A MODEL STUDY AND OPTIMUM DESIGN OF GEOTHERMAL DOWNHOLE HEAT EXCHANGERS

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ABSTRACT

A heat Transfer model of the Annular type downhole heat exchanger (DHE) is presented. Based on this model and the 'U' tube type model described by Pan (1983), an optimum design of the DHE using Powell's method is programmed and described in this paper. It is analysed qualitatively and some parameters of DHE and well affects on the heat output are shown.

INTRODUCTION

Most of our country's geothermal wells are low temperature and non-artesian. For non-artesian wells the conventional means of circulating the geothermal fluids is to use a downhole turbine pump, with the geo-fluids being either returned to the reservoir by a second reinjection well or rejected as waste water. The later is more common, with results, in some cases, of excessive drawdown of the reservoir and consequent environmental problems such as chemical or thermal pollution.

The downhole heat exchanger (DHE), consisting of a pipe system suspended in the well, can alleviate the above mentioned problems. Tap water or clean secondary water is circulated inside the DHE system absorbing downhole heat only from the well no geothermal fluid is withdrawn. Basically, there are two types of DHE, namely the 'U' tube type (Fig. 1 & 2) and the Annular (Coaxial) or pipe in pipe type (Fig. 3). The former is the most popular in heating istallations since it is easier to install, consisting generally of standard lengths of mild steel pipe joined together by screwed couplings.

There are more than 500 homes in Klamath Falls, Oregon U.S.A. using DHE system installed in single wells. Typically these consist of a 254 mm diameter wells with an average depth of 100 m, cased to depth with 203 mm diameter steel casing perforated just below the static water level, where the temperature is about 10°C to the bottom of the aquifer where the temperatures are about 95°C. Two DHE's, using a 50.8 mm diameter loop are used for the heating system and a 19.5 mm diameter pipe is used for the hot water system. City water is circulated through both loops to supply space heating devices and the domestic hot water (Bloomquist, R.G. et al. 1980).

At Moana Reno, Nevada U.S.A. Allis (1981) there are reports of a large increase in the rate of drilling new geothermal wells using DHE for domestic heating purposes, at a rate of about one every 1-2 weeks (1981). A typical installation consists of a 200 mm diameter well drilled to a depth between 50 to 200 m dependant upon the thickness of the hot water aquifer. They use a 20-50 mm 'U' tube type DHE extending the full length of well in which city supplied water is circulated.

In New Zealand several DHE installations can be found at Rotorua and Taupo a city of over 400 geothermal wells (Lorentz & Mountford 1980). Three wells have been drilled to about 200 mm and fitted with DHE's to supply heat for space heating and hot water at the Tauhara College, Taupo.

At Tauhara College, Taupo, New Zealand, three DHE systems preheat the water to the oil fired boiler serving the school heating system. The boiler comes into operation to boost the temperature of the water on very cold days. In 1981, the first year of full operation, the geothermal heating system cut fuel oil usage from 40,000 to 2,650 litres, at a savings of $16,000. If fuel oil costs continue to escalate the anticipated payback period for this scheme is about 3 years (Shannon, R.J. 1982).

Ryan, G.P. (1982) shows the economical feasibility of a DHE system for heating and cooling. Based on the predicted performance of DHE, a closed hot water circulating system that furnishes space heating and heats domestic hot water is technically and economically feasible. Based on a total incremental capital cost of $31,000 the savings in the cost of natural gas and electricity that the conventional system would consume would generate a simple payback of 4 years. During the 20 year system life used in the evaluation, the DHE system would save approximately 74,000 therms of natural gas and 500,000 kilowatt hours of electricity.

The literature on DHE is not extensive. Most of the earlier work has originated from the studies of Culver and Reistad (1978), centered on the GeoHeat Center at Klamath Falls and the University of Oregon, U.S.A..

The later work in which laboratory tests of full scale experimental work and a computer analogy for a 'U' tube type DHE have been outlined by Pan et al. (1982) and Freestone & Pan (1983,1985). The work was carried out by the Geothermal Institute and the Department of Mechanical Engineering at the University of Auckland, New Zealand.

These papers describe a mathematical derivation on Annular type DHE in which a heat transfer governing equation is derived in order to establish a convection heat transfer model. The computer simulation presented in this paper is based on the 'U' tube type DHE model described by Pan (1983). The major work of this simulation are (1) A model of convective heat transfer for Annular DHE and its design is linked to the simulation. (2) Powell's algorithm is used in this program as a method to achieve the optimum DHE parameter computation. The principal application of the simulation leads to an optional geometric parameter design for 'U' tube type and Annular type DHE's, which is very useful to obtain the maximum energy output for given well conditions. Also, the simulation can be used to design the thermodynamic optimum DHE system before drilling to meet a given duty.

A HEAT TRANSFER MODEL OF ANNULAR TYPE DHE

A heat transfer model of 'U' tube type DHE is originally reported in Culver & Reistad (1978). This work was extended and developed by Pan (1983) for the design and evaluation of a 'U' tube type DHE. A heat transfer model of an Annular type DHE is described in this section. Some typical experiments for an Annular type DHE were presented by Pan et al. (1982). The conclusions show that the performance of this type DHE is comparable to that of the 'U' tube type. Although experiments have been done and investigated, the heat transfer mechanism has not been analyzed in great detail.

In order to evaluate and design the Annular type DHE parameters, a heat transfer model is required and a governing equation has to be derived. The governing equations of a full length Annular DHE and a DHE that does not extend the entire distance from the top to the bottom of the geothermal wells are derived by applying conservation of mass, monentum, and energy to nods and paths. The model discussed below corresponds to the following points:

(1) The reservoir temperature and the inlet temperature of the DHE circulating fluids are assumed.

(2) The heat loss from the bore hole wall is negligible.

(3) The natural circulating mass flow rate in the well is unrestricted.

Considering an Annular DHE that does not extend the entire distance from the top to the bottom perforations, the configuration is shown in Figure 3. The reservoir flow is up the outside casing and returns back down inside casing by thermosyphon, however, the circulating water in the DHE is down the annulus and returns back up the centre tube as this direction of DHE flow has achieved better performance than the reverse flow (Freestone & Pan 1983).

Since the perforation at the botton of casing would allow the cooler brine from inside the casing to mix with that of the reservoir before it goes up the outside of the casing, a mixing ratio is being used in the following equations. The actual temperature of the brine flowing up the outside of the casing would therefore be less than that of the reservoir. A Schematic Network model for DHE that does not extend the entire distance from the top to the bottom perforations is shown in Figure 4. The Network model is based on modeling the well and DHE system as a Network with two types of flow: fluid flow and heat flow alone. The governing equation for the Network will now be established:

Mass Conservation:

$$X_m \cdot \dot{M} + (1 - X_m) \cdot \dot{M} = \dot{M} \qquad \langle 1 \rangle$$

Energy Conservation:

$$X_m \cdot T_3 + (1 - X_m) \cdot T_3 = T_2 \qquad \langle 2 \rangle$$

The Mixing Ratio (X_m) is determined by:

$$X_m = 1 - \frac{\dot{M}_{add}}{\dot{M}}$$

$\dot{M}$ is circulating mass flow in the convection cell, and $\dot{M}_{add}$ is the mass of the hot reservoir flow additional to the convection flow. $\dot{M}_{add}$ also can be expressed:

$$\dot{M}_{add} = \frac{\dot{M}_h (T_{out} - T_{in})}{T_s - T_3}$$

Convective motion from Culver & Reistad (1978)

$$\frac{2}{2-a} (W1 + W2 + W3) \dot{M}^2 = T_2 - T_3 \qquad \langle 3 \rangle$$

where: $$W1=\frac{F_{f1}}{2d^2gA_1^2D_{e1}B}$$

$$W2=\frac{F_{f2}}{2d^2gA_2^2D_{e2}B}(1-a)$$

$$W3=\frac{F_{f3}}{2d^2gA_3^2D_{e3}B}\ a$$

$$a=\frac{\text{DHE length}}{\text{total length of perforated interval}}$$

Heat absorbed by water in the DHE, Q_H is given by:

$$Q_H=\dot{M}_hC_p(T_{out}-T_{in}) \quad \langle 4\rangle$$

The mass flow rate of the convective current can be calculated by taking temperature measurement within the well and the DHE and by making an energy balance. This leads to the following relationship:

$$\dot{M}_hC_p(T_{out}-T_{in})=e\cdot\dot{M}C_p(T_2-T_{in}) \quad \langle 5\rangle$$

$$e=\frac{T_2-T_3}{T_2-T_{in}}$$

Here the e refers to heat transfer effectiveness.

During the Annular DHE operating process, the DHE flow down the annulus (See Figure 3) to the bottom absorbs heat and its temperature will reach a relatively higher value. During the flow up the inner tube to the top of the DHE, some heat is lost due to the temperature difference between the annulus colder inlet water and the inner tube hot outlet water of the DHE. The equation can be written by:

$$Q_H=\frac{U_aA_a}{2}(T_1+T_{1a}-T_{in}-T')-\frac{U_bA_b}{2}(T_{out}-T_{in}) \quad \langle 6\rangle$$

The gain of heat transfer can also be expressed by

$$Q_H=Q_{2-2a}-Q_{2a-1}+\dot{M}C_p(T_1-T_3) \quad \langle 7\rangle$$

where:

$$Q_{2-2a}=\frac{U_cA_c(1-a)}{2}(T_2+T_{2a}-T_{1a}-T_3)$$

$$Q_{2a-1}=\frac{U_eA_ca}{2}(T_{2a}-T_{1a})$$

and $T_2-T_{2a}=T_3-T_{1a}=Q_{2-2a}/\dot{M}C_p$

Combine the equations mentioned above to give a single equation for M

$$[\frac{2C_p}{U_aA_a}+\frac{U_bA_b}{2M_h^2C_p}+\frac{U_bA_b}{U_aA_aM_h}+\frac{1}{M_h}]\dot{M}^3+\frac{1+X_m}{1-X_m}\dot{M}^2+$$

$$[\frac{2U_cA_c(1-a)}{C_p}+\frac{U_eA_ca}{2C_p}]\dot{M}-(2-a)\frac{T_2-T_{in}}{W1+W2+W3}=0 \quad \langle 8\rangle$$

when equation <8> is solved for M, the other unknowns can be determined from the following:

$$Q_H=\frac{2C_p}{2-a}(W1+W2+W3)\dot{M}^3 \quad \langle 9\rangle$$

$$T_{out}=T_{in}+Q_H/\dot{M}_hC_p \quad \langle 10\rangle$$

$$T_3=T_2-Q_H/\dot{M}C_p \quad \langle 11\rangle$$

THEORETICAL METHOD OF OPTMUM DESIGN

In the previous section, two kinds of DHE heat transfer models were described. In this section, theoretical method is discussed for both 'U' tube and Annular type DHE of an optimum design based on the models.

The objective of the design is to find an optimum set of DHE parameters. The heat output value for the DHE system is maximumized under this set of DHE parameters. However, the relationship between the heat output value and the DHE parameters for a given condition is very complex and its derivative is not available. Optimization of the algorithum by calculating the derivatives is unsuitable, but, Powell's algorithum without calculating derivatives is very applicable. Powell's algorithum has been described in detail by Powell (1964), Avrel (1973, 1976). Essentially, Powell's algorithum is a conjugate directions method and is the most efficient means in direct search methods.

It is assumed that X(1), X(2),···, X(n) are parameters of a DHE system to be optimized, F(X)=1/Q(X), where Q(X) is the heat output for the system, $X=[X(1),X(2),\cdots,X(n)]^T$. The scheme of optimum design for the system is obtain by the following steps.

<i> Let K=o, $X_0^{(o)}\in R^n$ be the starting estimated point of X and n linearly independent directions $S_1^{(o)}, S_2^{(o)}, \cdots S_n^{(o)}$ given (the coordinate directions are usually closer , i.e,

$S_i^{(o)}=e_i\ \ i=1, 2,\cdots,n)$

<ii> Find numbers g_i^* , such that

$$f(X_{i-1}^{(k)}+g_i^*\cdot S_i^{(k)})=\min_{g_i} f(X_{i-1}^{(k)}+g_i\cdot S_i^{(k)})$$

for i=1, 2, ···, n and define

$$X_i^{(k)}=X_{i-1}^{(k)}+g_i^*\cdot S_i^{(k)} \quad i=1,2,\cdots n$$

<iii> Find g^*_{n+1}, such that

$$f(X_o^{(k)}+g^*_{n+1}(X_n^{(k)}-X_o^{(k)}))=\min_{g_{n+1}} f(X_o^{(k)}+g_{n+1}\cdot(X_n^{(k)}-X_o^{(k)}))$$

and let

$$X_{n+1}^{(k)}=X_o^{(k)}+g^*_{n+1}(X_n^{(k)}-X_o^{(k)})$$

<iv> Find the index m, such that

$$f(X_{m-1}^{(k)})-f(X_m^{(k)})=\max_i[f(X_{i-1}^{(k)})-f(X_i^{(k)}),\ i=1,2,\cdots,n]$$

<v> If $|g^*_{n+1}|<[\frac{f(X_o^{(k)})-f(X_{n+1}^{(k)})}{f(X_{m-1}^{(k)})-f(X_m^{(k)})}]^{\frac{1}{2}}$

set $S_i^{(k+1)}=S_i^{(k)} \qquad i=1,\cdots,n$

otherwise, set $S_i^{(k+1)}=S_i^{(k)} \qquad i=1,\cdots,m-1$

$$S_i^{(k+1)}=S_{i+1}^{(k)} \qquad i=m,\cdots,n-1$$

$$S_n^{(k+1)}=(X_n^{(k)}-X_o^{(k)})/\|X_n^{(k)}-X_o^{(k)}\|$$

<vi> IF $\frac{|f(X_o^{(k)})-f(X_{n+1}^{(k)})|}{|f(X_o^{(k)})|}\leqslant a_1$

and $\frac{\|X_o^{(k)}-X_{n+1}^{(k)}\|}{\|X_o^{(k)}\|}\leqslant a_2$

where $a_1>o$ and $a_2>o$ are some predetermined numbers, stop. otherwise, let

$$X_o^{(k+1)}=X_{n+1}^{(k)}$$

$$f(X_o^{(k+1)})=f(X_{n+1}^{(k)})$$

$$K=K+1$$

then Return to (ii)

In the above precedure., the quadratic method is adoped for one-dimensional line searchs.

COMPUTER SIMULATION

Culver & Reistad (1978) developed a computer analysis in which it was demostrated the influence of a number of geometric and fluid/thermal parameters on the 'U' tube type DHE output for a typical installation of Klamath Fall's wells. It has been modified and extended to enable the design of a 'U' type DHE to match given well parameters (Freeston & Pan 1985).

The major improvement of the computer simulation discussed in this paper is that the optimum design of Annular type DHE is considered and Powell's algorithum is adopted as mentioned above.

The computer simulation programme is written for the IBM-XT computer using FORTRAN, under the given geothermal well geometry and temperature conditions. With this data the DHE parameters and performance can be evaluated. Alternatively, the programme can design the optimum DHE system including the well to meet a given duty. In addition, the programme can go to four branches which are: (1) 'U' tube type DHE with perforated casing; (2) 'U'tube type DHE with convective promoter; (3) Annular type DHE with perforated casing and (4) Annualar type DHE with convective promoter.

A method for calculating liquid density as a function of temperature is included, based on Reynolds (1981). Viscosity, specific heat, and Prandtl number is also calculated as a function of temperature using a routine based on the parabolic least squares fitting of the data presented in Rogers and Mayhew (1980). Friction factor are calculated using formulas by Churchill, S.W. (1977).

Calculation of velocities, Reynolds number, friction factors, Stanton number and heat transfer coefficients can be achieved by setting a value for the convection mass flow through the casing and annulus or promoter and annulus, from which the various non-dimensional values are computed. The mass flow is set initially at 4 Kg/s, then the heat transfer coefficients using appropriate physical properties are calculated. Finally, the W function of Culver & Reistad (1978) are calculated. All the above are calculated in a loop, the variable for the DHE, casing and well are calculated respectively.

Overall heat transfer coefficients from Holman (1976) (p.387) are determined using the film coefficients from Perry & Chilton (1973) (pp. 10-13,15). The coefficients for the cubic equation (Culver & Reistad (1978), pp.44, V36 and equation <8> in this paper) are then calculated. Heat output and DHE outlet temperature are obtained by assuming convection mass flow and equation <9> and <10>. The cubic equation is then solved using Newton's method (Yie & Wang 1986 p.36) to obtain a new convection mass flow. The new mass flow is compared with initial assumed. If the error is greater than 1.0E-6 The new value is inserted and the programme recalculated from the velocity, Reynolds number, etc. If the value of mass flow is satisfactory, the final DHE heat output is obtained. The above procedure is written as a subroutine that is used frequently in the procedure of DHE optimum design.

RESULTS ANALYSIS

According to computer simulation, we can divide parameters into three classes by influence on the DHE heat output. These classes are:

(1) Well parameters: well temperature (or well temperature profile), well and casing parameters, mixing ratio (or permeability of the reservoir) and casing thermal conductivity,

(2) DHE parameters: DHE length, number of loops, diameter and thermal conductivity and,

(3) Operation conditions: mass flow rate of DHE circulating water and DHE inlet water temperature.

In general, the higher reservoir temperatures and larger well diameters are, the higher the energy extraction rate. Moreover, there exist a DHE length and a well casing diameter under which other fixed parameter can be used to calculate the maximum heat production. Increasing the mass flow rate through the DHE increases the energy extraction rate, but, it requires larger pumping power due to the increase pressure drop, and decrease of the outlet temperature which affects the users unfavorably.

The above performances are also confirmed with experimental analysis shown by Pan (1983).

The optimum DHE design program can be used for either computing DHE opertion performances such as heat output, pressure drop etc., for a set of given data or used to obtain a DHE system optimum design.

RECOMMENDATIONS

The mixing ratio introduced in the original work of Culver & Reistad (1978) is an attempt to account for the drawdown of the reservoir temperature due to extraction of heat by the DHE. It is associated with the permeability of the reservoir and heat extraction rate of the DHE. But, unfortunately available data is too little from these shallow geothermal reservoirs to make a definitive mixing ratio as a function of permeability. Although the ratio is assumed as a matter of experience, the simulation results are satisfactory. It is recommended that a fundamental study in this area be undertaken.

NOMENCLATURE

A_1 :Cross sectional area of annulus outside casing
A_2 :Cross sectional area inside casing without DHE
A_3 :Cross sectional area inside casing with DHE
A_a :DHE annulus surface area
A_b :DHE centre tube surface area
A_c :Casing surface area
B :Thermal expension coefficient
C_p :Specific heat
d :Density of fluid
D_{e1} :Hydrolic diameter of annulus outside casing
D_{e2} :Hydrolic diameter inside casing without DHE
D_{e3} :Hydrolic diameter inside casing with DHE
F_{f1} : Fanning friction factor for outer annulus
F_{f2} :Fanning friction factor for inside casing without DHE
F_{f3} :Fanning friction factor for inside casing with DHE
g :Acceleration due to gravity
M :Mass flow rate through casing
M_h :Mass flow rate through DHE
T_j :Water temperature at node j for Network model
T_s :Initial reservoir temperature
T_{in} :Inlet temperature to DHE
T_{out}:Outlet temperature from DHE
U :Overall heat transfer coefficient
U_a :U through the DHE annulus
U_b :U through the DHE centre tube
U_c :U through the casing without DHE
U_e :U through the casing with DHE

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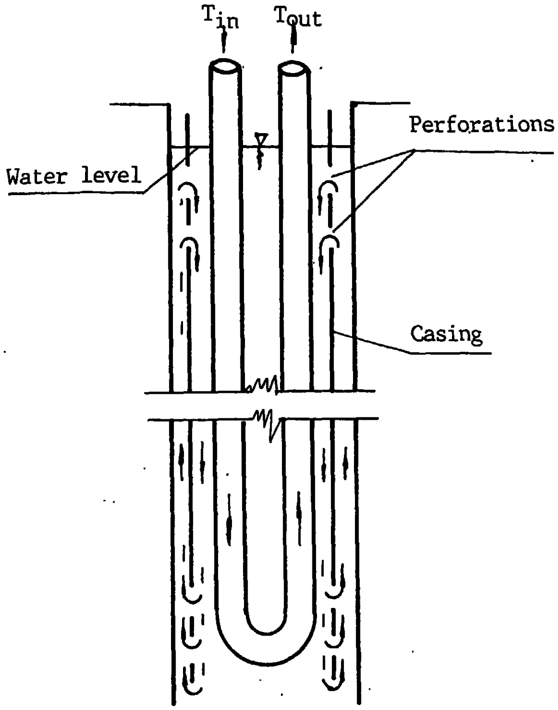

Figure 1. 'U' type DHE with perforated casing

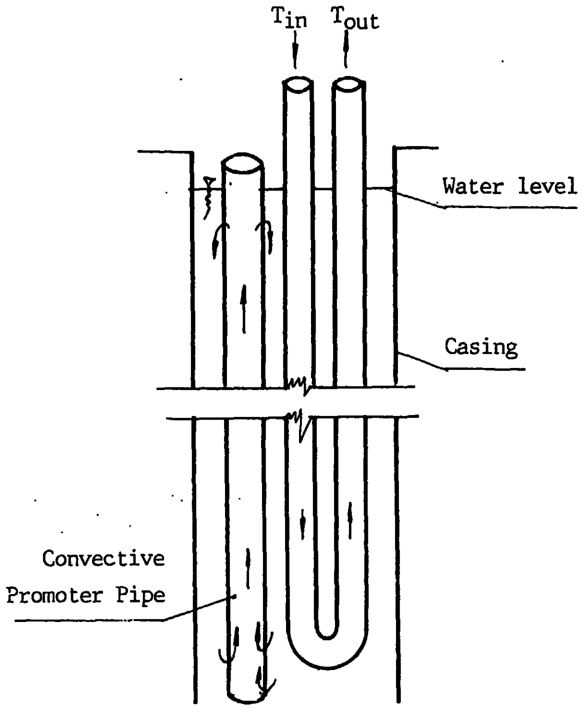

Figure 2. 'U' type DHE with convective promoter

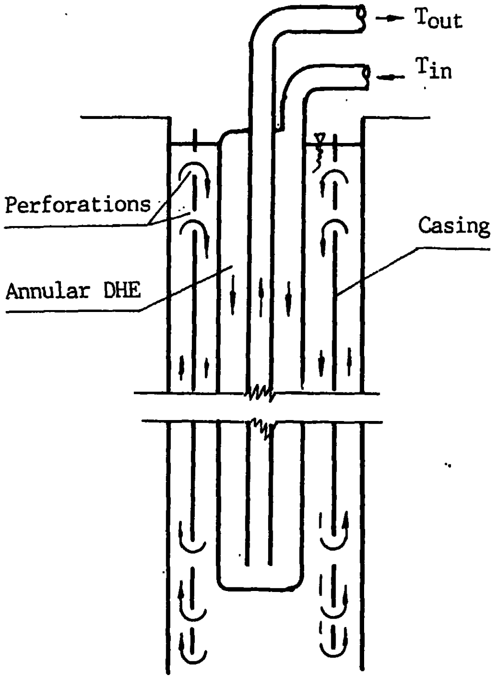

Figure 3. Annular DHE

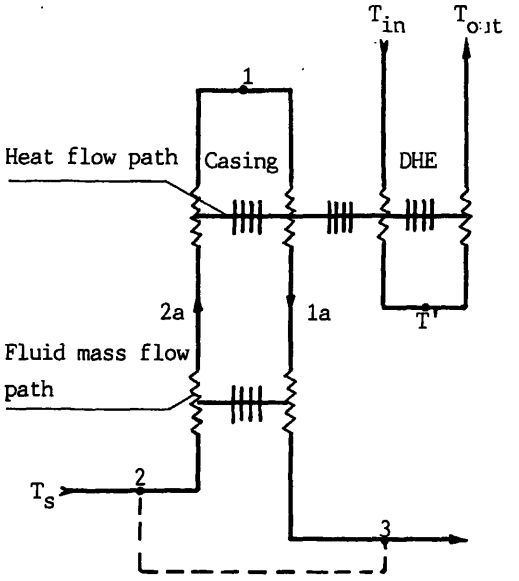

Figure 4. Network Model with Recirculation