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## MATERIAL BALANCE EQUATION IN GEOPRESSURED RESERVOIRS

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## ABSTRACT

A material balance equation (MBE) is developed for geopressured reservoirs which takes into account the water and rock expansion and pore volume reduction as pressure declines in the reservoir.

Data from a typical geopressured reservoir was used both in the MBE and in a four component, three-dimensional reservoir simulator developed by ARCO Oil & Gas Co. which was modified for use in a geopressured reservoir. Few example problems for different critical gas saturations are presented. The results of the proposed MBE are within 4% of those predicted after 27 years of production by the reservoir simulator.

Ignoring the rock and fluid compressibilities reduces the new MBE to the one introduced by schilthuis. Using schilthuis equation in a geopressured reservoir gives misleading results of up to 7 times higher gas and water in-place, or much lower recovery factors.

## INTRODUCTION

Geopressured brines in the Gulf Coast area of the United States exist at depths of greater than 8000 ft. The pressure gradients in these Cenozoic and Mesozoic sedimentary rocks are higher than hydrostatic (0.465 psi/ft), and with increasing depth they approach to lithostatic pressure gradient (1.0 psi/ft). The temperature gradients in geopressured zones are also higher than the gradients in normally pressured zones. These geopressured formations are potential sources for dissolved methane recovery, thermal energy, and hydraulic energy<sup>2</sup>.

The economics of methane recovery from geopressured-geothermal aquifers in the Texas and Louisiana gulf coast area has been evaluated by Doscher et al.<sup>1</sup>

In an earlier study<sup>1</sup>, the performance characteristics of these reservoirs were analyzed by a detailed reservoir modeling.

In particular, sensitivity studies were made to determine effects of various parameters that might enhance the economic production of methane from such aquifers.

Material Balance Equation

The effective rock compressibility is defined as

$$C_f = \frac{1}{V_{pi}} \frac{\Delta V_p}{P} = \frac{V_{pi} - V_p}{\Delta P} \quad (1)$$

From above equation an expression for pore volume can be obtained after some fluid is produced from the formation.

$$V_p = V_{pi}(1 - C_f \Delta P) \quad (2)$$

In a normally pressured reservoir the change in pore volume,  $V_p$ ,  $C_f$ ,  $\Delta P$ , is negligible since the pressure drop is not too great to affect the very low value of  $C_f$ , but in formations with fluid pressures higher than hydrostatic, especially the ones with pressures close to lithostatic, the fractional change in pore volume can reach up to 5%.

According to Hall<sup>4</sup> the rock compressibility decreases as porosity increases. A range of 3-5 x 10<sup>-6</sup> psi for  $C_f$  should be reasonably accurate since geopressured reservoirs have high fluid pressures and high porosities. According to Geerstma<sup>5</sup>, the effective value of compressibility should be of the order of one-half that measured in the laboratory under triaxial loading. Although no laboratory data on the compressibility of cores from geopressured reservoirs are available in the literature, 10 x 10<sup>-6</sup> psi<sup>-1</sup> is the highest value for most reported laboratory measurements.

The expansion of water inside the aquifer at lower pressures can be shown as

$$V_w c_w \Delta P = (W - W_p) B_w c_w \Delta P \quad (3)$$

The compressibility of water is a function of pressure, temperature, the amount of impurities (salts) dissolved in water, and the solubility of gas (assumed to be all

methane) in water. The solubility of methane in water at high pressures<sup>6</sup> were used to extrapolate the available formation volume factor data for water<sup>7</sup> to the required higher pressure ranges. From which the water compressibility were calculated by the following equation

$$C_w = \frac{B_w - B_{wi}}{B_{wi} \Delta P} \quad (4)$$

The initial condition for all the simulation studies were assumed to have no free gas in the reservoir and the water to be at bubble point inside the aquifer. then the initial volume of water in-place is the same as the initial pore volume available in the reservoir.

$$V_{wi} = V_{pi} = W B_{wi} \quad (5)$$

After some water and solution gas are produced the pressure inside the formation declines and free gas comes out of solution.

$$V_w = V_p - V_f \quad (6)$$

It can be assumed that  $V_w$ , the present volume of water inside the porous media, will expand inside the reservoir at lower pressures, or

$$V_w = (W - W_p) B_w \quad (7)$$

$V_f$ , the free gas volume can be expressed as

$$V_f = G_f B_g \quad (8)$$

The Scf of free gas inside the aquifer is equal to Scf of gas initially in-place minus the Scf of gas presently in solution minus the Scf of gas produced.

$$G_f = W R_{swi} - (W - W_p) R_{sw} - W_p R_{pw} \quad (9)$$

substituting Eqs. 2, 7, 8, and 9 into Eq. 6 gives:

$$(W - W_p) B_w = V_{pi} (1 - c_f \Delta P) - B_g [W R_{swi} - (W - W_p) R_{sw} - W_p R_{pw}] \quad (10)$$

using Eq. 5 in 10 and after some manipulation and simplification the following will be obtained:

$$W = \frac{W_p [B_w + B_g (R_{pw} - R_{sw})]}{B_w - B_{wi} (1 - c_f \Delta P) + B_g (R_{swi} - R_{sw})} \quad (11)$$

or by applying Eq. 4 in 11

$$W = \frac{W_p [B_w + B_g (R_{pw} - R_{sw})]}{\Delta P B_{wi} (C_w + C_f) + B_g (R_{swi} - R_{sw})} \quad (12)$$

The recovery factor for water can be found from

$$r_w = \frac{W_p}{W} = \frac{\Delta P B_{wi} (C_w + C_f) + B_g (R_{swi} - R_{sw})}{B_w + B_g (R_{pw} - R_{sw})} \quad (13)$$

The recovery factor for gas can be found from

$$r_g = \frac{G_p}{G_{swi}} = \frac{W_p R_{pw}}{W R_{swi}} = r_w \frac{R_{pw}}{R_{swi}} \quad (14)$$

knowing the PVT of water and gas ( $B_w, B_g, c_w, R_{sw}$ ), some production data ( $W_p, G_p, W \Delta P$ ), and a knowledge of formation compressibility ( $c_f = f(\phi)$ ) then  $W$ , the water in-place can be estimated from Eqs. 11 or 12, from which the Scf of initial gas in-place can be found from

$$G_{swi} = W R_{swi} \quad (15)$$

The amount of gas left in the reservoir at a lower pressure can be estimated from

$$G_{left} = G_{swi} - G_p = W R_{swi} - G_p = G_f + (W - W_p) R_{sw} \quad (16)$$

Eq. 11 reduces to the Schilthuis MBE if formation compressibility is ignored.

$$W = \frac{W_p [B_w + B_g (R_{pw} - R_{sw})]}{B_w - B_{wi} + B_g (R_{swi} - R_{sw})} \quad (17)$$

The results of the simulation study<sup>1</sup> confirm that critical gas saturation was the most influencing parameter in the economical production of methane. Table 1 shows the production and reservoir properties of a typical geopressured aquifer<sup>3</sup> used in the simulation study. Figs. 1 and 2 indicate that for the typical parameters used the critical gas saturation must be less than 0.5% before any increased gas production is obtained.

The initial producing GWR is the solution value of 34 Scf/STB for all critical gas saturations. This indicates that the gas produced initially is only the dissolved gas. As time progresses, formation pressure drops, gas is liberated and accumulates within the reservoir rocks, and the producing GWR decreases. For a critical gas saturation of 0.1%, the average gas saturation in the producing well block reaches 0.1% at the end of 3 years, and the free gas begins to be produced. After 6 years, the gas saturation has exceeded the critical gas saturation throughout the reservoir, and the producing GWR increases linearly with time until  $P_{wf}$  falls to 6000 psia (the limit of BHP). After 10 years of production  $P_{wf}$  has decreased to 6000 psi for critical gas saturations of 0.5% and more but still the liberated gas (free gas) inside the aquifer has not reached the critical value.

Fig. 3 shows the decline in average

reservoir pressure with gas production for different critical gas saturations. The pressure declines linearly as long as the free gas is immobile while the water volume is expanding and the pore volume is decreasing with pressure. Once the free gas starts to move, due to its high relative permeability it reduces the production of water and hence keeps the reservoir pressure high.

For strong water-drive reservoirs, where pressures remain above the bubble point of the gas in water, or when the critical gas saturation is more than one percent, the gas recovery factor will be the same as that for water.

#### Example 1

The following initial conditions and the data presented in Table 1 were used in the reservoir simulator.

$$\begin{aligned} P_i &= 11043 \text{ psia} \\ R_i &= 0.034 \text{ Mcf/STB} \\ B_{wi}^{swi} &= 1.0434 \text{ bbl/STB} \end{aligned}$$

For the case of  $S_{gc} = 0.1\%$ , after 27 years of production the following data were obtained from the simulator.

$$\begin{aligned} \bar{P} &= 5524 \text{ psia} \\ P_{wf} &= 3086 \text{ psia} \\ S_{wf} &= 0.62\% \text{ in the center grid} \\ W_p^g &= 182 \times 10^6 \text{ STB} \\ G_p^p &= 16.646 \times 10^6 \text{ Mcf} \end{aligned}$$

The PVT used at above pressure are:

$$\begin{aligned} R &= 0.02343 \text{ Mcf/STB} \\ B_{sw}^{sw} &= 1.0427 \text{ bbl/STB} \\ B_g^w &= 0.68219 \text{ bbl/Mcf} \end{aligned}$$

The cumulative gas-water ratio and the total pressure drop inside the formation are calculated from:

$$R_{pw} = \frac{(16.646 \times 10^6)}{0.0915 \text{ Mcf/STB}} / (182 \times 10^6) =$$

$$\Delta P = 11043 - 5524 = 5519 \text{ psi}$$

Using above data in Eqs. 11, 13, 14, and 15

$$W =$$

$$\frac{(182 \times 10^6) [1.0427 + 0.68219(0.0915 - 0.0234)]}{1.0427 - 1.0434(1 - 5 \times 10^{-6} \times 5519) + 0.68219(0.034 - 0.0234)}$$

$$= 5614 \times 10^6 \text{ STB}$$

$$r_w = \frac{182 \times 10^6}{5614 \times 10^6} = 3.24\%$$

$$r_g = (0.0324)(0.0915 / 0.034) = 8.7\%$$

$$G_{swi} = 5614 \times 10^6 \times 0.034 = 191 \times 10^6 \text{ Mcf}$$

Above data were obtained by using the modified MBE; the reservoir simulator gave the following results:

$$\begin{aligned} W &= 5857 \times 10^6 \text{ STB} \\ G_{swi} &= 199.5 \times 10^6 \text{ Mcf} \\ r_w &= (182 \times 10^6) / (5857 \times 10^6) = 3.11\% \\ r_g &= (16.646 \times 10^6) / (199.5 \times 10^6) = 8.34\% \\ \text{error} &= (5857 - 5614) / 5857 = 4.1\% \end{aligned}$$

If formation compressibility is ignored and the general schilthuis MBE is used then:

$$W = \frac{(182 \times 10^6) [1.0427 + 0.68219(0.0915 - 0.0234)]}{1.0427 - 1.0434 + 0.68219(0.034 - 0.0234)} = 30.41 \times 10^9 \text{ STB}$$

which overestimates the water and solution gas in-place and underestimates the fractional gas recovery by a factor of 5.

#### Example 2

The same PVT and reservoir data were used in this run, only  $S_{gc}$  was 2%. After 14 years of production the following data were obtained from the simulator.

$$\begin{aligned} \bar{P} &= 8024 \text{ psia} \\ W_p &= 97.97 \times 10^6 \text{ STB} \\ G_p &= 2.98 \times 10^6 \text{ Mcf} \end{aligned}$$

The PVT used at above pressure were

$$\begin{aligned} R_{sw} &= 0.0288 \text{ Mcf/STB} \\ B_w &= 1.0431 \text{ bbl/STB} \\ B_g &= 0.5439 \text{ bbl/Mcf} \end{aligned}$$

The cumulative gas-water ratio and the total pressure drop inside the formation are calculated from

$$R_{pw} = \frac{(2.98 \times 10^6)}{0.0304 \text{ Mcf/STB}} / (97.97 \times 10^6) =$$

$$\Delta P = 11043 - 8024 = 3019 \text{ psi}$$

Using above data in Eq. 11

$$W = \frac{(97.97 \times 10^6) [1.0431 + 0.5439(0.0304 - 0.0288)]}{1.0431 - 1.0434(1 - 5 \times 10^{-6} \times 3019) + 0.5439(0.034 - 0.0288)} = 5596 \times 10^6 \text{ STB}$$

Ignoring the formation compressibility gives:

$$W = \frac{(97.97 \times 10^6) [1.0431 + 0.5439(0.0304 - 0.0288)]}{1.0431 - 1.0434 + 0.5439(0.034 - 0.0288)} = 40.45 \times 10^9 \text{ STB}$$

which is 7 times higher than the value obtained by the modified MBE.

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Table 1  
The Base Case Parameters

A, sq mile	40
$c_f$ , psia <sup>-1</sup>	$5 \times 10^{-6}$
D, ft	13,000
h, ft	162
k, md	20
$p_i$ , psia	11,000
$p_{wh}$ , psia	500
$q_w$ , B/D	20,000
$r_w$ , ft	0.2
C, ppm	30,000
$S_{gi}$ , %	0.00
$T_r$ , °F	240
$\phi$	0.18

## Nomenclature

A	Drainage area, sq. miles (Km <sup>2</sup> )
$B_g, B_w$	Formation volume factor for gas and water, bbl/Mcf & bbl/STB (m <sup>3</sup> /stock-tank m <sup>3</sup> )
C	Salinity of salt, ppm
$c_f, c_w$	Formation and water compressibilities, psi <sup>-1</sup> (KPa <sup>-1</sup> )
D	Depth of well from midpay zone to surface, ft(m)
$G_f$	Free gas volume inside the reservoir, Mcf (Std m <sup>3</sup> )
$G_p$	Cummulative produced gas, Mcf (Std m <sup>3</sup> )
$G_{sw}$	Volume of gas dissolved in water, Mcf (Std m <sup>3</sup> )
h	Net formation thickness, ft(m)
K	Single-phase formation permeability, md
$\bar{P}$	Average formation pressure, psi (KPa)
$P_{wf}$	Bottomhole flowing pressure, psi (KPa)
$P_{wh}$	Wellhead pressure, psi (KPa)
$q_w$	Water flow rate, B/D (m <sup>3</sup> /d)
$r_g, r_w$	Recovery factor for gas and water, %
$R_{pw}$	Cummulative produced gas-water ratio, Mcf/STB (Std m <sup>3</sup> /stock-tank m <sup>3</sup> )
$R_{sw}$	Solution gas-water ratio, Mcf/STB (Std m <sup>3</sup> /stock-tank m <sup>3</sup> )
$S_g$	Gas saturation, %
$T_r$	Reservoir temperature, °F (°C)
$V_f$	Free gas volume inside the reservoir, bbl (m <sup>3</sup> )
$V_p$	Reservoir pore volume, bbl (m <sup>3</sup> )
$V_w$	Volume of water inside the reservoir, bbl (m <sup>3</sup> )
W	Initial volume of water inside the reservoir, STB (stock-tank m <sup>3</sup> )
$W_p$	Cummulative produced water, STB (stock-tank m <sup>3</sup> )

$\phi$  Porosity, %  
 $\Delta$  Difference

Subscripts

i Initial  
 g Gas  
 w Water

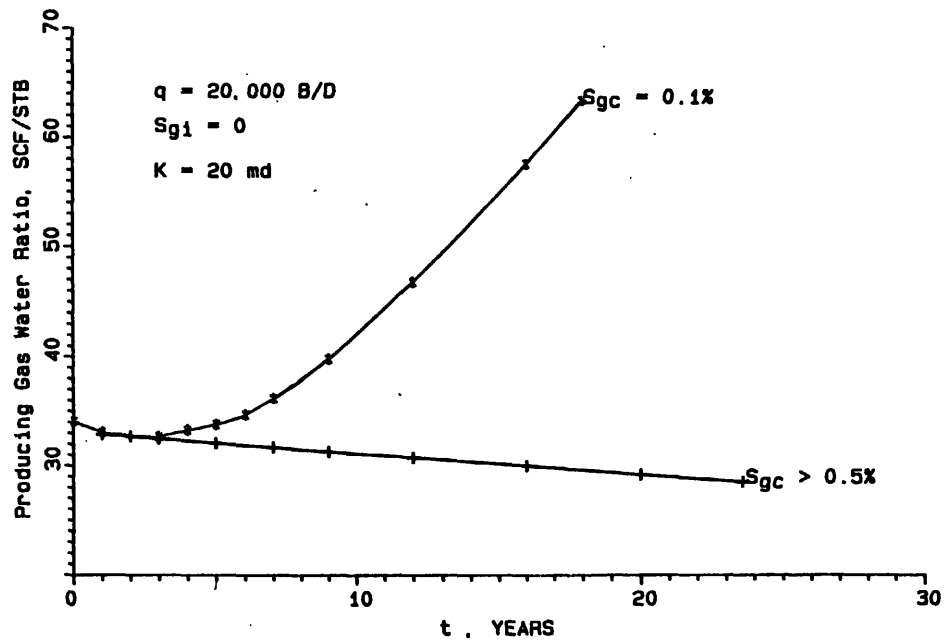


Fig. 1 - Effect of critical gas saturation on producing gas-water ratio.

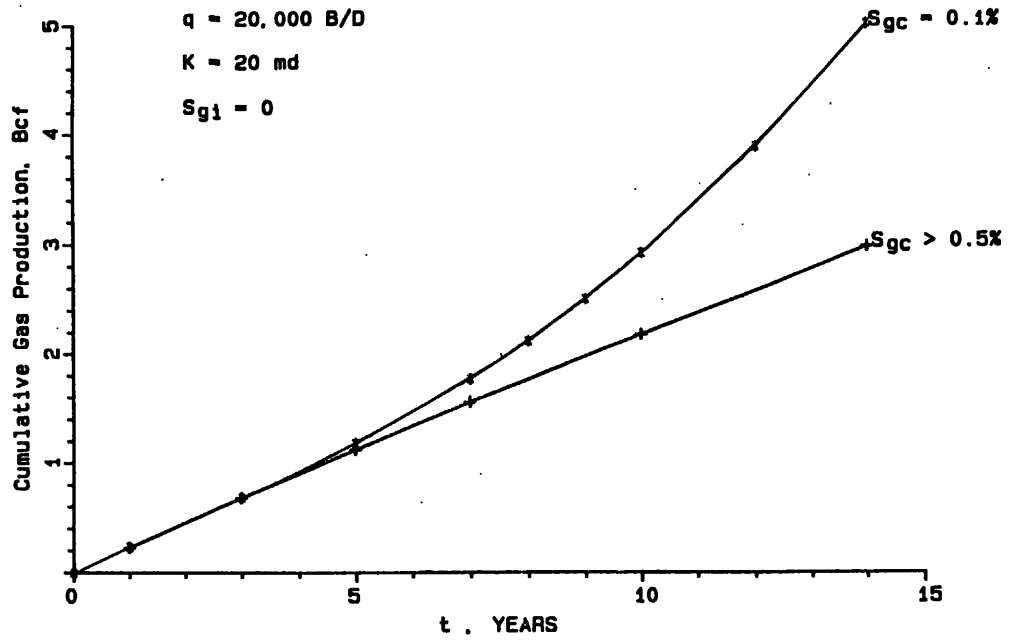


Fig. 2 - Effect of critical gas saturation on cumulative gas recovery.

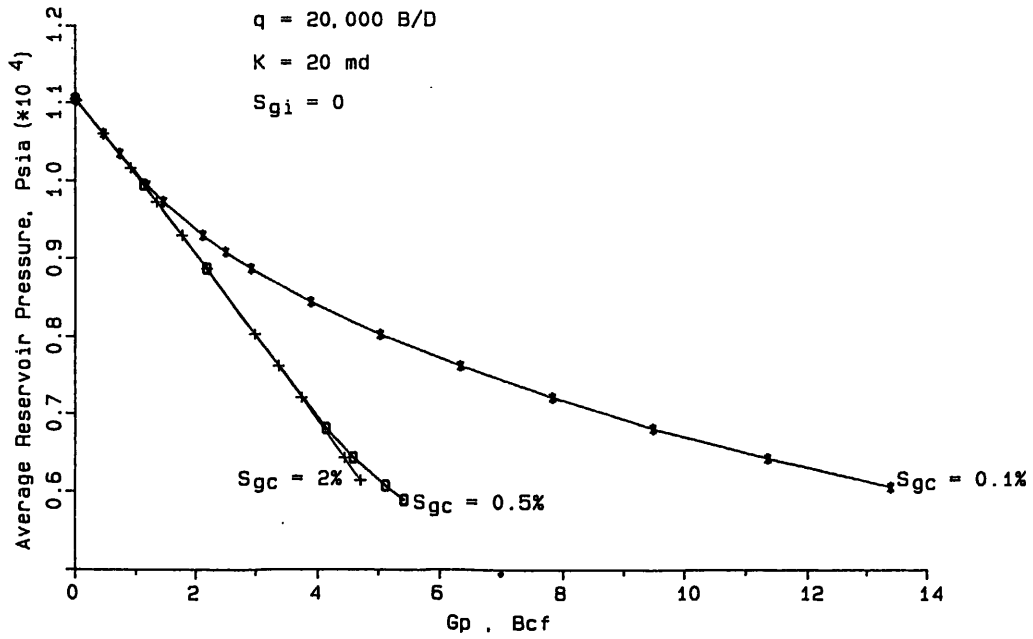


Fig. 3 - Effect of critical gas saturation on reservoir pressure.