experiments were conducted under the conditions that S_1 was fixed as 20 MPa and S_2 was smaller than S_1. An example of the experiment is shown in fig.5 where the results given in fig.4(b) is also shown for comparison. The result show that the plot of dT/dP vs P is almost same as the plot for the case that $S_1 = S_2$, and it can be clearly divided into two segments. In each segment, the plot fit to a straight line, and the magnitude of the pressure at the intersection of the two lines is close to the magnitude of S_2. The result show that, even if S_1 is not equal to S_2, stage II does not appear explicitly at least on the plot of dT/dP vs P. Therefore, independently of S_1 and S_2, the tectonic stress normal to the crack plane, S_v, can be estimated as the pressure at the intersection of the two lines which fit to the plot of dT/dP vs P. Figure 6 summarizes the results of application of the method to all of the hydraulic fracturing experiments conducted in this study. The results show that the present method yields a good estimate of the applied compressive stress S_v, although the estimated magnitudes have a tendency to be slightly higher than the magnitude of the applied stress.

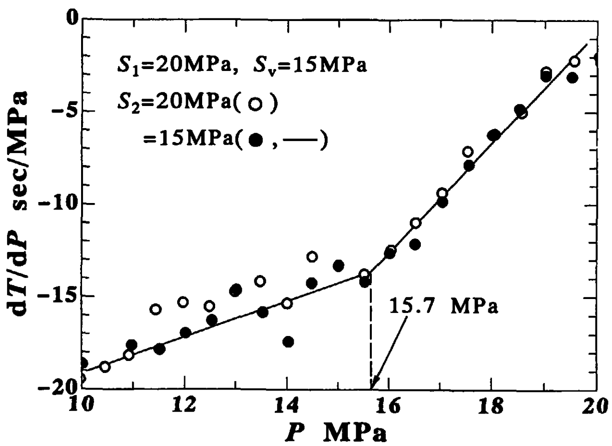

Figure 5. An example of the results of laboratory experiments for the case that $S_1 \neq S_2$.

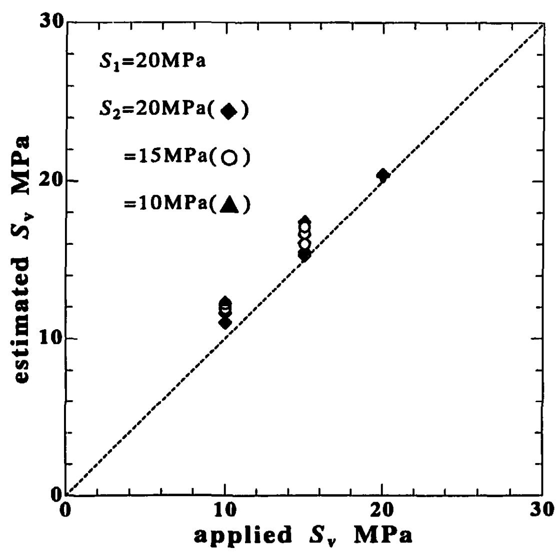

Figure 6. Comparison between the applied vertical stress S_v and the stress S_v estimated by the present method.

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APPENDIX

Let us consider the elastic problem of a penny shaped crack which is induced perpendicularly to the wellbore axis (fig.1), where the wellbore axis is assumed to be located at the center of the crack. Also, it is assumed that one of the principal axis of the tectonic stresses coincides with the wellbore axis, and the magnitudes of the principal tectonic stresses acting in the plane normal to the wellbore axis, are same. Then, the problem is formulated by means of integral transforms and reduced to solving a singular integral equation (Keer et al., 1977). By using the expressions of Keer et al.(1977), the singular integral equation is given by

$$\frac{2}{\pi}\int_{r_B}^{r_B+L} \tau f(\tau)\{R(r,\tau)+S(r,\tau)\}d\tau = P - S_v \qquad (r_B < r < r_B + L) \qquad (A1)$$

where R and S are known functions (Keer et al., 1977), r is radius, and f is an unknown function which is related to the crack aperture w as follows:

$$w(r) = \frac{4}{E'}\int_{r_B+L}^{r} f(\tau)d\tau \qquad (A2)$$

From eq.(A1), it is readily understood that the function f is directly proportional to $P - S_v$. Therefore, the function f becomes zero independently of r, when $P - S_v = 0$. Thus, it is shown from eq.(A2) that the crack aperture w also becomes zero independently of r, when $P - S_v = 0$. It means that the whole of the crack closes simultaneously when the pressure in the crack is equal to the tectonic stress normal to the crack plane.