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A METHOD FOR PREDICTING RESERVOIR PERFORMANCE OF A LUMPED-PARAMETER MODEL

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ABSTRACT

A prediction method of the reservoir performance for a lumped-parameter reservoir model with unsteady water influx from an infinite aquifer is presented. In this method, it is assumed that a short time fluid production and a short time water influx are alternately carried out.

Symbols

A : thermal equivalent of work, kJ/m^{3}MPa
Cr: specific heat of rock, kJ/kg°C
C_t: compressibility of water, 1/MPa
Gp: total flow rate of fluid produced, kg/hr
Gr: flow rate of reinjected water, kg/hr
Ga: flow rate of influx water, kg/hr
ip: enthalpy of fluid produced, kJ/kg
ir: enthalpy of reinjected water, kJ/kg
ia: enthalpy of influx water. kJ/kg
k : permeability of aquifer, m^2
L : distance, m
P : pressure, MPa
Pf: initial pressure, MPa
Pr: pressure at L=0, MPa
S : area where influx water inflows to reservoir, m^2
t : time, hr
u : internal energy, kJ/kg
V_o: pore volume of reservoir, m^3
v : specific volume of fluid, m^3/kg
W : mass of fluid, kg
x : steam quality
α : ratio of pore volume to rock volume of reservoir
γ_r: density of rock, kg/m^3
γ_w: density of water, kg/m^3
θ : temperature, °C
μ : viscosity, MPa hr
ϕ_a: porosity of aquifer

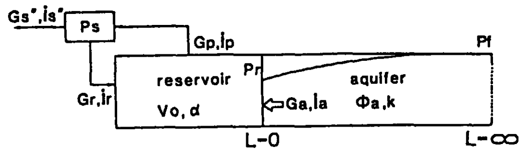
Superscripts
" : vapor phase
' : liquid phase

Subscript
s : separator

INTRODUCTION

The main purpose of the reservoir simulation is to adopt an optimum development condition for a real reservoir, by deducing physical behaviors of a reservoir model. The simulation work, therefore, starts with making reservoir model, which will range widely from the homogeneous one called lumped-parameter model or tank model to the heterogeneous one called block model, and it depends on the amount of informations on the real reservoir that which type of model can be applied.

The authors(1985) have presented a method which is available to predict reservoir temperature and pressure changes for a lumped-parameter model with such conditions that flow rates of steam produced and influx water from aquifer are constant. This paper deals with a similar model but with different conditions from the previous report.



Analytical model

ANALYTICAL MODEL

The analytical model dealt with in this paper is shown in the figure, where followings are assumed;

1) the total flow rate of fluid G_p is constant
2) the separator outlet pressure P_s is fixed
3) the separated water, flow rate of G_r and enthalpy of i_r, is reinjected into the reservoir
4) the aquifer adjacent to the reservoir has an infinite extent
5) the fluid in aquifer is in the compressed water condition and flows linearly to the reservoir in accordance with Darcy's law
6) the enthalpy of influx water ia is constant

THEORY

A mass and an energy balance equations in the above model will, respectively, be

$$-G_p+G_r+G_a=\frac{dW}{dt} \tag{1}$$

and

$$-G_p\cdot i_p+G_r\cdot i_r+G_a\cdot i_a=\frac{dW_u}{dt}+\alpha V_o\gamma_r C_r\frac{d\theta}{dt} \tag{2}$$

where the internal energy is defined as

$$u=i-APv \tag{3}$$

From Miller's report(1962), the flow rate of water influx from the infinite aquifer is given by

$$G_s=(P_f-P_r)\sqrt{\frac{\phi_s\cdot k\cdot C_t}{\pi\mu t}}\,S\gamma_w \qquad (4)$$

Although the steam production and the water influx occur simultaneously, it is assumed here, for convenience'sake, that a short time production and a short time influx are alternately carried out. Therefore, following four cases are separately discussed as; production only is carried out under the compressed water condition, influx only under the same condition, production only under the saturated condition and influx only under the same condition, respectively.

Compressed water condition

1. Production

In the case where only production is carried out, the balance equations are

$$-G_p+G_p(1-x)=-G_p x=\frac{dW}{dt} \qquad (5)$$

and

$$-G_p\{i_p-(1-x)i_r\}=\frac{dW_w}{dt}+\alpha V_s\gamma_r C_r\frac{d\theta}{dt} \qquad (6)$$

where x is a steam quality at the separator outlet, and

$$W_v=V_0 \qquad (7)$$

Assuming that G_p and i_r are constant, and x is also constant of x_{m-1} for a short time production, the integration of eq.(5) with the condition of $W=W_{m-1}$ at t=0 gives a mass change in the reservoir by a production for a time t

$$-G_p x_{m-1}t=\bar{W}-W_{m-1} \qquad (8)$$

and from eq.(7)

$$-G_p x_{m-1}t=V_0\left(\frac{1}{\bar{v}}-\frac{1}{v_{m-1}}\right) \qquad (9)$$

where W_{m-1} and v_{m-1} are a mass and a specific volume of fluid before the production starts. Substituting eqs.(3) and (7) into eq.(6) gives

$$-\frac{G_p}{V_0}\{i_p-(1-x)i_r\}=\frac{d}{dt}\left(\frac{i}{v}-AP\right)+\alpha\gamma_r C_r\frac{d\theta}{dt} \qquad (10)$$

As mentioned in the author's previous report, (i/v-A) can be approximated as

$$\left(\frac{i}{v}-AP\right)=(5776.9661\theta-408917.9690)-(2582.5715\theta-508053.5620)\times10^{3}\bar{v} \qquad (11)$$

Then, eq.(10) becomes

$$-\frac{G_p}{V_0}\{i_p-(1-x)i_r\}=(\alpha\gamma_r C_r+5776.9661-2582.5715\times10^{3}\bar{v})\frac{d\theta}{dt}+(508053.5620-2582.5715\theta)\times10^{3}\frac{d\bar{v}}{dt} \qquad (12)$$

From eq.(9)

$$\bar{v}=\frac{V_0}{G_p x_{m-1}}\cdot\frac{1}{\dfrac{V_0}{G_p x_{m-1}}\dfrac{1}{v_{m-1}}-t} \qquad (13)$$

and

$$\frac{d\bar{v}}{dt}=\frac{V_0}{G_p x_{m-1}}\cdot\frac{1}{\left\{\dfrac{V_0}{G_p x_{m-1}}\dfrac{1}{v_{m-1}}-t\right\}^2} \qquad (14)$$

Substituting eqs.(13) and (14) into eq.(12) and arranging it, following equation may be obtained.

$$\frac{d\theta}{dt}+\frac{A_{m-1}}{(N_{m-1}-t)\{A_{m-1}-(N_{m-1}-t)\}}\theta=\frac{B_{m-1}+C_{m-1}(N_{m-1}-t)^2}{(N_{m-1}-t)\{A_{m-1}-(N_{m-1}-t)\}} \qquad (15)$$

where

$$A_{m-1}=\frac{V_0}{G_p x_{m-1}}\cdot\frac{2582.5715\times10^{3}}{\alpha\gamma_r C_r+5776.9661} \qquad (16)$$

$$B_{m-1}=\frac{V_0}{G_p x_{m-1}}\cdot\frac{508053.5620\times10^{3}}{\alpha\gamma_r C_r+5776.9661} \qquad (17)$$

$$C_{m-1}=\frac{G_p}{V_0}\cdot\frac{i_p-(1-x)i_r}{\alpha\gamma_r C_r+5776.9661} \qquad (18)$$

$$N_{m-1}=\frac{V_0}{G_p x_{m-1}}\cdot\frac{1}{v_{m-1}} \qquad (19)$$

Solving eq.(15) with the condition of $\theta=\theta_{m-1}$ at t=0 gives a temperature

$$\theta=\left(\frac{B_{m-1}+A_{m-1}C_{m-1}N_{m-1}}{N_{m-1}\{A_{m-1}-(N_{m-1}-t)\}}-C_{m-1}\right)t+\left(1-\frac{A_{m-1}t}{N_{m-1}\{A_{m-1}-(N_{m-1}-t)\}}\right)\theta_{m-1} \qquad (20)$$

and the pressure at the temperature θ is given by*

$$P=(9.6987\times10^{-3}\underline{\theta}^5-1.2191\underline{\theta}^4+81.0377\underline{\theta}^3-1536.2709\underline{\theta}^2+19254.9544\underline{\theta}-98062.2800)\times10^{3}v-(1.1747\times10^{-3}\underline{\theta}^5-1.4859\underline{\theta}^4+74.7705\underline{\theta}^3-1871.8986\underline{\theta}^2+23412.6654\underline{\theta}-118656.8748) \qquad (21)$$

where

$\underline{\theta}$ = θ/10

2. Water influx

Writing pressure drops in the reservoir after the first production and influx as $\Delta P_1(=P_f-P_1)$, the second as $\Delta P_2(=P_1-P_2)$,.., the (m-1)th as $\Delta P_{m-1}(=P_{m-2}-P_{m-1})$ and the mth as $\Delta P'_m(=P_{m-1}-P'_m$, P'_m is the pressure after the mth production is finished), the balance equations will be

$$\gamma_w B\left(\frac{\Delta P_1}{\sqrt{t}}+\frac{\Delta P_2}{\sqrt{t-t_1}}+\cdots+\frac{\Delta P'_m}{\sqrt{t-t_{m-1}}}\right)=\frac{dW}{dt} \qquad (22)$$

and

$$\gamma_w B\left(\frac{\Delta P_1}{\sqrt{t}}+\frac{\Delta P_2}{\sqrt{t-t_1}}+\cdots+\frac{\Delta P'_m}{\sqrt{t-t_{m-1}}}\right)i_s=\frac{dW_w}{dt}+\alpha V_s\gamma_r C_r\frac{d\theta}{dt} \qquad (23)$$

where

$$B=\sqrt{\frac{\phi_s k C_t}{\pi\mu}}\,S$$

Assuming that $\Delta P'_m$ is constant for a short time influx, cumulative influx water for $t-t_{m-1}$ can be given by the integration of eq.(22)

$$\gamma_w B\left(\Delta P_1\int_{t_{m-1}}^{t}\frac{dt}{\sqrt{t}}+\Delta P_2\int_{t_{m-1}}^{t}\frac{dt}{\sqrt{t-t_1}}+\cdots+\Delta P'_m\int_{t_{m-1}}^{t}\frac{dt}{\sqrt{t-t_{m-1}}}\right)$$
$$=2\gamma_w B\{\Delta P_1(\sqrt{t}-\sqrt{t_{m-1}})+\Delta P_2(\sqrt{t-t_1}-\sqrt{t_{m-1}-t_1})+\cdots+\Delta P'_m\sqrt{t-t_{m-1}}\}$$
$$=W-\bar{W}_{m-1}=V_0\left(\frac{1}{v}-\frac{1}{\bar{v}_{m-1}}\right) \qquad (24)$$

where $\bar{W}_{m-1}$ and $\bar{v}_{m-1}$ are a mass and a specfic volume of fluid after the (m-1)th production is finished, and

$$v=\frac{V_0}{2B\gamma_w}\cdot\frac{1}{F(t)+\dfrac{V_0}{2B\gamma_w}\dfrac{1}{\bar{v}_{m-1}}} \qquad (25)$$

and

$$\frac{dv}{dt}=-\frac{V_0}{2B\gamma_w}\cdot\frac{\dfrac{dF(t)}{dt}}{\left\{F(t)+\dfrac{V_0}{2B\gamma_w}\dfrac{1}{\bar{v}_{m-1}}\right\}^2} \qquad (26)$$

where

$$F(t)=\Delta P_1(\sqrt{t}-\sqrt{t_{m-1}})+\Delta P_2(\sqrt{t-t_1}-\sqrt{t_{m-1}-t_1})+\cdots+\Delta P'_m\sqrt{t-t_{m-1}}$$

Eq.(23) can be rewritten like eq.(12) as

$$\frac{2B\gamma_w i_s}{V_0}\frac{dF(t)}{dt}=(\alpha\gamma_r C_r+5776.9661-2582.5715\times10^{3}v)\frac{d\theta}{dt}-(508053.5620-2582.5715\theta)\times10^{3}\frac{dv}{dt} \qquad (27)$$

Substituting eqs.(25) and (26) into eq.(27) and

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* In the previous report, P is approximated as a equation of the third order of $\underline{\theta}$.

arranging it,

$$\frac{d\theta}{dt}+\frac{a_0\frac{dF(t)}{dt}}{\{F(t)+n_{m-1}\}\{F(t)+n_{m-1}-a_0\}}\theta=\frac{\frac{dF(t)}{dt}\left(b_0+2c_0\{F(t)+n_{m-1}\}^2\right)}{\{F(t)+n_{m-1}\}\{F(t)+n_{m-1}-a_0\}} \tag{28}$$

will be obtained, where

$$a_0=\frac{V_0}{2B\gamma_w}\frac{2582.5715\times10^3}{\alpha\gamma_rC_r+5776.9661} \tag{29}$$

$$b_0=\frac{V_0}{2B\gamma_w}\frac{508053.5620\times10^3}{\alpha\gamma_rC_r+5773.9661} \tag{30}$$

$$c_0=\frac{B\gamma_w}{V_0}\frac{i_a}{\alpha\gamma_rC_r+5776.9661} \tag{31}$$

$$n_{m-1}=\frac{V_0}{2B\gamma_w}\frac{1}{\bar{v}} \tag{32}$$

Solving eq.(28) with the condition of $\theta=\theta_{m-1}$ at $t=t_{m-1}$ gives

$$\theta=\left(2c_0+\frac{b_0+2a_0c_0n_{m-1}}{n_{m-1}\{F(t)+n_{m-1}a_0\}}\right)F(t)+\left(1-\frac{a_0F(t)}{n_{m-1}\{F(t)+n_{m-1}-a_0\}}\right)\theta_{m-1} \tag{33}$$

The pressure is given by eq.(21).

<u>Saturated condition</u>

<u>1. Production</u>

From relations

$$W''v''+W'v'=V_0 \tag{34}$$

$$W_u=W''u''+W'u' \tag{35}$$

$$u''=i''-APv'' \qquad u'=i'-APv' \tag{36}$$

$$W=W''+W' \tag{37}$$

the first term of the right side in eq.(6) will be

$$\frac{dW_u}{dt}=V_0\frac{d}{dt}\left(\frac{i''-i'}{v''-v'}\right)+W\frac{d}{dt}\left(\frac{i'v''-i''v'}{v''-v'}\right)+\left(\frac{i'v''-i''v'}{v''-v'}\right)\frac{dW}{dt}-AV_0\frac{dP}{dt} \tag{38}$$

Substituting eq.(8) into eq.(38) gives

$$\frac{dW_u}{dt}=V_0\frac{d}{dt}\left(\frac{i''-i'}{v''-v'}\right)-AV_0\frac{dP}{dt}+W_{m-1}\frac{d}{dt}\left(\frac{i'v''-i''v'}{v''-v'}\right)-G_p\chi_{m-1}\left(\frac{i'v''-i''v'}{v''-v'}\right)-G_p\chi_{m-1}\frac{d}{dt}\left(\frac{i'v''-i''v'}{v''-v'}\right)t \tag{39}$$

Then, eq.(6) can be written by the substitution of eqs.(39) and (9) as

$$-G_p\left\{i_p-(1-\chi_{m-1})i_r-\chi_{m-1}\left(\frac{i'v''-i''v'}{v''-v'}\right)\right\}=V_0\frac{d}{dt}\left(\frac{i''-i'}{v''-v'}\right)-AV_0\frac{dP}{dt}+\frac{V_0}{v_{m-1}}\frac{d}{dt}\left(\frac{i'v''-i''v'}{v''-v'}\right)+\alpha V_0\gamma_rC_r\frac{d\theta}{dt}-G_p\chi_{m-1}\frac{d}{dt}\left(\frac{i'v''-i''v'}{v''-v'}\right)t \tag{40}$$

Eq.(40) can be rewritten as

$$\frac{G_p\chi_{m-1}}{V_0}\left\{\left(\frac{i'v''-i''v'}{v''-v'}\right)-\frac{i_p-(1-\chi_{m-1})i_r}{\chi_{m-1}}\right\}=\left(\left\{\frac{d}{dP}\left(\frac{i''-i'}{v''-v'}\right)-A\right\}+\frac{1}{v_{m-1}}\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)+\alpha\gamma_rC_r\frac{d\theta}{dP}-\frac{G_p\chi_{m-1}}{V_0}\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)t\right)\frac{dP}{dt} \tag{41}$$

For ranges of 200°C≤θ≤300°C and 0.7MPa≤P≤10MPa, following relations will be easily obtained.

$$\theta=179.8890P^{0.238} \tag{42}$$

$$\frac{d\theta}{dP}=42.8136P^{-0.762} \tag{43}$$

$$\left(\frac{i'v''-i''v'}{v''-v'}\right)=751.0391P^{0.236} \tag{44}$$

$$\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)=177.2452P^{-0.764} \tag{45}$$

$$\frac{d}{dP}\left(\frac{i''-i'}{v''-v'}\right)-A=8199.7385P^{-0.194} \tag{46}$$

Writing the pressure P as

$$P=P_{m-1}(1+p) \tag{47}$$

eqs.(43)~(46) can be rewritten as

$$\frac{d\theta}{dP}=\left(\frac{d\theta}{dP}\right)_{m-1}(1+p)^{-0.762} \tag{48}$$

$$\left(\frac{i'v''-i''v'}{v''-v'}\right)=\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}(1+p)^{0.236} \tag{49}$$

$$\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)=\left\{\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)\right\}_{m-1}(1+p)^{-0.764} \tag{50}$$

$$\left\{\frac{d}{dP}\left(\frac{i''-i'}{v''-v'}\right)-A\right\}=\left\{\frac{d}{dP}\left(\frac{i''-i'}{v''-v'}\right)-A\right\}_{m-1}(1+p)^{-0.194} \tag{51}$$

Substituting eqs.(47)~(51) into eq.(41) and arranging it,

$$\frac{dt}{dp}+\frac{H_{m-1}(1+p)^{-0.764}}{(1+p)^{0.236}-M_{m-1}}t=\frac{1}{(1+p)^{0.236}-M_{m-1}}\left\{J_{m-1}(1+p)^{-0.194}+K_{m-1}(1+p)^{-0.764}+L_{m-1}(1+p)^{-0.762}\right\} \tag{52}$$

is obtained, where

$$H_{m-1}=\frac{\left\{\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)\right\}_{m-1}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}}P_{m-1} \tag{53}$$

$$J_{m-1}=\frac{V_0}{G_p\chi_{m-1}}\frac{\left\{\frac{d}{dP}\left(\frac{i''-i'}{v''-v'}\right)-A\right\}_{m-1}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}}P_{m-1} \tag{54}$$

$$K_{m-1}=\frac{V_0}{G_p\chi_{m-1}}\frac{1}{v_{m-1}}\frac{\left\{\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)\right\}_{m-1}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}}P_{m-1} \tag{55}$$

$$L_{m-1}=\frac{\alpha V_0\gamma_rC_r}{G_p\chi_{m-1}}\frac{\left(\frac{d\theta}{dP}\right)_{m-1}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}}P_{m-1} \tag{56}$$

$$M_{m-1}=\frac{1}{\chi_{m-1}}\frac{i_p-(1-\chi_{m-1})i_r}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}} \tag{57}$$

Solving eq.(52) with the condition of p=0 at t=0 results

$$t=\frac{1}{(1+p)^{0.236}-M_{m-1}}\left(\frac{J_{m-1}\{(1+p)^{0.806}-1\}}{0.806}+\frac{K_{m-1}\{(1-p)^{0.236}-1\}}{0.236}+\frac{L_{m-1}\{(1+p)^{0.238}-1\}}{0.238}\right) \tag{58}$$

The temperature is given by eqs.(47) and (42).

<u>2. Water influx</u>

In this case, the differential equation will be

$$\frac{dF(t)}{dP}+\frac{\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)}{\left(\frac{i'v''-i''v'}{v''-v'}\right)-i_a}F(t)=-\frac{V_o}{2\gamma_w B}\frac{\frac{d}{dP}\left(\frac{i''-i'}{v''-v'}\right)-A}{\left(\frac{i'v''-i''v'}{v''-v'}\right)-i_a}$$

$$-\frac{V_o}{2\gamma_w B}\frac{1}{v_{m-1}}\frac{\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)}{\left(\frac{i'v''-i''v'}{v''-v'}\right)-i_a}-\frac{\alpha V_o\gamma_r C_r}{2\gamma_w B}\frac{\frac{d\theta}{dP}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)-i_a} \quad (59)$$

Writing

$$h_{m-1}=-\frac{V_o}{2\gamma_w B}\frac{\left\{\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)\right\}_{m-1}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}}P_{m-1} \quad (60)$$

$$j_{m-1}=-\frac{V_o}{2\gamma_w B}\frac{\left\{\frac{d}{dP}\left(\frac{i''-i'}{v''-v'}\right)-A\right\}_{m-1}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}}P_{m-1} \quad (61)$$

$$k_{m-1}=-\frac{V_o}{2\gamma_w B}\frac{1}{\bar{v}_{m-1}}\frac{\left\{\frac{d}{dP}\left(\frac{i'v''-i''v'}{v''-v'}\right)\right\}_{m-1}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}}P_{m-1} \quad (62)$$

$$l_{m-1}=-\frac{\alpha V_o\gamma_r C_r}{2\gamma_w B}\frac{\left(\frac{d\theta}{dP}\right)_{m-1}}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}}P_{m-1} \quad (63)$$

$$m_{m-1}=\frac{i_a}{\left(\frac{i'v''-i''v'}{v''-v'}\right)_{m-1}} \quad (64)$$

and with eqs.(47) (51), eq.(59) becomes similar equation to eq.(52), and solving it with the condition of F(t)=0 at p=0

$$F(t)=\frac{1}{(1+p)^{[illegible]}-m_{m-1}}\left(\frac{j_{m-1}\{(1+p)^{0.866}-1\}}{0.866}+\frac{k_{m-1}\{(1+p)^{0.238}-1\}}{0.238}+\frac{l_{m-1}\{(1+p)^{0.238}-1\}}{0.238}\right) \quad (65)$$

can be obtained.

CALCULATION PROCEDURE

Calculations are made by shifting the initial reservoir conditions one after another. The procedures are detailed as follows;

Compressed water condition

The first step

1. Determine the initial enthalpy i_0 and the specific volume of fluid in the reservoir at given initial temperature θ_0 and pressure P_f.
2. Calculate steam quality x_0 at the separator outlet pressure P_s by

$$x_o=\frac{i_o-i_s'}{i_s''-i_s'}$$

[Production for Δt is carried out]

3. Substituting x_0 into x_{m-1}, v_0 into v_{m-1}, Δt into t in eq.(13), θ_0 into θ_{m-1} in eq.(20) and 1 into m in eqs.(16)~(19), calculate $\bar{v}_0$, $\bar{\theta}_0$ and $\bar{P}_0$ by eq.(21) respectively.

[Water influx for Δt is carried out]

4. Substituting $\bar{\theta}_0$, $\bar{v}_0$ and $\Delta P'_1\sqrt{\Delta t}$ into θ_{m-1}, $\bar{v}_{m-1}$ and F(t), calculate v_1 by eq.(25), θ_1 by eq.(33) and P_1 by eq.(21).
5. Determine i_1 at θ_1 and P_1.

The second step

1. Giving P_1, θ_1, i_1 and v_1 to the initial conditions, calculate $\bar{P}_1$, $\bar{\theta}_1$ and $\bar{v}_1$ similarly to the prodecures 1~4.

[Water influx for Δt]

2. Substituting (P_f-P_1) into ΔP_1, $(P_1-\bar{P}_1)$ into $\Delta P'_2$, $\bar{\theta}_1$ into θ_{m-1}, $\bar{v}_1$ into v_{m-1} and $\Delta P_1\sqrt{2\Delta t}+(\Delta P'_2-\Delta P_1)\sqrt{\Delta t}$ into F(t), calculate v_2 by eq.(25), θ_2 by eqs.(29)~(32) and eq.(33), and P_2 by eq.(21), respectively.
3. Determine i_2 at θ_2 and P_2

Repeat above procedures until the reservoir fluid becomes saturated condition.

Saturated condition

The (n+1)th step

[Production for Δt]

1. Substituting the last reservoir conditions in the compressed water condition, P_n, θ_n and v_n into eqs.(53)~(57) and giving Δt to t in eq.(58), find p, that is $\bar{P}_{n+1}=P_n(1+p)$.
2. Substituting $\bar{P}_{n+1}$ into P in eq.(42), calculate $\bar{\theta}_{n+1}$.
3. Calculate $\bar{v}_{n+1}$ by eq.(13).
4. Determine $\bar{i}_{n+1}$ at $\bar{P}_{n+1}$ or $\bar{\theta}_{n+1}$.

[Water influx for Δt]

5. Substituting $\bar{P}_{n+1}$ into P_{m-1} and $\bar{v}_{n+1}$ into $\bar{v}_{m-1}$ in eqs.(60)~(64) and giving $F[\Delta t(n+1)]$ into F(t), find p, that is, $P_{n+1}=\bar{P}_{n+1}(1+p)$.
6. Substituting P_{n+1} into P in eq.(42), calculate θ_{n+1}.
7. Determine i_{n+1} at P_{n+1} or θ_{n+1}.
8. Substituting $F[\Delta t(n+1)]$ into F(t) and $\bar{v}_{n+1}$ into $\bar{v}_{m-1}$, calculate v_{n+1}.

Repeat above procedures.

SUMMARY

For the reservoir system as shown in the figure, the reservoir performance can be predicted by eqs.(20), (21), (33), (42) and (58), by considering that fluid production and water influx are alternately carried out. Taking each time step Δt as short as possible, accurate results will be obtained, especially in the compressed water condition.

REFERENCES

Fukuda, M. and Itoi, R., 1985, A Prediction Method of Geothermal Reservoir Temperature and Pressure Changes, G.R.C. Transactions, Part II.

Miller, F.G., 1962, Theory of Unsteady-State Influx of Water in Linear Reservoirs, Journal Institute of Petroleum, 48, 365.