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ANALYSIS OF PRODUCTION DATA FROM FRACTURED LIQUID DOMINATED GEOTHERMAL RESERVOIRS IN ICELAND

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ABSTRACT

The functional relation between production mass flow and the resulting pressure decline furnishes important data on various reservoir properties. In conjunction with particular interpretational models these relations provide estimates of reservoir parameters. Since such results are contingent upon the models selected, we refer to the estimates as apparent values of the parameters. Using available production data from liquid-dominated geothermal systems in Iceland we have obtained some estimates of the reservoir parameters of the Icelandic flood-basalts. Based on simple free liquid surface interpretational models, the inferred apparent permeability of the flood-basalts of north-central Iceland is less than 10^{-15} m^2. The values for the south-west of Iceland are an order of magnitude higher.

INTRODUCTION

The principal physical parameters of fracture dominated reservoirs, in particular, the permeability and the porosity, are invariably poorly defined. Surface exploration provides practically no information on these parameters and observations on core samples provide limited information on the permeability. Moreover, standard well-tests tend to yield biased estimates that depend on the overall test procedures. The results can only represent weighted averages of the parameters over some vaguely defined formation volumes. In other words, the resulting estimates will have to be regarded as apparent values contingent upon the particular test procedure applied.

As a consequence, production from fracture dominated reservoirs very frequently has to be initiated without much reliable information on subsurface conditions. In fact, the most useful information is gradually revealed in the form of pressure decline histories as production is continued over longer periods of time. Upon proper interpretation of the observational data and model simulation, the past can then be applied to estimate important parameters and to predict future reservoir responses.

Depending on conditions, various methods are available to extract information on reservoir properties from pressure decline data. We have developed some rather simple analytical methods for the case of unconfined liquid dominated reservoirs. These methods are applicable to cases where the pressure decline data are limited to one or a very few observational boreholes and additional data on subsurface conditions are scarce. In such cases more detailed modeling would hardly be appropriate. They can, moreover, be used to obtain the first large-scale averages before more detailed modeling is attempted. The principal purpose of the present paper is to discuss the application of these procedures to a few field examples of fracture dominated geothermal reservoirs in Iceland.

FREE LIQUID SURFACE MODELS

Pressure decline in geothermal reservoirs involves transient processes that result from capacitive effects in the systems. There are mainly two types of such effects, that is, liquid/formation compressibility and the mobility of free liquid surfaces and other similar interfaces. Signals resulting from compressibility effects have a very short relaxation time (Bodvarsson, 1984) when measured on the time scale of long-term production. Since we are mainly interested in the interpretation of pressure decline data that have been obtained over periods of years we will disregard the compressibility effects. On the other hand, the field examples to be set forth below involve liquid dominated reservoirs that have a free liquid surface such that specific interpretational models have to be developed to account for this type of capacitance.

Phenomena resulting from the movement of interfaces such as a free liquid surface are of a non-linear nature and lead to mathematical difficulties. Bodvarsson (1977, 1984) has, however, demonstrated that cases with a small free liquid surface amplitude can be linearized and the quantitative treatment is then much simplified. Since very many practical cases involve such conditions, pertinent results of the linearized theory will be presented in the following.

We first consider the testing of a half space that is composed of a liquid saturated permeable heterogeneous and anisotropic formation. We assume that there is a free liquid surface at some depth below the surface of the permeable formation. The field setup is such that until time t = 0 the liquid is in equilibrium with a horizontal free surface. At time t = 0 the injection of a constant mass flow of liquid q is being initiated at the injection port Q. The resulting flow pressure field causes a gradual rise of the free liquid surface in a region above Q. Observing the flow pressure p(P,t) at a field point P and time t, we can define the unit step response of the system

$$(1)\quad u(P,Q,t) = p(P,t)/q$$

The simplest interpretational models that are pertinent to this situation are homogeneous and isotropic models. Assuming a half-space of this type and using a linearized surface boundary condition, Bodvarsson (1977) has derived the following results.

Place a coordinate system with the (x,y) plane in the equilibrium liquid surface and the z-axis vertically down. The injection port is placed at Q = (0,0,d), that is, on the z-axis at a depth d below the equilibrium surface. The geometry of the situation is displayed in Figure 1. Let $c=k/\nu$ be the liquid conductivity of the half-space with k the permeability of the formation and ν the kinematic viscosity of the fluid. Furthermore, let ϕ be the porosity of the half-space, g be the acceleration of gravity and define a characteristic velocity $w = cg/\phi = kg/\phi\nu$. At the conditions described, the unit step response u(P,Q,t) of the liquid pressure is

$$(2)\quad u(P,Q,t) = (1/4\pi c)\left[(1/r_{PQ}) + (1/r_{PQ'}) - (2/r_{PM})\right]$$

where r_{PQ} is the distance between the two ports, $r_{PQ'}$ is the distance from P to the reflection image of Q over the plane z=0 and M=(0,0,-(d+bt)) is a moving image and hence

$$(3)\quad r_{PM} = \left[x^2 + y^2 + (z + d + wt)^2\right]^{1/2}$$

The moving image accounts for the evolution of the free liquid surface.

Turning back to the heterogeneous and anisotropic half-space, we assume now that u(P,Q,t) has been measured for a pair of ports. Equation (2) can then be applied to obtain estimates of the conductivity c and characteristic velocity w. These estimates are referred to as apparent values contingent upon the interpretation on the basis of a homogeneous and isotropic half-space free liquid surface model.

The development above reflects the simplest possible situation that would apply for a reservoir with a free liquid surface in the absence of any additional information of geological or physical nature on the permeable formation under consideration. Where the only observational data is a pressure decline function, the obvious choice of model is a homogeneous and isotropic half-space.

Many field situations are strongly anisotropic such that the flow is practically unidirectional. To obtain the pertinent modification of the above equations, we assume now that the flow is parallel to the x-axis and that Q represents a line-source of unit density mass flow placed parallel to the y-axis at a depth d. As shown in Figure 2, this is now a two-dimensional situation where the unit step response is independent of the y coordinate. The expression for this function follows from equation (2) by an integration with respect to y over the entire y-axis, resulting in

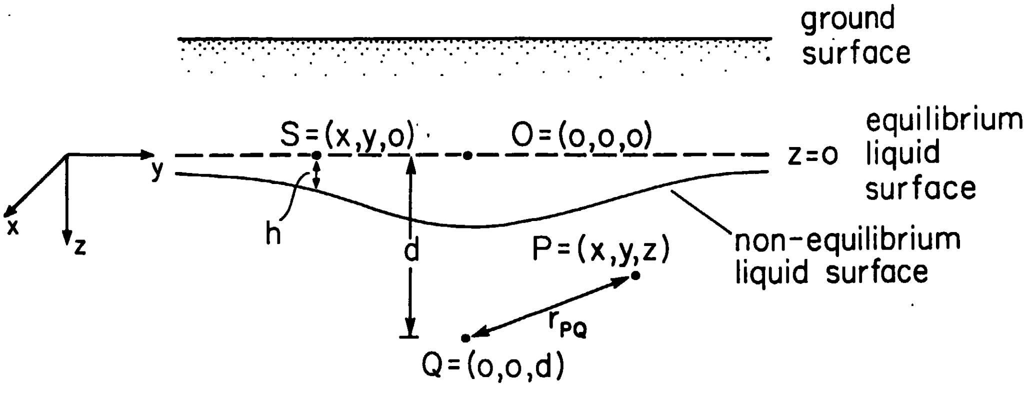

Figure 1. The unconfined half space model.

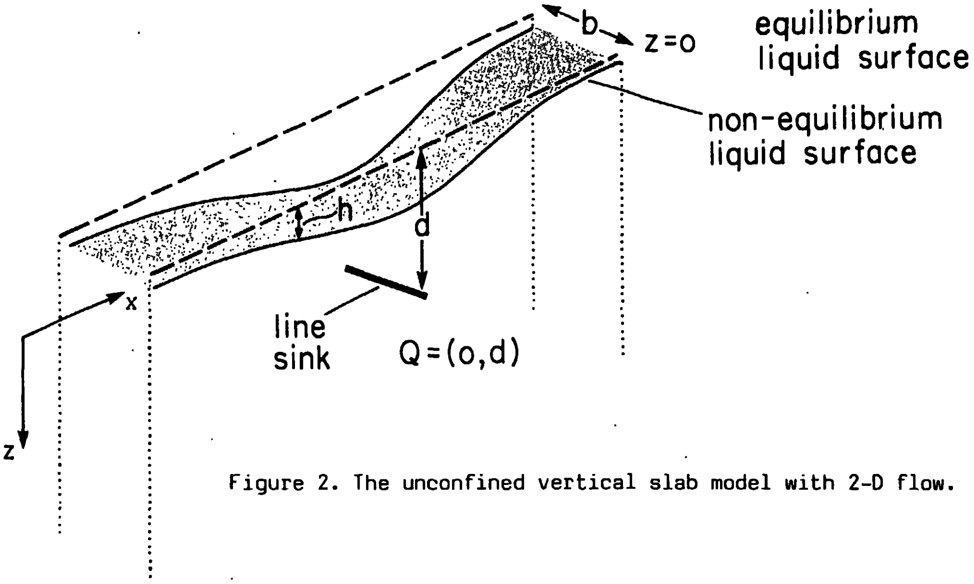

Figure 2. The unconfined vertical slab model with 2-D flow.

$$u(P,Q,t) = (1/2\pi c)\left[\log(r_{PQ})+\log(r_{PQ'})-2\log(r_{PM})\right] \quad (4)$$

where Q' and M are now line images and distances are the perpendiculars from P to the line sources such that, for example,

$$(5)\quad r_{PM} = \left[x^2 + (z+d+wt)^2\right]^{1/2}$$

The above results apply equally well to the whole anisotropic half-space as to a vertical slab of constant width b that is parallel to the (x,z) plane. In the case of a total line-source mass flow q into the slab of width b, the factor in the front of the brackets in equation (4) has to be adjusted to ($q/2\pi cb$).

APPLICATION OF THE FREE SURFACE MODELS

Consider now the case of a heterogeneous and anisotropic liquid dominated reservoir with a free liquid surface where we have a source flow history q(Q,t) for the source point Q and have observed the pressure response p(P,Q,t) at a field point P. The interest centers on obtaining apparent values of the porosity ϕ and the permeability k from the pair of functions. In the case of variable production rate q(t) at the source point Q, we first have to derive the unit step response for the pair of ports. We assume causal conditions, that is, equilibrium for the time $t \le 0$. The following integral equation for the causal response function G(P,Q,t) has to be solved

$$(6)\quad p(P,Q,t) = \int_0^t G(P,Q,(t-\tau))q(\tau)d\tau$$

The unit step response is then

$$(7)\quad u(P,Q,t) = \int_0^t G(P,Q,\tau)d\tau$$

An attractive method for solving equation (6), based on linear programing techniques, has been given by Coats et al. (1964).

Suppose that geology indicates that the reservoir is of such extent and depth that it can be represented by the entire half-space. The apparent values of the characteristic velocity w and the conductivity c are then obtained by an iterative non-linear least-squares numerical matching (Menke, 1984) of expression (2) to the observed unit step function. In more detail w is determined by the curvature of the response and the amplitude of the response determines c.

A few remarks are called for. First, because compressibility has been neglected, equations (2) and (4) are not applicable during a short interval of time following the onset at t = 0. It is, however, a simple matter to show that this interval is very short and does in very many practical cases not interfer with the above matching process. Second, it is important that the formation porosity enters our equations only through a surface boundary condition (Bodvarsson, 1984). The value of the apparent porosity determined by the above procedure would thus only be relevant to the surface region where the free liquid surface is positioned.

ESTIMATES OF APPARENT RESERVOIR PARAMETERS

The procedure outlined above will now be applied to the estimating of apparent reservoir parameters for three low-temperature and one high-temperature liquid dominated geothermal systems in Iceland. Practically all geothermal systems in Iceland are embedded in flood basalts with a fracture-dominated permeability. In this type of formation, horizontal fluid conductivity is provided by openings along lava bed interfaces and through interbed layers of scoria and tuffs. Vertical conductivity is mainly provided by basaltic dikes and faults that are very numerous in the formation. There are swarms of parallel dikes and faults that result in a pronounced flow anisotropy. A brief overview of the physical conditions in the flood basalts has been given by Bodvarsson (1983).

The geothermal fields are listed in Table I and their locations are given in Figure 3. The fields have been in commercial production for a considerable time such that long-term data is available on the performance of the reservoirs (Palmason et al., 1983). Table I provides information on each of the individual areas. The production data and the unit step response functions derived on the basis of equation (6) are presented by Axelsson (1985). In all of the four cases long term pressure decline data is only available from one well and information on subsurface conditions rather limited. Examples of production data from two of the fields, Svartsengi and Laugarnes, are displayed in Figures 4 and 5 whereas the unit step response functions are displayed in Figures 6,7,8 and 9.

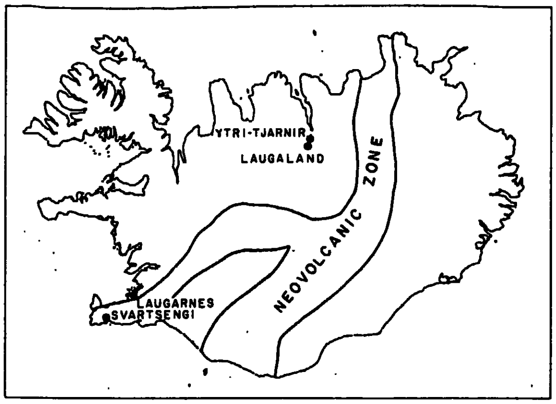

Figure 3. Location of geothermal fields listed in Table I.

In applying the procedures outlined above to the data from the four fields, the following principal simplifying assumptions are made. Because of the distribution of the production wells, we conclude in the case of the homogeneous/isotropic half-space model that the production region within the reservoir is sufficiently small that the flow can be assumed to be derived from a point-sink. Although there are no problems in considering distributed sinks, a numerical test indicates that this assumption is justified. In the case of the dike-swarm model we make the analog assumption that the production is obtained from a line-sink.

TABLE I List of geothermal fields being analyzed

Geothermal Field	Location & Classification	In Production Since	Production Depth	Reservoir Temperature
Svartsengi	SW Iceland HT/liquid-dominated	1977	800-1800 m	240°C
Laugarnes	SW Iceland LT-field	1960	700-1000 m	125°C
Laugaland	N Central Iceland LT-field	1978	1000-1500 m	95°C
Ytri-Tjarnir	N Central Iceland LT-field	1980	1000-1500 m	80°C

LT = low-temperature

HT = high-temperature

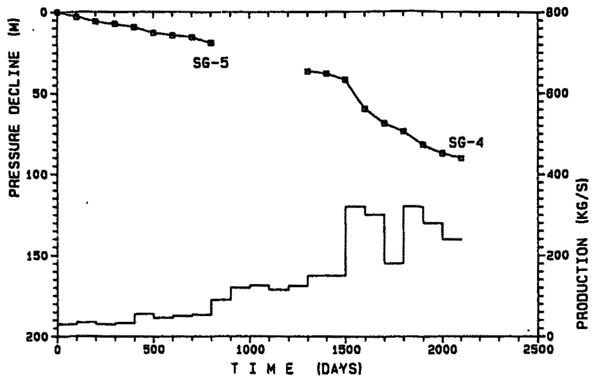

Figure 4. Production from the Svartsengi field and the resulting pressure decline in wells SG-4 and SG-5 since late 1976.

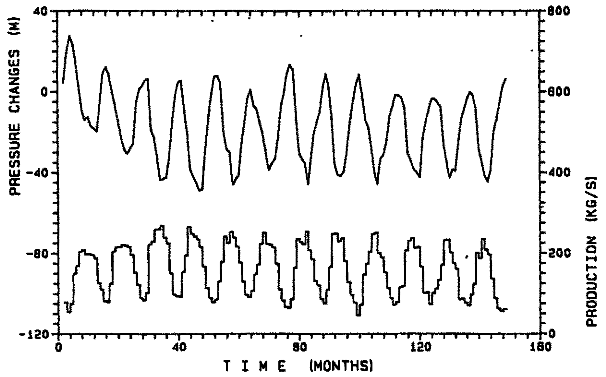

Figure 5. Production from the Laugarnes field and the resulting pressure changes in well RV-7 since June 1966.

The observational pressure decline data are obtained from non-producing boreholes that are located within the main production zones in the individual geothermal fields. It is clear that these boreholes are sufficiently well connected to the main aquifer systems that their water level displays the liquid pressure at some level within the systems. An inspection of equation (2) indicates that the amplitude, and hence the conductivity c, is not too sensitive to the depth of the field point P. The curvature, and hence the characteristic velocity w, is more sensitive to the location of P. We therefore make the assumption that the boreholes reflect the true ground-water level above the production zone. As a consequence, we place the field point P in equations (2) and (4) at the surface P = (r,0).

Based on these assumptions, the processing of the data on the basis of the procedures described above is now straight forward. Details of the methodology are given by Axelsson (1985). The unit step response functions are matched with the theoretical response functions based on equations (2) to (5), for both the half-space (3-D) and dike-swarm (2-D) models. The results are displayed in Figures 6,7,8 and 9. The optimum fit physical parameters are listed in Tables II and III. These are our principal results.

The quality of the apparent parameter estimates is difficult to assess accurately since the matching process is non-linear. Rough bounds on the apparent parameter estimates have therefore been found visually and the results indicate that the apparent conductivity c is estimated within 10-15% whereas the apparent characteristic velocity w is estimated within 20-30%.

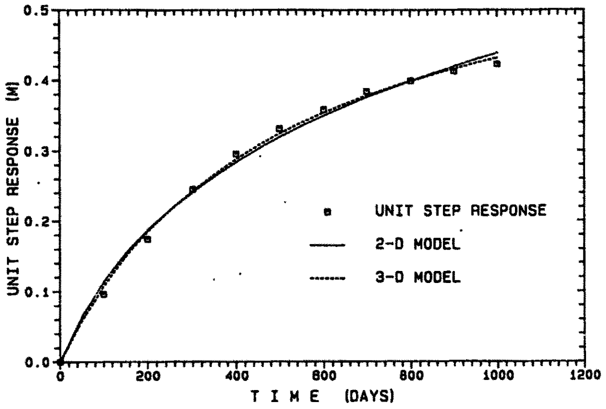

Figure 6. Unit step response of the Svartsengi field based on the data in Figure 4 along with the best fitting theoretical responses for the 2-D and 3-D free surface interpretational models.

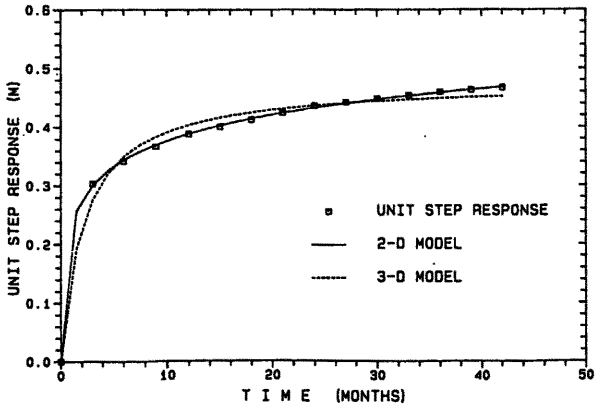

Figure 7. Unit step response of the Laugarnes field based on the data in Figure 5 along with the best fitting theoretical responses for the 2-D and 3-D free surface interpretational models.

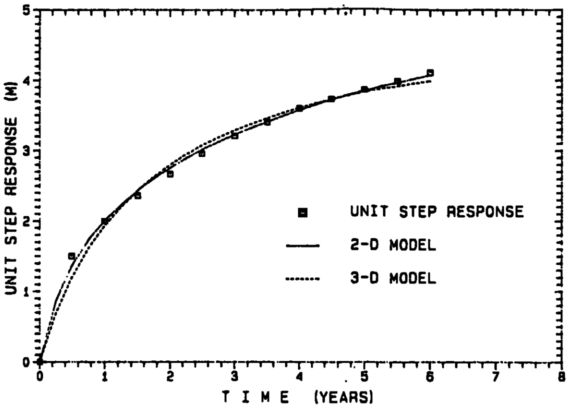

Figure 8. Unit step response of the Laugaland field along with the best fitting theoretical responses for the 2-D and 3-D free surface interpretational models.

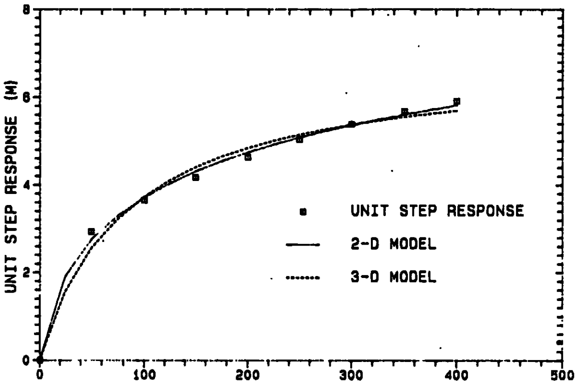

Figure 9. Unit step response of the Ytri-Tjarnir field along with the best fitting theoretical responses for the 2-D and 3-D free surface interpretational models.

DISCUSSION

The apparent permeability values set forth above furnish some information on the hydrological properties of the Icelandic flood-basalts. At this juncture, it is appropriate to underline that the results have to be viewed from the vantage point of the rather strong assumptions made directly or implicitly in the two interpretation models applied.

In discussing geothermal reservoir properties, a clear distinction has to be made between liquid reservoirs and reservoirs of thermal energy. In the individual areas, the extent of each type of reservoir depends on local geological and physical conditions. In the present context, where hydrological properties are in the focus of interest, it is the liquid reservoir that is of principal concern.

Clearly, the homogeneous/isotropic half-space model is a gross oversimplification that can only provide rather vaguely defined apparent or average values of the formation permeability in the particular locations of the individual geothermal fields. Since observations on the general results of drilling for thermal fluids in Iceland indicate that the thermally active reservoirs have a higher permeability than the non-active country rock, the apparent permeabilities listed in Table II are most likely higher than corresponding values for the non-active flood-basalts.

From the geological point of view, the dike-swarm with a predominantly parallel permeability is a more realistic model of the flow structure in the geothermal fields of Iceland. The observational data at our disposal yield only permeability widths, that is permeability by swarm-width products. Since it is frequently difficult to estimate the widths of the swarms from surface observations, the inferred permeability widths are not readily converted into more local average permeabilities. The parallel permeability widths will have to be regarded as the relevant result.

TABLE II Apparent permeability and apparent porosity based on the 3-D interpretational model with a field point at P=(r,o). The table is based on the assumption of a kinematic viscosity of 1.5×10^{-7} m^2/s for Svartsengi and 3×10^{-7} m^2/s for the other fields.

	depth d (m)	distance r (m)	permeability k (m^2)	porosity ϕ
Laugarnes	800	300	13×10^{-15}	2.7×10^{-3}
Laugaland	1200	300	0.82×10^{-15}	1.1×10^{-3}
Ytri-Tjarnir	1000	30	0.74×10^{-15}	1.8×10^{-4}
Svartsengi	1000	300	4.5×10^{-15}	1.2×10^{-2}

TABLE III Apparent permeability width bk, and apparent porosity width $b\phi$, based on the 2-D interpretational model with a field point at P=(r,o). The table is based on the assumption of a kinematic viscosity of 1.5×10^{-7} m^2/s for Svartsengi and 3×10^{-7} m^2/s for the other fields.

	d(m)	r(m)	bk(m^3)	$b\phi$(m)
Laugarnes	800	300	160×10^{-12}	0.5
Laugaland	1200	300	7.9×10^{-12}	1.8
Ytri-Tjarnir	1000	30	6.4×10^{-12}	0.2
Svartsengi	1000	300	30×10^{-12}	21

It is interesting to convert the observed permeability widths into estimates of the widths of the quasi-vertical fractures that provide the flow channels. Consider an extended fracture of uniform width h. In unidirectional laminar liquid flow in direction x along the fracture, the mass flow q per unit length across the flow is (Lamb, 1932)

$$(8) \quad q = -(h^3/12\nu)dp/dx,$$

where dp/dx is the pressure gradient in the direction of the flow. Hence, referring to the definition of permeability on the basis of Darcy's law, we infer that the permeability width of the fracture is $h^3/12$. In the case of a dike-swarm with n parallel fractures of the same width, the permeability width is thus

$$(9) \quad bk = nh^3/12.$$

Applying equation (9) to the four cases under study we obtain the results in Table IV. Considering observations on flows from fractures into boreholes in the geothermal areas (Bodvarsson, 1983), these values are not unreasonable.

We draw the following general conclusions from our results. Contingent upon the use of the homogeneous and isotropic half-space interpretation model, we infer that the global apparent permeability of the non-active flood basalts in central north Iceland is less than one millidarcy while the corresponding value for the southwest is an order of magnitude higher. The flow in the geothermal fields is controlled by quasi-vertical fractures along dikes with dike-swarm permeability widths of the order of a few to 100 x 10^{-12} m^3. Due to the very heterogeneous fracture-dominated nature of the flood-basalts, these results represent large-scale averages but the values have no local significance.

TABLE IV Fracture widths for n parallel fractures based on the permeability width estimates in Table III

n =	1	5	10
Laugarnes	1.2 mm	0.7	0.6
Laugaland	0.5	0.3	0.2
Ytri-Tjarnir	0.4	0.2	0.2
Svartsengi	0.7	0.4	0.3

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