DESIGN CONSIDERATIONS OF A DOWN-HOLE COAXIAL GEOTHERMAL HEAT EXCHANGER

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ABSTRACT

This report is an analysis of the performance of a coaxial heat exchanger used for the extraction of heat from a geothermal well, for use in space or process heating. The calculations are based on conductive heat transfer into the well and are therefore a lower bound on the performance of such a system, since in most cases heat will be transferred to the well convectively. The analysis makes use of full solution to the problem including temperature variation with depth, transfer between inner and outer tubes, friction and the cooling of the formation with time.

FORMULATION

The analysis of the heat transfer in a coaxial down-hole heat exchanger is closely related to the determination of heat exchange due to mud circulation during drilling (see for example Keller, Couch and Berry, 1973, and Tanaka and Yoshida, 1979). The heat exchanger is somewhat simpler since there is no heat transfer due to rotation of the string or from tool heating. The formulation of the solution can be separated into two parts--determination of the heat transfer between the two coaxial flows, and calculation of the heat transfer in the formation.

Considering first the heat transfer between the tubes, the configuration may be represented as in figure 1. The flow shown is down the center tube and returning back up the annulus; however the reverse flow may also be considered by substituting a negative velocity into the analysis.

Conservation of energy allows the formulation of the equations governing the upflowing and downflowing temperatures, namely:

$$\frac{dT_2}{dx} = F + \frac{2}{\rho c_v U_1 (R^2 - r^2)} [h_1 R (T_1 - T_2) - h_2 r (T_2 - T_3)] \quad \ldots(1)$$

and

$$\frac{dT_3}{dx} = -F - \frac{2h_2}{\rho c_v U_2 r} (T_2 - T_3) \quad \ldots(2)$$

where

$$F = \frac{1}{\rho c_v} \frac{dp}{dx} \quad \ldots(3)$$

is the heating effect due to friction.

These equations can be solved simultaneously and yield an expression for the exit temperature.

$$T_2^* = T_1^* - [(1 - \frac{\lambda_1}{\alpha}) A_1 e^{\lambda_1 \ell} + (1 - \frac{\lambda_2}{\alpha}) A_2 e^{\lambda_2 \ell}] + \frac{F}{\alpha r} \quad \ldots(4)$$

where

$$A_1 = - \frac{\lambda_2 \phi_0 + (M - F) e^{\lambda_2 \ell}}{\lambda_1 e^{\lambda_2 \ell} - \lambda_2 e^{\lambda_1 \ell}}$$

$$A_2 = \frac{\lambda_1 \phi_0 + (M - F) e^{\lambda_1 \ell}}{\lambda_1 e^{\lambda_2 \ell} - \lambda_2 e^{\lambda_1 \ell}}$$

$$\phi_0 = (T_1^* - T_3^*) - \frac{M}{\alpha} + \frac{F}{\alpha r}(1+\gamma)$$

$$\lambda_1, \lambda_2 = \tfrac{1}{2} \alpha \gamma (-1 \pm \sqrt{1 + 4/\gamma}$$

$$\alpha = \frac{2h_2 \Pi r}{\rho c_v Q}, \quad Q = \pi r^2 U_2$$

$$\gamma = \frac{h_1 R}{h_2 r}$$

Here the * refers to wellhead conditions and M is the geothermal gradient. It should be noted that h_1, the overall transfer coefficient between the outer tube and the earth, is a function of time. However the time rate of change is considered to be sufficiently small that the problem is quasi-steady. The variation with respect to time can be considered by changing the value of h_1 in the solution. The value of h_1 is determined by solving the heat diffusion equation in the earth:

$$K[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r}] = \frac{\partial T}{\partial t}$$

with boundary condition $k \frac{\partial T}{\partial r} = h^* (T - T_\infty)$

where k and K are the thermal conductivity and diffusivity of the earth and h^* is the purely convective heat transfer coefficient. T is the wall temperature (i.e. wall temperature at time zero). The effective heat transfer coefficient (h_1) in terms of distant earth temperatures is therefore

$$h_1 = h^* \frac{T-T_\infty}{T_0-T_\infty}$$

Where T_0 is the formation temperature at time zero. T is the solution to equation 5, which may be determined in Laplace transformed space, where

$$\left[\frac{T - T_\infty}{T_0 - T_\infty}\right]_L = \frac{1}{s}\left\{1 - \frac{\frac{h^*}{k} K_0\left(\sqrt{\frac{s}{K}}\, a\right)}{\sqrt{\frac{s}{K}}\, K_1\left(\sqrt{\frac{s}{K}}\, a\right) + \frac{h^*}{k} K_0\left(\sqrt{\frac{s}{K}}\, a\right)}\right\} \quad (6)$$

where s is the Laplace transform variable, a is the wall radius and K_0 and K_1 are modified Bessel functions.

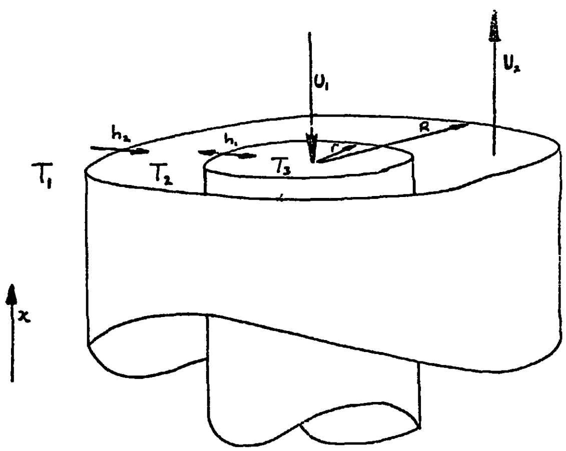

Figure 1: Problem configuration

The inversion of this transform may be done numerically, providing a value for h_1 (generally in the order of 10 W/m^2 ^{0}C).

The inner heat transfer coefficient may be determined as that relevant to a double-pipe heat exchanger namely:

$$\frac{1}{h_2} = \frac{r_o^2}{r_i^2}\frac{1}{h_i} + \frac{r_o}{k_s}\ln\frac{r_o}{r_i} + \frac{1}{h_o} \quad (7)$$

where r_i and r_o refer to the inside and outside radii of the inner pipe, k_s is the thermal conductivity of the pipe, and h_i and h_o the inside and outside convective heat transfer coefficients are given typically by:

$$h_i = \frac{k_f}{r_i}\, 0.0395\, (Re_i)^{0.75} \quad (8)$$

where k_f is the thermal conductivity of the fluid and Re is the Reynolds number of the flow. This empirical correlation is for turbulent flow in vertical pipes (Holman, 1968, p. 146).

RESULTS

The energy output of a downhole heat exchanger is a function of many variables depending on the configuration and formation conditions. The output is governed by the rate and direction of flow and is also a function of time as heat is extracted from the formation. As a test case, formation conditions typical of the 200m well at Tauhara College Taupo will be used:

$$(T_1{}^* = 100^0C,\ M = 0.1^0C/m) \quad (9)$$

The results summarized in Table 1 indicate the dependence on a) input temperature, b) frictional heating, c) inner tube radius, d) outer tube radius, e) flow rate and f) flow direction. The time variation may be excluded by considering the output at a given time (the listed energy output is after about 1 week).

These variations are illustrated in figure 2, and can be essentially summarized as follows: To maximize energy transfer

(a) Inner tube should be as small as possible
(b) Outer tube should be as large as possible
(c) Flow is down the annulus and up the inner tube.

The frictional heating and input temperature do not greatly affect the energy transfer. To achieve optimum transfer the flow rate should be maximized, however it should be remembered that the temperature difference achieved will be reduced, and also that friction in the pipes may restrict the magnitude of the flow rate.

OPTIMIZATION

If it is assumed that the maximum achievable velocity of flow is restricted to (say) 5m/sec, then it is impossible to achieve the two desirable goals of minimizing inner pipe radius and maximizing flow rate. With this restriction, the energy transfer as a function of inner pipe diameter will be as summarized in Table 2. It is clear then the beneficial effect of increasing the flow rate outweighs the effect of decreasing the inner pipe radius. It should also be noted that the final example considered (r = 0.1272) results in the maximum velocity also being achieved in the annulus, and there is an enhancement of heat exchange with the outside.

TABLE 1 - Thermal output as a function of heat exchanger parameters

a) Input Temperature

$r = 0.0572m$; $R = 0.1699m$; $Q = 0.03\ m^3/s$

Inlet temp°C	0	10	20	30	40
Energy (kw)	47.5	47.5	47.5	47.4	47.4

b) Frictional Heating

$r = 0.0572m$; $R = 0.1699m$; $Q = 0.03m^3/s$; $T_3^* = 40°C$

Pressure drop MPa	0	2	5	10	50
Energy (kw)	47.4	47.4	47.4	47.4	47.4

c) Inner tube radius

$R = 0.1699m$; $Q = 0.01m^3/s$; $T_3^* = 40°C$

Inner radius r(m)	0.0172	0.0372	0.0572	0.0872	0.1272
Energy (kw)	58.1	26.7	20.9	18.5	18.2

d) Outer tube radius

$r = 0.0572$; $Q = 0.03\ m^3/s$; $T_3^* = 40°C$

Outer radius R(m)	0.1099	0.1499	0.1699	0.1899
Energy (kw)	34.0	42.9	47.4	52.0

e) Flow rate

$r = 0.0572$; $R = 0.1699\ m$; $T_3^* = 40°C$

Flow rate (m^3/sec)	0.001	0.005	0.01	0.02	0.03	0.04
Energy (kw)	5.43	13.3	20.9	34.5	47.4	60.0

f) Flow direction

$r = 0.0572\ m$; $R = 0.1699m$; $T_3^* = 40°C$

Flow rate (m^3/sec)	0.001	0.01	0.02
Energy (forward flow)	5.43	20.9	34.5
Energy (reverse flow)	7.68	22.7	36.0

$R = 0.1699m$; $T_3^* = 40°C$; $Q = 0.01m^3/sec$ (reverse)

Inner radius (m)	0.0172	0.0372	0.0572	0.0872	0.1272
Energy (kw)	61.0	28.9	22.7	20.1	19.8

CONCLUSION

(a) If the purpose of the downhole heat-exchanger is to provide maximum energy transfer, the optimum configuration is one which has the same velocity (or Reynolds number) in the upward and downward flows. The flow can then be maximized to the capacity of the pumps, after which the energy transfer will also be a maximum.

(b) It is recognized that the maximum heat transfer may in fact give rise to an insufficent temperature difference between inlet and outlet. In this case there is some advantage in reducing the size of the inner pipe.

(c) The "reverse" flow configuration in which fluid flows down the annulus and back up the inner pipe results in slightly greater heat transfer.

(d) The outer tube of the heat exchanger should be as large as the well permits.

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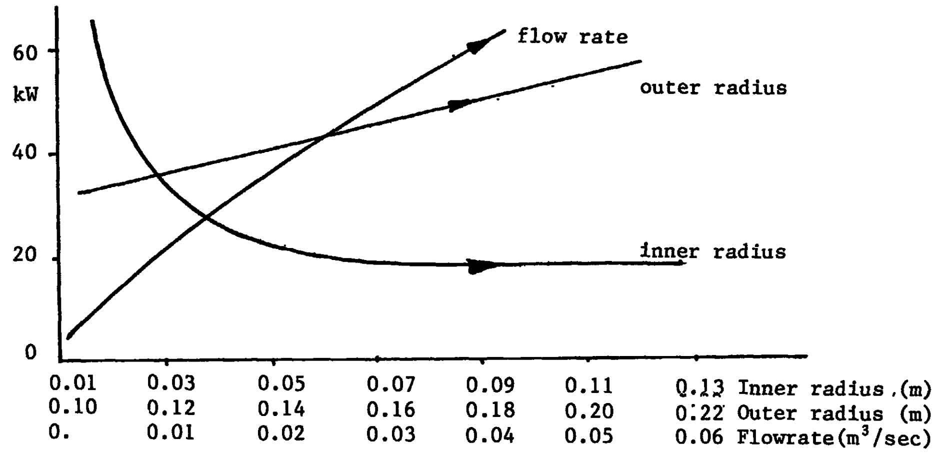

Figure 2: Heat exchanger output as a function of configuration and operational parameters

Inner pipe radius (m)	0.0172	0.0372	0.0572	0.0872	0.1272
Maximum allowable flow rate (m^3/sec)	0.0008	0.0116	0.0350	0.0936	0.216
Energy transfer (kw)	6.94	30.2	53.7	89.0	340

Table 2 - Heat exchanger output for maximum flow velocity 5m/sec