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AN EFFICIENT PROCEDURE FOR THE NUMERICAL SIMULATION OF
GEOHERMAL RESERVOIRS

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ABSTRACT

Much computing time is spent when a numerical simulator is used to study a geothermal reservoir. An effort was made to develop an efficient methodology to solve the non linear system of equations in each time step of the initial value problem. Advantage was taken from the experience reported in the literature and some improvements were introduced: an algebraic manipulation to reduce the number of equations, a different treatment to equations for regions under and above the saturation curves and a simple procedure to shift from one treatment to the other. The resulting system of equations is almost linear and its exact solution is approached through Newton-Ramphson corrections to the main variables and updating of other variables in each iteration. A computer code was developed and it was validated applying it to the six problems posed by the US Department of Energy in 1980. Computing times were significantly lower than those of other codes.

INTRODUCTION

A great deal of work has been done in the numerical simulation of geothermal reservoirs and some computer codes proved to be reliable in solving the mass momentum and energy balance equations to simulate the flow of hot water in one or two phases in a permeable medium. This was made clear in the workshop held at Stanford (Standord University, 1980) to compare results from several computer codes in the solution on six problems with varying difficulty posed by the U.S. Department of Energy. Some conclusions from the workshop were: "The studies presented at this workshop clearly indicated that these simulators can solve a wide variety of geothermal problems... they can describe the physical processes as well as we now understand them... Model efficiency is important because models are becoming more complicated and therefore costly to implement; a large number of runs are normally required in analyzing a typical field problem; model efficiency is critical to the use of Monte Carlo methods of risk assessment.... It is infortunate indeed that program efficiency was not documented".

In the previous experience of the author in using an available code and in developing another

(Cruickshank, 1982) to study specific reservoirs, much computing time was employed in the simulation runs; this, in most computing facilities, is inconvenient and causes project delays because many runs have to be made through the model calibration, sensitivity analysis and alternative testing stages of the study of any reservoir. Therefore it was considered important to dedicate effort to develop efficient procedures of geothermal reservoir simulation.

There have been claims about procedure efficiency in the literature (Coats, 1977, Cheng and Crichlow, 1980) and that experience with some further developments resulted in the methodology described here which is thought to be both flexible and efficient and of acceptable accuracy comparing its results with those of other simulators.

GOVERNING EQUATIONS

The basic equations to be solved are the conservation laws of mass, momentum and energy which expressed for a control volume Ω bounded by a surface Γ have the general form:

$$\int_{\Gamma} q_i (F + F_D) \cdot \bar{n} d\Gamma + \frac{\partial}{\partial t} \int_{\Omega} q_i \rho d\Omega = \int_{\Omega} Q_i d\Omega \quad (1)$$

where q_i is a mass associated (intensive) property.

The geothermal reservoir is considered to be a porous and permeable rock filled with the geothermal fluid. In the present development only one fluid component (water) in two phases (liquid and vapor) is considered. In such a medium, the mass conservation equation is applied only to the fluid in the void fraction of the rock; with $q=1$ and the integral of the dispersive term equal to zero, eq. 1 becomes

$$\int_{\Gamma} \bar{F} \cdot \bar{n} d\Gamma + \frac{\partial}{\partial t} \int_{\Omega} \phi \rho d\Omega = \int_{\Omega} M d\Omega \quad (2)$$

Also, the momentum equation may be seen to reduce to Darcy's law:

$$\bar{F}_j = - \frac{|k|k_j}{\mu_j} \rho_j (\nabla P - \rho_j \bar{g}); j = \text{liquid, vapor} \quad (3)$$

The conservation of energy is applied to the bulk of rock and fluid; with $q=u$ the internal energy only since the kinetic energy is very small, eq. 1 is:

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$$\int_{\Omega} (\bar{F} + \bar{F}_D) \cdot \bar{n} d\Gamma + \frac{\partial}{\partial t} \int_{\Omega} U d\Omega = \int_{\Omega} E d\Omega \quad (4)$$

with

$$U = \rho \phi u + (1 - \phi) \rho_r u_r$$

Energy source terms include work done by external forces, mainly pressure and gravity, and the energy taken in or out of the system by the mass sources. Neglecting the work of gravity the pressure work may be added to the convective transport of internal energy, leaving only energy attached to mass sources in the right hand side of eq.4.

The dispersive transport integral may no longer be taken as null and is normally considered to be directly proportional to the gradient of the mass associated property in balance. Two mechanisms contribute to this term in the case of energy; heat conduction and mechanical dispersion. The last term is usually neglected although in fractured and layered formations may be larger than conduction. Assuming that mechanical dispersion may be expressed as a diffusivé quantity, the dispersive transport term may be expressed as:

$$\int_{\Gamma} h \bar{F}_D \cdot \bar{n} d\Gamma = \int_{\Gamma} K_D \nabla T \cdot \bar{n} d\Gamma \quad (5)$$

where K_D includes thermal conductivity and dispersivity.

Calling

$$\bar{G} = -K_D \nabla T + \sum_j h_j \bar{F}_j \quad (j = \text{liquid, vapor})$$

eq.4 is transformed to

$$\int_{\Omega} \bar{G} \cdot \bar{n} d\Gamma + \frac{\partial}{\partial t} \int_{\Omega} U d\Omega = \int_{\Omega} E d\Omega \quad (6)$$

Eqs. 2 and 6 are exactly those reported in the literature, except for the inclusion here of the mechanical dispersion term.

Apart from the conservation laws, equations of state are needed to describe the thermodynamic state of the fluid within the pores of fractures of the rock formation. These equations relate fluid properties like density, internal energy, viscosity, etc, to state variables, mainly pressure, temperature and phase concentration. These relations may be obtained from standard thermodynamic tables or from equations that fit the tables. The equations obtained by Mercer and Faust (1977) are used here. Some simplifying assumptions are a local thermal equilibrium between phases including the rock and a small influence of surface tension; the first one is acceptable for granular media and even for fractured media with distances among fractures of the order of some meters in view of the mesh sizes and time integration steps normally used to simulate a geothermal reservoirs; according to some experiments, surface tension influence appears to be very low for temperatures above 100°C (Faust and Mercer, 1979).

SPACE AND TIME INTEGRATION

The simulation of the behaviour of a geothermal

reservoir is achieved by solving the initial value problem posed by equations 2 and 6 in the region of interest with the corresponding time dependent boundary conditions. With variable medium characteristics and complex boundaries the solution is conveniently obtained by discretizing the regions, that is to say, subdivide it into subregions and apply the conservation equations to each one of them; the solution (determination of state variables) is then obtained for representative points within the region; these may be nodal points at the boundaries of the subregions as in the finite element method, at their center as in the integrated finite difference method (IFD) or at mesh points as in the finite difference method. Here, the IFD method was selected because its simplicity, its adaptability to irregular geometries and its ability to manage problems from zero to three dimensions without change in the algorithmic formulation; these advantages are explained in more detail by Preuss and Narashiman (1985). The main relative disadvantage is that the lines connecting adjacent elements must be coincident with a principal axis of permeability.

Eqs. 2 and 6 should be integrated with respect to time; this may be done using the mean value theorem and a mean value which is a linear combination of the integrand extreme values in the interval:

$$\int_{t_k}^{t_{k+1}} I(t) dt = [(1-\theta)I(t_k) + \theta I(t_{k+1})] \Delta t \quad (7)$$

The time integral of the storage terms is simply the difference between its final and initial values.

At any point in space and time, the state of the fluid is fully determined by two state variables: if only one phase is present these variables are pressure and temperature; in a two phase mixture they are phase concentration (liquid or vapor) and either pressure or temperature since in thermic equilibrium there is a unique relation among them. It has been usual in previous works to resort to internal energy or enthalpy as a substitute of temperature to avoid the duality of state variables in one and two phases; nevertheless the new variable is dependent on phase concentration. The need for two state variables is the reason for the use of two time dependent balance equations to determine the development of a geothermal system.

With the space discretization and time integration by eqs. 7 the simulation problem reduces to the solution of a set of 2 N simultaneous non linear equations in 2 N unknowns which are the N couples of state variables at the end of the time interval and at N internal discretization points of the region. This system of equations has to be solved through iterative methods among which the Newton-Raphson technique has proved to be reliable and efficient provided time steps are not taken too large when transition from one to two phases or viceversa occurs. This is because the more important non linearity of the geothermal process lies precisely in the phase change.

This fact points to the possibility that a special iterative procedure in phase transitions might be a first step to more efficient solution algorithms.

Some researchers (Chen and Chrichlow, 1980) have taken advantage of the fact that in one phase regions the change in temperature (or enthalpy) is much smaller than that in pressure, allowing the uncoupling of the mass and energy equations between iterations or from one time step to the next. This allows to write the mass balance equations as a quasi linear system of equations in p^{k+1} and the energy balance equations as a system in h^{k+1} and solving them successively by iterations to take into account their mutual influence. The two phase region is treated similarly but solving the energy equation in the first place because enthalpy changes are larger than those of pressure.

Yet another alternative to diminish the computational effort is the use of the alternating direction iteration or the line successive overrelaxation methods of solution of the system of non linear equations in two and three-dimensional regions. The need for too many iterations to meet some specific error tolerance could handicap these methods in front of the direct solution of the complete system.

THE PROPOSED METHODOLOGY

Probably the most important among the above considerations in problems involving phase changes and coexistence of one and two phases is that of a particular procedure to deal with the phase change. The second important question is a different treatment for each region because its obvious different behaviour which is reflected in the variation change of the state variables. This is the case for water which, as mass is extracted or injected within a given volume, experiences large pressure changes in one phase state and large enthalpy or vapor concentration variations in the two phase region.

Based on this last observation a step to reduce the number of equations is possible by ranking the importance of the state variables changes during the time integration interval.

For the monophasic flow, pressure is the variable with larger changes and they may be evaluated through the mass balance (eq. 2) in its Newton-Ramphson with line successive overrelaxation form as:

$$\begin{aligned} 0\Delta t \left(\sum_{\ell=1}^n M_{\ell}^{k+1} \delta p_i - M_{i-1}^{k+1} \delta p_{i-1} - M_{i+1}^{k+1} \delta p_{i+1} \right) \\ = (1-0)\Delta t \sum_{\ell=1}^n M_{\ell} (p_{\ell}^k - p_i^k + g\rho_{\ell}^k dz_{\ell}) + \theta\Delta t \sum_{\ell=1}^n M_{\ell}^{k+1} \\ (p_{\ell}^{k+1} - p_i^{k+1} + g\rho_{\ell}^{k+1} dz_{\ell}) - (\rho_i^{k+1} \phi_i^{k+1} - \rho_i^k \phi_i^k) + S_m \end{aligned} \quad (8)$$

which forms a tridiagonal system of N_1 equations in the correction to the pressure p^{k+1} for each iteration, where N_1 is the number of elements with one phase flow; since ρ^{k+1} and μ^{k+1} suffer little variation they are simply updated in these equations at each iteration. Temperature varies little also and may be explicitly obtained from a combination

of the energy and mass equations where new variable values are updated through iterations:

$$\Delta T = \frac{S_e + R_e - U^k (S_m + R_m)}{\rho^{k+1} \phi^{k+1} c_{v\ell} + (1-\phi) \rho_r c_{vr}} \quad (9)$$

For a two phase flow the property that undergoes larger changes is liquid (or vapor) fraction of the mixture; in order to reduce the number of equations it is convenient to eliminate it using the mass and the energy conservation equations. From the latter on gets:

$$\begin{aligned} \Delta C_{\ell} = \{ S_{\ell} + R_{\ell} - [C_{\ell}^k \phi_{\ell}^{k+1} \rho_{\ell}^{k+1} c_{v\ell} + (1-\phi) \rho_r c_{vr}] \Delta T \\ - (1 - C_{\ell}^k) \Delta(\phi \rho_v u_v) - C_{\ell}^k u_{\ell}^k \Delta(\phi \rho_{\ell}) \} / \left[\phi^{k+1} (\rho^{k+1} u^{k+1} - \rho_v^{k+1} u_v^{k+1}) \right] \end{aligned} \quad (10)$$

Using the Clausius-Clapeyron relation for changes of P and T in the two phase region ΔT is transformed to ΔP in eq. 10 and the value of ΔC_{ℓ} is substituted in the mass equation:

$$S_m + R_m = \Delta(\phi \rho_v) + C_{\ell}^k \Delta(\phi(\rho_{\ell} - \rho_v)) + (\rho_{\ell}^{k+1} - \rho_v^{k+1}) \phi^{k+1} \Delta C_{\ell} \quad (11)$$

This gives rise again to a tridiagonal system of N_2 equations in p^{k+1} only where the final values of ρ_{ℓ} , ρ_v , u_v , ϕ which have little change are updated through iterations; ΔC_{ℓ} is obtained explicitly from eq. 10.

With this manipulations at every time interval and iteration, one has to solve a tridiagonal system of only $N=N_1+N_2$ equations in p^{k+1} , thus reducing considerably the computing times.

Every time interval is integrated with each element in its initial state be it one, or two phase and the new set of state variables is tested to find out if there has been a change of state; if that is the case for some element the calculation is taken over again starting from the saturation curve at the point that is crossed by the initial path of integration. This is illustrated in fig 1 with a P-h diagram. If at the end of the time interval the final condition lies under the saturation line, the crossing point is determined using the slope of the saturation line and that of the integration path

$$P_s = p^{k+1} - \frac{h_s^{k+1} - h_s^{k+1}}{\left(\frac{dh_s}{dp} \right)^{k+1} - J} \quad (12)$$

with

$$J = \frac{h^{k+1} - h^k}{p^{k+1} - p^k}$$

The computation is restarted with initial conditions at the points P_s , h_s with an integration time interval for that element of

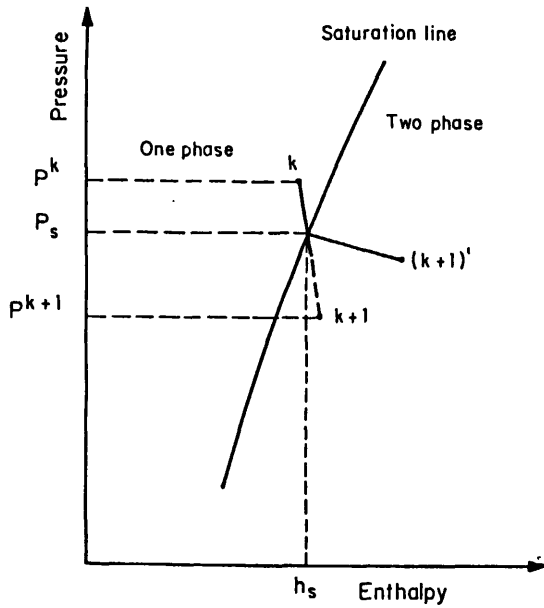


Figure 1. Crossing the saturation line

$$\Delta t' = \left(1 - \frac{p^k - p_s}{p^k - p^{k+1}} \right) \Delta t \quad (13)$$

which ends up at the point (k+1)'

This simple procedure is used in any direction the change of phase state takes place, be it from one to two phase or viceversa, and from or to liquid or vapor single phase.

MODEL VALIDATION AND COMPARISON

A computer code was built in FORTRAN using the described algorithm, where, as was already mentioned, the IFD discretization and mainly the constitutive equations by Faust and Mercer (1977) were used.

In order to test if the algorithm had any undesired numerical error the code was applied to the six problems used by the US Department of Energy to test geothermal simulator codes from different make. The results are in all coincident with results from other simulators and are reported elsewhere (Cruickshank, 1987).

Processing time comparison has been made with a code of the US Geological Survey (Faust and Mercer, 1977) solving a two-dimensional aquifer where CPU time was reduced from 60 min to 3 min in a Burroughs 6700, and with a previously developed code (Cruickshank, 1982) which for the same problem and machine used 25 min CPU. Computing time was saved both by efficiency in solving the system of equations (rarely more than two iterations were needed) and by the possibility of employing large time intervals. Code comparison with more modern available codes is in course in collaboration with

the Instituto de Investigaciones Eléctricas (México).

CONCLUSIONS

In an attempt directed towards reducing computing effort in the numerical simulation of geothermal reservoirs, a new procedure was developed which assembles methodologies reported in the literature to have had success in geothermal problems and some new ones that further improved them. The procedure was validated comparing its results with those of other simulators applied to the same problems and the processing time employed was significantly lower than two other available codes.

It is not argued that this is the best methodology to numerically simulate geothermal reservoirs, but it is thought that some of the underlying ideas might help to improve some of the existing models.

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LIST OF SYMBOLS

- A_ℓ area of interphase between the volume element and its ℓ adjacent element, L^2
- $c_{v,j}$ specific heat (constant volume) of phase j L^2/t^2T
- C_ℓ liquid phase fraction
- d_ℓ distance between nodal point of the element and its ℓ adjacent element, L
- dz_ℓ vertical projection of d_ℓ , L
- E intensity of energy source, L^2/t^3
- \bar{F} mass flux vector, m/L^2t
- \bar{F}_D mass dispersion vector, mL^2/t
- \bar{g} gravitational vector, L/t^2
- \bar{G} energy flux vector, m/t^3
- h_j specific enthalpy of phase j , L^2/t^2
- I function integrated in time
- k tensor of absolute intrinsic permeabilities, L^2 ; time superscript
- k_j relative permeability of phase j
- K_D tensor of thermal diffusivities, mL/L^3T
- M intensity of mass source m/tL^3
- M_ℓ mobility between the element and its ℓ adjacent element, $M_\ell = k_\ell A_\ell / \mu_\ell d_\ell$, L^4t/m
- R_i inward transport term for the element, $i=mass, energy$

$$R_m = - \int_{t_k}^{t_{k+1}} \int_{\Gamma} \bar{F} \cdot \bar{n} d\Gamma dt; R_e = - \int_{t_k}^{t_{k+1}} \int_{\Gamma} \bar{G} \cdot \bar{n} d\Gamma dt$$

S_i source term for the element, i =mass, energy

$$S_m = \int_{t_k}^{t_{k+1}} \int_{\Omega} M d\Omega dt \quad S_e = \int_{t_k}^{t_{k+1}} \int_{\Omega} E d\Omega dt$$

t time, t

T temperature, T

u_j specific internal energy of phase j , L^2/t^2

U internal energy contained in the rock-fluid mixture per unit volume of the medium L^2/t^2

Γ closed surface bounding volume Ω , L^2

δV Newton-Ramphson method correction between iterations for variable V

ΔV time increment of variable V

μ viscosity, mL/t

ρ mass density, m/L^3

ϕ porosity

θ time weighting coefficient

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