OPTIMIZATION OF INJECTION SCHEDULING IN GEOTHERMAL FIELDS

James Lovekin and Roland N. Horne

Stanford Geothermal Program

Abstract

This study discusses the application of algorithms developed in Operations Research to the optimization of brine reinjection in geothermal fields. The injection optimization problem is broken into two sub-problems: (1) choosing a configuration of injectors from an existing set of wells, and (2) allocating a total specified injection rate among chosen injectors. The allocation problem is solved first. The reservoir is idealized as a network of channels or arcs directly connecting each pair of wells in the field. Each arc in the network is considered to have some potential for thermal breakthrough. This potential is quantified by an arc-specific breakthrough index, b_{ij}, based on user-specified parameters from tracer tests, field geometry, and operating considerations. The sum of b_{ij}-values for all arcs is defined as the fieldwide breakthrough index, B. Injection is optimized by choosing injection wells and rates so as to minimize B subject to constraints on the number of injectors and the total amount of fluid to be produced and reinjected. The use of the various methods is demonstrated with reference both to hypothetical data and an actual data set from the Wairakei Geothermal Field in New Zealand.

1. INTRODUCTION

As the geothermal industry matures, the need for a method to optimize a program of reinjection becomes more important. Fluid injection holds the promise of increased recoveries from geothermal fields by providing a medium to absorb a greater percentage of the heat in the reservoir rock. Unfortunately, injection into geothermal fields also has potential for decreasing thermal recovery by extracting heat unevenly. If the injected fluid travels too directly to producing wells without contacting a large volume of the reservoir rock, premature thermal breakthough may occur and the economic life of the field may be cut short. This is all the more likely because flow patterns in geothermal reservoirs are often controlled by fractures. Tracer surveys provide a powerful tool for gaining insight into these flow patterns. It is not uncommon for more distant wells to show stronger tracer response than wells which are closer to the injector.

The purpose of this study is to provide a systematic approach for using such tracer data, together with information about field geometry and operating conditions, in the optimization of injection scheduling in geothermal fields.

The operator of a geothermal field is interested in two questions:

(1) Which wells should be made injectors?

(2) How should the total required injection rate be distributed?

The first question may be called the problem of *configuration*, and the second the problem of *allocation*. In practice, the operator typically solves the configuration problem first. Often, operational considerations dictate the solution. It turns out, however, that the solution to the allocation problem provides a straightforward approach to solving the configuration problem for an unrestricted case. For this reason, this study will address the allocation problem first.

The inspiration for this study's approach to the allocation problem grows directly out of the observation that, because of fracturing in geothermal fields, tracer response between two wells is often unrelated to how close the wells are to each other. In this approach, the geothermal reservoir is idealized as a network of direct connections or arcs between every pair of wells. Each arc is presumed to have some potential for thermal breakthrough. This potential is assigned a numerical value called a breakthrough index, b_{ij}, which is some function of operating rates for the wells on either end of the arc and an arc cost, c_{ij}, based on tracer test parameters and field geometry. The sum of b_{ij}-values for all arcs is defined as the fieldwide breakthrough index, B. Injection is optimized by choosing injection rates so as to minimize B, subject to constraints on individual well capacities and total injection requirements. The allocation problem bears a striking resemblance to problems of linear programming (LP) which are studied in the field of Operations Research. The thesis of this paper is that algorithms from Operations Research, used in conjunction with tracer tests, provide a useful method of optimizing geothermal injection.

2. BACKGROUND

The bulk of field experience indicates that rapid thermal breakthrough and large tracer recoveries correlate strongly.[1,2,3] Tracer test results have been published for a number of fields, including the Wairakei and Broadlands Fields in New Zealand,[4] the Geysers Field in California,[5] the Larderello Field in Italy,[6] the Kakkonda, Onuma, Hatchobaru, and Otake Fields in Japan,[7] and the Klamath Falls Field in Oregon.[8]

In treating a geothermal field as a network of direct connections, the current study builds on Horne's idea of a connectability map. The reservoir is considered as a network of pipes, each with some physical parameter (analagous to a diameter or a Reynold's number) expressing the ease with which a tracer slug or a thermal front could pass through. To gain insight into the optimization of such a system, the literature of optimizing pipe networks was reviewed. Linear programming (LP) was found to be a commonly employed technique.[9,10,11,12,13,14] In these studies of pipe network optimization, the objective function to be minimized was typically some combination of installment costs and discounted operating costs, while the system constraints were provided by linearized flow equations, network geometry, water supply limitations, and outlet flow requirements. The decision variables in these formulations were usually the dimensions (lengths, diameters, or both) of the pipes to be installed.

The current study's approach to optimizing geothermal injection draws an analogy to the transportation problem. As will be discussed in greater detail later, the cost associated with each arc is based in part on parameters from tracer tests. Several authors have discussed methods of inferring fracture apertures and other reservoir properties from tracer tests by applying a non-linear, least-squares method of curve-fitting to plots of produced tracer concentration versus time.[16,17,18] The method proposed here allows the use of any reservoir properties so inferred. However, since it builds on the assumption (supported by field experience) that tracer response and thermal response are strongly correlated, it does not require a solution of the inverse problem. Rather than make inferences about what the geothermal reservoir actually is, the proposed method makes operational decisions directly based on what the reservoir actually does.

3. ALLOCATION PROBLEM

3.1 Analogy to Transportation Problem

Figure 1 illustrates the analogy between the classis "transportation problem" and the injection optimization problem. In this analogy, nodes 1 and 2 could represent injection wells, and nodes 3, 4, and 5 could represent production wells. The arcs in the network represent the potential fluid flow paths from each injector to each producer. However, these arcs do not imply anything about the actual geometry of fluid flow. Each arc has associated with it some "cost" per unit of fluid transmitted, where the cost is an expression of the increased likelihood of thermal breakthrough, as assessed based on tracer tests, field geometry, and operational considerations. The problem is to minimize the likelihood of thermal breakthrough throughout the field, while meeting constraints on the injection capacity of individual wells and satisfying total injection requirements.

The following LP formulation for the injection optimization problem illustrates the parallels with the transportation problem for the case of N_1 injectors and N_2 producers:

$$\text{Minimize} \qquad B = \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} b_{ij} = \sum_{i=1}^{N_1}\sum_{j=1}^{N_2} c_{ij}\, q_{ri}$$

$$\text{Subject to} \qquad q_{ri} \le q_{rimax}, \qquad i = 1, N_1$$

$$\sum_{i=1}^{N_1} q_{ri} = Q_{rtot}$$

$$q_{ri} \ge 0, \qquad i = 1, N_i$$

The decision variables, q_{ri}, are the reinjection rates for each injection well, i. The arc costs, c_{ij}, express the increased chance of thermal breakthrough resulting from movement of a unit of fluid from each injector to each producer. The product of an injection rate and an arc cost constitutes the breakthrough index, b_{ij}, for a particular arc. The summation of breakthrough indices for all arcs constitutes the fieldwide breakthrough index, B, which is the objective function to be minimized. The supply constraints for the injection optimization problem simply express the requirement that each injector has to operate at a rate less than its capacity, q_{rimax}. The demand constraint requires that the summation of all injection rates must equal the specified fieldwide total injection rate, Q_{rtot}.

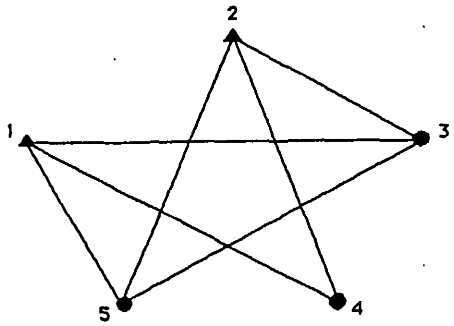

Figure 1. Idealized network of arcs.

Finally, the non-negativity constraint ensures that none of the injectors are operating at "negative rates", *i.e.*, that they are not acting as producers.

3.2 Definition of Arc Costs

In the injection optimization problem, the arc costs are expressed in terms of increased likelihood of premature thermal breakthrough. The relation between known reservoir parameters and thermal breakthrough is difficult to quantify. For instance, it would be difficult to calculate the time required for a given percentage drop in the enthalpy of produced fluids without detailed knowledge of reservoir properties and operating conditions. For the purposes of optimization, however, such detailed knowledge is not necessary. All that is required is a *relative* assessment of the "cost" of injection into different wells. For the optimization process to be valid, it is only necessary that the likelihood of thermal breakthrough be assessed on the same terms for each injector/producer pair. On this basis, one can consider any known parameter that relates injectors and producers and decide, in relative terms, whether it has a direct or an inverse relationship with the likelihood of thermal breakthrough. This allows one to weight one's definition of arc costs according to whatever data are available or whatever factors one considers important.

In the computer algorithms prepared for this study, the weighting factors composing the arc costs have been drawn from three sources: tracer tests, field geometry, and operating conditions. Table 1 describes the various weighting factors used. Each of these factors will be discussed in the following paragraphs.

Table 1. Weighting factors for arc costs

Factor	Definition	Relation to Arc Cost		
t_i	Initial tracer arrival time	c	α	$1/t_i$
t_p	Peak tracer arrival time	c	α	$1/t_i$
C_p	Peak tracer concentration	c	α	C_p
$\int$	Fractional tracer recovery	c	α	$\int$
q_{pt}	Producing rate during tracer test	c	α	$1/q_{pt}$
q_{rt}	Injection rate during tracer test	c	α	$1/q_{rt}$
q_p	Producing rate under operating conditions	c	α	q_p
L	Horizontal distance between wells	c	α	L^2
H	Elevation change from producing zone	c	α	e^{SH}

The tracer test parameters considered in this study are based on a slug-type tracer test. In this type of test, a certain quantity or "slug" of tracer is released instantaneously in an injection well. This gives rise to a characteristically spiked tracer response profile at the production wells, as illustrated in Figure 2. One parameter which may be directly interpreted from such a profile is the initial tracer response time, t_i. Intuitively, t_i should be inversely correlated with the likelihood of thermal breakthrough. That is, the longer it takes for the tracer to break through from a given injector to a given producer, the less likely it is that premature thermal breakthrough will be a problem between those two wells. Therefore, t_i enters into the calculation of arc costs as a reciprocal, as shown in Table 1.

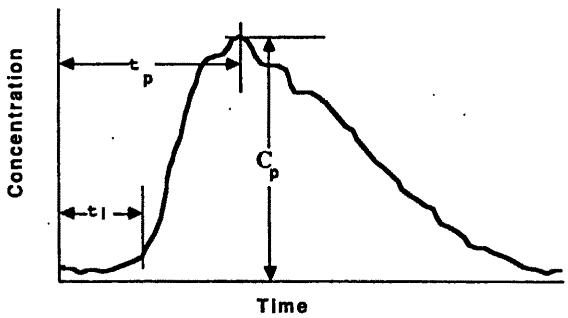

Figure 2. Schematic response from slug-type tracer test.

Two other weighting factors which are available from a tracer response profile are the peak tracer concentration, C_p, and the fractional tracer recovery, f. C_p is simply the concentration at time t_p. To obtain a value for f, one must first calculate the mass of tracer recovered by integrating the area under the tracer response curve and multiplying by the producing rate (assumed constant) during the tracer test. (If the producing rate was not constant during the tracer test, one may multiply each concentration measurement by its respective producing rate to produce a curve of the amount of tracer recovered per unit time, and then calculate the total tracer recovered by integrating under this curve.) The f-value is then just the mass of tracer produced divided by the mass of tracer injected. Both C_p and f are positively correlated with the likelihood of premature thermal breakthrough, and they may therefore be used directly as weighting factors in calculating arc costs.

In the category of weighting factors from field geometry, the most accessible parameter is the horizontal distance between wells (L). It is important to recognize that L has no predictable relation with thermal breakthrough in the case of fractured reservoirs. However, in the case of porous-media-type reservoirs or reservoirs in which high permeability zones approximate horizontal planes (*e.g.*, at contacts between lava flows), the flow of injected fluid away from injection wells in the reservoir may be radial. In this case, the surface area available for heat exchange from the rock to the cooler fluid grows in proportion to the square of L.[19] Therefore, the likelihood of thermal breakthrough may be considered inversely proportional to L^2, which may enter into the calculation of arc costs as a reciprocal. It should be emphasized, though, that distance between wells is not a reliable substitute for tracer test data in the common case of fractured geothermal fields.

The other accessible parameter in terms of field geometry is the difference in elevation (H) between producing and injecting zones. Tracer test data from Wairakei suggest that tracer breakthrough is much more likely in deep producing wells.[4] This makes physical sense, because cooler injected fluids are more dense than reservoir fluids and would be expected to sink within the reservoir. However, H itself is not appropriate as a weighting factor, because it may be either positive or negative. To calculate a weighting factor based on H, this study has used an exponential function, because it is strictly positive and because it increases or decreases the arc cost based on whether H is positive (producing zones below injecting zones) or negative (producing zones above injecting zones). When H is zero (producing and injecting zones at the same elevation), the exponential of H is 1, and the arc cost is unaffected. To keep the exponential term from dominating all other weighting factors, the exponent, H, has been multiplied by a scaling factor, S. Thus, elevation enters into the calculation of arc costs as the weighting factor e^{SH}. For elevation differences on the order of hundreds of meters, an S-value of 10^{-3} keeps this weighting factor in a range between 0.37 and 2.72.

The following equation illustrates how the various weighting factors discussed so far could be combined in calculating the breakthrough index for each arc:

$$b = c\,q = \left[\frac{1}{t_i}\,\frac{1}{t_p}\,C_p\,f\,L^2\,e^{SH}\,\frac{q_p}{q_{pt}}\,\frac{1}{q_{rt}}\right] q_r$$

The expression in parentheses represents one formulation of the arc cost in expanded form. Because the different weighting factors apply in different situations, an actual optimization run would probably use only some subset of these factors. For example, t_i and t_p would usually not both be used. It should also be noted that the list of weighting factors is not exhaustive: other weighting factors could be included, based on the developer's knowledge of the reservoir and operating requirements. Further, although this study has applied a scaling factor only in the case of elevation differences, scaling factors could easily be applied to other arc cost components as well, depending on which factors the developer considers important.

3.3 Computer Program Descriptions

Four separate algorithms were formulated in this work, as summarized in Table 2. These methods are described in detail in reference 23, which also contains program listings.

Table 2. Summary of computer programs to optimize injection

Program Name	Application	Data Entry Program
LPAL1	Linear programming allocation for injection rates only	LPIN1
LPAL2	Simultaneous linear programming allocation for both injection and production rates	LPIN2
LPAL3	Alternating linear programming allocation for both injection and production rates	LPIN3
QPAL	Quadratic programming allocation for both injection and production rates	QPIN
INCON	Injector configuration chooser	CONDAT

A general summary of the similarities and differences between the programs is provided in Table 3. As the table shows, all the LP programs operate by providing an explicit ranking of wells based on cost coefficients and throttling back one well at a time, from most to least damaging, until total rate requirements are just met. QPAL does not provide an explicit ranking of wells, but it uses a quadratic programming solver which generally yields the same rate allocations as LPAL3. All the programs except LPAL1 allocate both injection and production rates. All the programs except LPAL2 allow wells to be shut in, *i.e.*, to be assigned a zero rate. The well rankings provided by LPAL1 and LPAL2 do not vary with changes in total operating rates, because the arc costs used in calculating cost coefficients are all fixed. Effectively, this means that neither LPAL1 nor LPAL2 can take into account the mutual dependence of injection and production rates in determining the likelihood of thermal breakthrough. For this reason, LPAL3 is the most realistic of the LP programs presented. (Note that if producing well rates are predetermined, LPAL3 can be used to generate the same injection allocation as LPAL1 by simply setting the required total producing rate equal to the sum of the known well rates.) QPAL also accounts for the mutual dependence of injection and production rates and is the only program presented which explicitly assesses the quality of the solution by identifying indeterminate cases.

Table 3. Comparison of allocation programs

Program Feature	LPAL1	LPAL2	LPAL3	QPAL
Provides ranking of injectors	Yes	Yes	Yes	No
Solves for both injection and production rates	No	Yes	Yes	Yes
Allows wells to be shut in	Yes	No	Yes	Yes
Well ranking varies with total rates	No	No	Yes'	NA
Assesses quality of solution	No	No	No	yes

4. CONFIGURATION PROBLEM

4.1 Enumeration Approach

The computer programs discussed so far have addressed the problem of how to allocate a specified total injection rate among a pre-chosen set of injection wells. The configuration problem concerns how to choose this set of injectors from a group of pre-existing wells. The solution of the allocation problem provides a straightforward approach to the configuration problem. The end result of the allocation routines is not only a set of injection rates but a minimized value of the fieldwide breakthrough index, B. For a particular configuration, this value expresses in a single number the likelihood of premature thermal breakthrough for the entire field under optimal loading. Therefore, the configuration problem may be approached by enumeration, *i.e.*, by applying an allocation algorithm to each possible injector configuration and selecting the configuration with the lowest minimized B-value as optimal.

The theoretical upper limit on the number of configurations which an enumerative approach would have to consider is given by the expression

$$\frac{N!}{N_1!\, N_2!}$$

where N = total number of wells
N_1 = number of injectors
N_2 = number of producers

For the number of wells in a typical geothermal field, this value is small enough that consideration of all configurations would not require excessive computer time. Moreover, certain configurations could usually be removed from consideration because the sum of maximum rates for the wells involved would be insufficient to meet the total rates required. Thus, the actual number of configurations for which the allocation routine would need to be run would usually be less than the theoretical maximum.

It should be noted that the data requirements for such a configuration-choosing routine are much more extensive than for the allocation routines previously discussed. The data must characterize not just the arcs between designated injector/producer pairs (as in Figure 1) but between *all* well pairs. For directional information, data should be supplied in both directions. A complete set of tracer data, for example, requires that a separate tracer test be conducted on each well and that tracer response be monitored in all other wells. Further, rate limitations must be specified for each well both as an injector and as a producer.

4.2 Computer Program Description

This study has developed a configuration-choosing program called INCON, which uses the enumerative approach in conjunction with the QP allocation algorithm. The computer codes for INCON and its associated data-entry program (CONDAT) are described in reference 23 The input parameters for INCON include the total number of wells, the maximum allowable number of injectors, and the required fieldwide production and injection rates. For each possible injector configuration, the program checks to insure that the required fieldwide rates can be met. It then runs the QP allocation algorithm on each feasible configuration, and selects the configuration with the lowest B-value. A flow chart for INCON is presented in Figure 3.

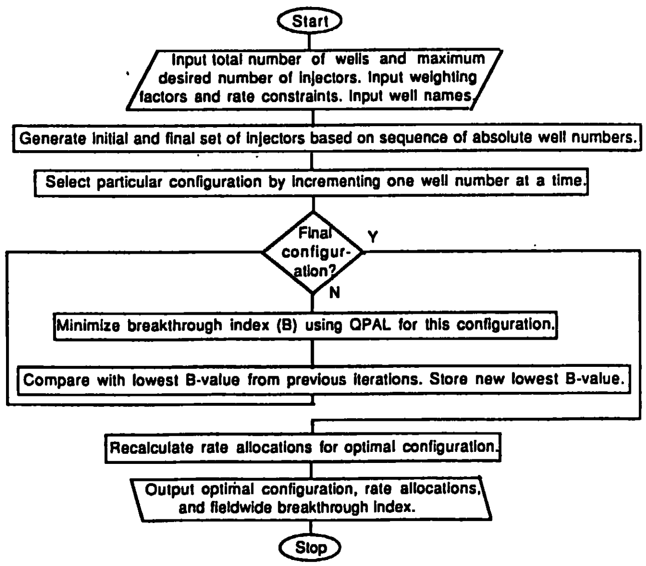

Figure 3. Flow chart for INCON.

5. APPLICATIONS TO WAIRAKEI

5.1 Background

The Wairakei Geothermal Field is a liquid-dominated field located near the town of Taupo on the North Island of New Zealand. A series of tracer tests were conducted in the field in 1979 and 1980. These tests took advantage of a downflow of cooler fluid in several wells from a zone above the reservoir. The tests were intended to determine where this cooler fluid was going and whether production from offsetting wells was being adversely affected. Glass vials of radioactive tracer (Iodine-131) were lowered into the downflowing wells and broken below the point of cool fluid entry. Producing wells were monitored continuously for tracer response. Measured tracer concentrations were normalized by dividing them by the amount of tracer injected to account for variation in the size of the tracer slugs in the different tests. McCabe *et al.* provide a detailed description of the testing procedures and results.[4]

The Wairakei tracer tests are a classic example of fracture-controlled flow in a geothermal reservoir. In several instances, wells which were further away from the tracer injection wells exhibited stronger response than closer wells. Because the Wairakei Field illustrates so well the notion of a geothermal reservoir as a network of direct connections between wells, and because it has several sets of tracer data to quantify these connections, it is an ideal test case to demonstrate the use of the injection optimization programs presented in this study.

Reference 23 summarizes the Wairakei data that have been used for the allocation programs. The tracer test parameters (t_i, t_p, C_p, and f) used were those reported by McCabe *et al.*[4] Production rates during the test (q_{pt}) were estimated from actual production rates as of December, 1976, as reported by Pritchett *et al.*[20] Injection rates during the tests (q_{rt}) for Wells WK-101 and WK-107 are those reported by Bixley.[21] Since no q_{rt}-value was available for Well WK-80, a value of 50 kg/s was estimated. These values of q_{pt} and q_{rt} were also used as the maximum well capacities (q_{rmax} and q_{pmax}). Values of the horizontal distance (L) between wells were determined from well maps, since the wells were all drilled vertically. To calculate values of the elevation change (H) between producing and injecting zones, the following assumptions were made: (1) The depth of the injection zone was taken as the depth at which the tracer was released. (2) The depth of the production zone was taken as the depth of the lowest fissure indicated on drill logs,[22] or, in the absence of reported fissures or logs, as the midpoint of the open interval.[20] (3) For Well WK-121, the elevation of the uphole perforations at 975 m (-532 m sub-sea) was used, since this was reported to be the primary source of production.[4]

For the configuration-choosing program, the maximum well capacities were assumed to apply for all wells both as injectors and as producers. A computer program (HGEN) was written to generate a set of elevation changes for a complete set of arcs using the injection and production zone elevations just described. A second computer program (LGEN) was written to calculate a complete set of horizontal distances from the surface well coordinates reported by Pritchett.[20] These calculated L-values differed only slightly from the measured L-values used with the allocation programs.

5.2 Optimal Rate Allocations

5.2.1 Sensitivity to Total Rate

All four allocation programs were run on the Wairakei data to investigate how varying total injection and production rates would affect the optimal rate allocation. The wells involved in the tracer tests included the three wells with cool fluid downflow (the "injection" wells) and nineteen producing wells. The fieldwide capacities for injection and production were 140 and 689 kg/s, respectively, based on the sum of individual well capacities. Sensitivity studies were run using a single weighting factor ($1/t_i$) to calculate cost coefficients. These sensitivity studies entailed fixing one of the total rates (either Q_{rtot} or Q_{ptot}) at a value below total capacity and varying the other from total capacity to a low rate.

For all the sensitivity studies, LPAL1 and LPAL2 established well rankings which were invariate with total rates. This was as expected for these two programs, because neither of them incorporates variable well rates into their calculations of cost coefficients. LPAL1 ranked WK-107 as the most prone to thermal breakthrough, followed by WK-101 and WK-80. LPAL2 ranked the injectors the same way and also provided a ranking of the producers. As total rates were cut back in the sensitivity studies, these programs throttled back one well at a time in the order of the predetermined rankings. In contrast, the ranking of wells by LPAL3 varied with total rates, and the rate allocations for one category of wells (producers or injectors) depended on the rates of wells in the other category. Further, the rate allocations from QPAL generally agreed with those from LPAL3, as expected. These points are illustrated by the following three sensitivity studies.

In the first sensitivity study, Q_{ptot} was fixed at 550 kg/s, and Q_{rtot} was varied from a capacity rate of 140 kg/s to 50 kg/s. Table 4 shows the sequence in which LPAL3 and QPAL shut wells in. (This table and all subsequent tables of sensitivity data present ranking for all three injectors; however, for the sake of brevity, the only producers listed are those with curtailed rates.) Several points are worth noting from Table 4. First, the cost coefficients of the producers shift continuously as the injection rate drops. Second, for marginal changes in the injection rate, the relative ranking of the producers stays the same. For example, as Q_{rtot} goes from 140 to 100 kg/s, the producers maintain their relative ranking and their allocated rates. As long as the producing rate allocations remain unchanged, the cost coefficients for the injectors also stay the same. However, when Q_{rtot} drops to the point that WK-107 is shut in entirely (90 kg/s), the ranking of the producers shift, which changes their allocated rates and alters the cost coefficients of the injectors. Table 4 also illustrates that the allocations by LPAL3 and QPAL generally agree. This agreement breaks down when Q_{rtot} is reduced to 50 kg/s, because the ranking of producers becomes indeterminate. LPAL3 chooses to curtail WK-121, but this choice is arbitrary because all remaining producers have the same cost coefficient (0.005). QPAL curtails a different set of producers, but labels the solution as non-unique. It should be noted, however, that even with a sparse data set, the problem of indeterminacy does not occur until only one injector remains active. This illustrates that LPAL3 and QPAL make use of all available data first in deciding which wells to cut back.

The second sensitivity study fixed Q_{rtot} at 100 kg/s and decreased Q_{ptot} from a capacity rate of 689 kg/s to 400 kg/s. In this case, the cost coefficients for the injectors shift continuously as Q_{ptot} is reduced. However, because WK-107 is ranked as the most damaging injector at all levels of Q_{ptot}, the injection rate allocation stays the same, and the cost coefficients for the producers stay the same as well. As Q_{ptot} is reduced, the producers are throttled back one at a time, according to rank. The allocations by QPAL agree with those by LPAL3, as before. At a Q_{ptot} of 400 kg/s, an

Table 4. Sensitivity of Wairakei well ranking to total rate.
Case 1. Q_{ptot} = 550 kg/s; Q_{rtot} varies

	LPAL3				QPAL	
Q_{rtot} kg/s	Injectors	Cost Coefficient	Curtailed Producers	Cost Coefficient	Injectors	Curtailed Producers
140	107	81	24a	250	107	24a
	101	20	48a	167	101	48a
	80	13	121a	33	80	121a
			116a	31		116a
			76b	29		76b
100	107b	81	24a	50	107b	24a
	101	20	48a	33.3	101	48a
	80	13	121a	33.3	80	121a
			116a	31.2		116a
			76b	28.5		76b
90	107a	312	121a	33	107a	121a
	80	8	116a	31	80	116a
	101	0.055	76a	29	101	76a
			103a	20		103a
			108b	9		108b
70	107a	312	121a	33	107a	121a
	80b	8	116a	25	80b	116a
	101	0.055	76a	24	101	76b
			103a	20		103a
			108b	5		108b
50	107a	308	116a	15	80a	18a,d
	101a	16	76a	13	101a	22a,d
	80	550	108a	9		24a,d
			121b,c	0.005		30a,d
						121a,d
						116b,d

a Total curtailment
b Partial curtailment
c Arbitrary allocation by LPAL3
d Non-unique allocation by QPAL3

indeterminate condition is reached when LPAL3 elects arbitrarily to curtail WK-68, which has the same cost coefficient as WK-70. QPAL makes the same allocation, but labels it as non-unique.

In the third sensitivity study, Q_{rtot} was fixed at a lower rate of 70 kg/s, and Q_{ptot} was again reduced gradually from an initial rate of 689 kg/s. The first point to note is that the ranking of producers differed from that of the previous sensitivity study because the injection allocation has changed (*i.e.*, WK-107 has been shut in). As additional producing wells are shut in, the ranking of the injectors shifts so that WK-80 rather than WK-101 is curtailed. This causes a corresponding realignment of the producing wells. At a Q_{ptot} of 500 kg/s, the problem becomes doubly indeterminate, first because the two remaining injectors have the same cost coefficient (0.05), and second because all the producers after WK-108 also have identical cost coefficients (0.007). For this indeterminate case, LPAL3 and QPAL make different allocations in both the injector and the producer categories.

In summary, the three sensitivity cases showed that LPAL3 and QPAL could optimize injection for a fixed injector configuration in a way that accounted for the interdependence of injection and production rates. The rate allocations provided by LPAL1 and LPAL2 were less satisfying because they were based on a fixed well ranking. The ranking provided by LPAL3 depended on total rates, though for marginal changes in total rates the relative ranking remained the same. The rate allocations of LPAL3 and QPAL agreed in all cases except when the optimal allocation was indeterminate.

5.2.2 Sensitivity to Different Weighting Factors

A sequence of runs applying a variety of different weighting factors to Wairakei data showed that tracer test parameters tended to yield similar rate allocations, whether used singly or in combinations. Further, elevation changes alone could be used to calculate allocations which were similar to those from tracer test parameters. On the other hand, using just the horizontal distance between wells yielded totally different rate allocations. This suggests that, for fractured reservoirs such as Wairakei, elevation changes are much more important than horizontal distances in determining optimal injection allocations.

5.3 Optimal Configuration

To demonstrate the application of the configuration-choosing program (INCON) to the Wairakei Field, two runs were made, the first based on elevation changes between producing and injecting zones (*H*-based), the second based on horizontal distances between wells (*L*-based). These two data sets were selected because, despite all the tracer data available for Wairakei, *H* and *L* were the only parameters that could provide a characterization for each arc. The runs assumed that Q_{ptot} and Q_{rtot} were to be 550 and 100 kg/s, respectively. The maximum number of injectors was specified as three. With a total of 22 wells, this meant that the maximum number of configurations to be considered was 1,540. However, because not all the combinations of wells could achieve the required total rates, the number of combinations for which INCON actually performed a rate allocation was only 1,122. Each execution of the program with these data sets required about 50 minutes of real time using a DEC VAX 11/750 computer.

As would be expected, the two configurations are quite different. The *H*-based configuration (see Table 5) places injection in deep wells near the center of the field, while the *L*-based configuration places injection in isolated wells at the field's southeast corner. Based on the parallels between *H*-based and tracer-based rate allocations discussed in the previous section, it might be reasonable to expect that the *H*-based injector configuration would be better in practise. However, it should be noted that the final *H*-based configuration depends not just on elevations but on rate constraints. If INCON had optimized on the basis of elevation alone, it would have simply chosen the three deepest wells (WK-48, WK-121, and WK-18) as injectors. Because these wells could not collectively produce 100 kg/s, the program designated WK-121 and WK-18 as inactive producers and chose the next two deepest wells (WK-24 and WK-55) as injectors instead. The combined maximum rates of these three injectors happened to be exactly 100 kg/s. It is clear, though, that slight changes in either the estimated capacities for individual wells or the required total rate could cause the optimal *H*-based configuration to change significantly.

6. CONCLUSIONS

6.1 The optimization of injection scheduling in geothermal fields may be accomplished by working in relative terms with data directly available from tracer tests, field geometry, and operating considerations.

6.2 Linear and quadratic programming may be used to allocate a specified total injection rate among pre-chosen wells. Such methods should allow for the interdependence of injection and production rates in determining the likelihood of thermal breakthrough.

6.3 The optimization techniques described in this study make use of all available data first in deciding which wells to eliminate as injectors. The techniques are not a substitute for efforts to understand reservoir behavior in a more physical sense, but

Table 5. Rate allocation for optimal well configuration in Wairakei, based on elevation differences between wells. Q_{rtot} = 100 kg/s; Q_{ptot} = 550 kg/s

Injector Name	Maximum Injection Rate	Assigned Injection Rate
WK-24	33	33
WK-48	20	20
WK-55	47	47
Producer Name	**Maximum Producing Rate**	**Assigned Producing Rate**
WK-18	11	0
WK-22	13	13
WK-30	44	44
WK-44	34	0
WK-67	50	40
WK-68	10	10
WK-70	40	40
WK-74	49	49
WK-76	45	45
WK-80	50	50
WK-80	52	0
WK-83	52	52
WK-88	53	0
WK-101	40	40
WK-103	28	28
WK-107	50	50
WK-108	51	51
WK-116	38	38
WK-121	19	0

they allow a geothermal developer to make beneficial use of whatever tracer return data are available.

6.4 For the Wairakei Geothermal Field, several different combinations of tracer test data yield the same allocations of injection and production rates. This suggests that the design of an optimal injection strategy does not depend critically on fine details of tracer response.

6.5 For fractured reservoirs such as Wairakei, elevation differences between production and injection zones are much more important than horizontal distances between wells in determining the optimal allocation of injection rates. The fact that large elevation differences tend to correlate with strong tracer response supports the theory that reinjected water moves rapidly downward within the reservoir.

6.6 The choice of an optimal injector configuration may be made by enumerating all feasible configurations, optimizing the rate allocation for each, and selecting the configuration with the lowest potential for premature thermal breakthrough. However, the solutions provided by such an approach are very dependent on specified rate constraints.

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