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DIMENSIONAL ANALYSIS OF AN ECONOMIC MODEL FOR A HOT-WATER GEOTHERMAL HEATING SYSTEM

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ABSTRACT

A complex economic model was developed for evaluation of geothermal hot-water heating for large institutional systems. The Buckingham Pi method of dimensional analysis was applied to this this model in order to simplify analysis and trade-off studies. The result is a greatly-simplified equation which compares the overall cost of a geothermal system to continued use of the present system using a minimum number of key dimensionless variables.

INTRODUCTION

As part of an early study, we developed an economic model for evaluating the feasibility of hot-water geothermal heating for large, existing institutions, such as university campuses (Kauffman and Houghton, 1979). The model, described below in more detail, is quite complex. It is not easy to estimate the effect of changes in values of various parameters on the overall economic feasibility of the system. It is not easy to separate out the effects of uncertainties in input data.

In order to obtain a simpler, more readily understood economic model, we applied the Buckingham Pi technique to our complex model. The Buckingham Pi method reduces a complex problem to a minimum, but not necessarily unique, set of dimensionless groups. It has been frequently applied to physical situations, such as complex fluid flow and heat transfer problems (Langhaar, 1951; Perry et al., 1963; Bennett and Myers, 1974).

The Buckingham Pi method is based on the premise that any equation representing the behavior of a given system must be dimensionally homogeneous; thus it is possible to write it in the form of a selected number of dimensionless terms. Using π_i as a symbol for a dimensionless group, then:

$$F = f(\pi_1, \pi_2, \ldots, \pi_j)$$

The number of groups, j, required for any system is given by:

$$j = n - r$$

where n is the total number of independent variables involved and r is the number of fundamental dimensions (mass, time, etc.) incorporated in the variables.

The groups are determined by first selecting a basic set of r variables. This selection may be based on intuition concerning physical realities, random selection, etc. Each of the r fundamental dimensions, however, must be represented in this set and be associated on a one-to-one basis. The j remaining variables, V_i, are then individually combined with the proper combination of the r basic variables to render them dimensionless. Values of π_i and corresponding values for F may then be found by varying the V_i over the appropriate range of interest. The relationship between F and π_i may then be determined. By careful combining of all the relationships between F and the π_i's, the total relationship may be determined. The relative effects of various π_i's may then be determined and insignificant groups discarded. The resulting equation then relates F to the important variables of the system.

ECONOMIC MODEL

Our economic model estimates the ratio of the cost of providing heat with a new geothermal system to the cost of providing an equivalent amount of heat with an existing fossil-fuel distributed steam system. There are three major parts to the model: geothermal capital costs, geothermal operating costs and system comparison.

Geothermal capital costs include estimated costs for a production well, a disposal well, a production well pump, a distribution pump, pipeline costs, building unit costs (heat exchangers, controls, individual building modifications, etc.), engineering fee and a contingency allowance. Operating costs include power for well and distribution pumps, maintenance, operating labor and water treatment costs. The final system comparison is given by:

$$R = \frac{C_c + PC_o}{PC_s}$$

where R is the system ratio, C_c and C_o are geothermal capital and annual operating costs, P is the desired pay-out period and C_s is the annual fuel cost for the existing system.

Needless to say, there are many parameters involved in estimating the terms in the R equation. Some are well defined; e.g., the heat capacity of water. Some can be estimated reasonably well through published correlations or other data sources; e.g., costs of pumps. Terms in these two categories were fixed in our model.

Sixteen terms, however, were either left as engineering design choices or were kept as independent variables to examine their effect on R. These terms are:

T = Water temperature delivered to buildings

N = Number of buildings in the system

A = Average floor area per building

L = Peak heating demand per unit floor area

S_F = System stream factor (total annual required use divided by total annual capacity)

G_R = Average geothermal gradient

D = Water table depth

X = Distance from users to well

I = Average interconnection pipeline distance

U = Pipeline cost per unit length

B = Building modification cost

W = Water treatment cost

G = Natural gas cost

F = Fuel cost (if not natural gas)

E = Electric power cost

P = Desired payout period.

DIMENSIONAL ANALYSIS

The sixteen variables listed above include six fundamental dimensions: dollars, energy, temperature, length, time and number of buildings. We selected ten (16-6) π's for our analysis, each with a variable for adjusting it without affecting other π's.

$\pi_1 = F/G \qquad V_1 = F$

$\pi_2 = E/G \qquad V_2 = E$

$\pi_3 = \dfrac{U}{GLP\sqrt{AN}} \qquad V_3 = U$

$\pi_4 = \dfrac{B}{GLPA} \qquad V_4 = B$

$\pi_5 = \dfrac{W\sqrt{AN}}{GLP} \qquad V_5 = W$

$\pi_6 = S_F \qquad V_6 = S_F$

$\pi_7 = \dfrac{G_R\sqrt{AN}}{T} \qquad V_7 = G_R$

$\pi_8 = \dfrac{D}{\sqrt{AN}} \qquad V_8 = D$

$\pi_9 = I\sqrt{N/A} \qquad V_9 = I$

$\pi_{10} = \dfrac{X}{\sqrt{AN}} \qquad V_{10} = X$

Values of R were calculated for a typical base case and for a large number of cases deviating from the base case in the value of one V_i. Typical results are shown in Figures 1 and 2 for π_4 and π_7.

Linear relationships were found for R vs π_i or R vs $1/\pi_i$ for all except π_7, which was found to fit an exponential function. We then were able to construct an overall equation for R:

$$R = \left(\frac{k_1}{\pi_i}\right)\left(1 + k_2\pi_2\right)\left(1 + k_3\pi_3\right)\left(1 + k_4\pi_4\right)$$
$$\times \left(1 + k_5\pi_5\right)\left(1 + \frac{k_6}{\pi_6}\right)\left(e^{k_7/\pi_7}\right)\ 1 + k_8\pi_8)$$
$$\times \left(1 + k_a\pi_a\right)\left(1 + k_{10}\pi_{10}\right)$$

We evaluated the k's and the size of the π's for reasonable physical and economic values. Terms involving π_3, π_5, π_8, π_9 and π_{10} were shown to be of negligible importance.

We then formulated a new function with new k's:

$$R = \left(\frac{k_1}{\pi_1}\right)\left(1 + k_2\pi_2\right)\left(1 + k_4\pi_4\right)\left(1 + \frac{k_6}{\pi_6}\right)\left(e^{k_7/\pi_7}\right)$$

This equation was found to fit all of our calculated case data to within ±10 percent over the ranges of V_i's which are of interest. This permits us to carry out sensitivity analyses and other trade-off studies with much greater ease than using the full, computerized model.

It should be pointed out that our selection of π's and the functional form of our simplified equation are not unique. A wide variety of choices could be available. In fact, we experimented with other sets of π's; overall results were similar to the set outlined above.

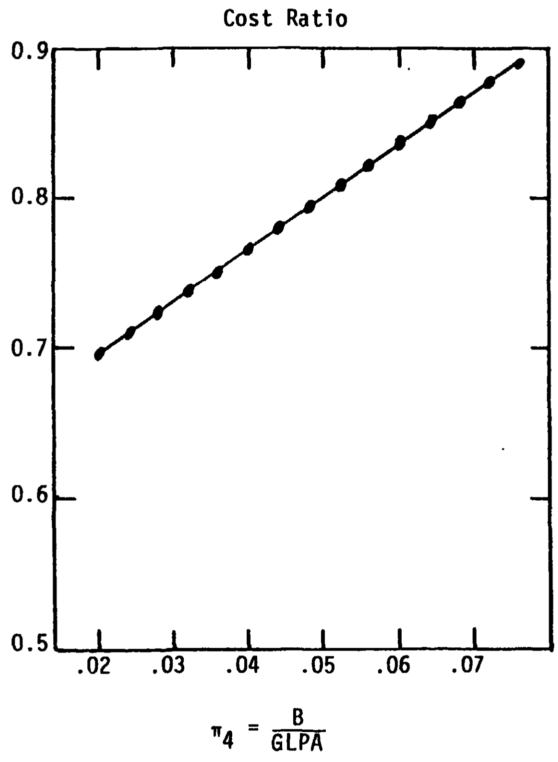

Fig. 1. Cost ratio as a function of π_4. Result is linear.

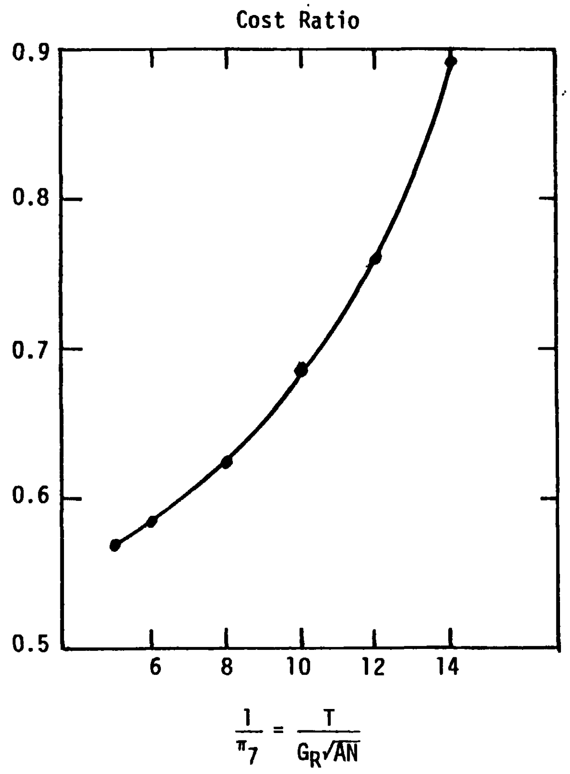

Fig. 2. Cost ratio as a function of π_7. Result is exponential in $1/\pi_7$.

CONCLUSIONS

The Buckingham Pi method can be used to analyze and simplify complex problems involving both engineering and economic factors. Considerable care and judgment are required, however, in order to select the most useful combinations for a given problem. The resulting simplified correlations can be used for rapid trade-off studies and other analyses.

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