THE CHARACTERIZATION OF CRACK STRUCTURE CONTROLLING PERMEABILITY USING VELOCITY-PRESSURE DATA

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Expressions for permeability and elastic moduli can be derived. Such expressions provide a means of modeling the microstructure controlling permeability and velocity and they elucidate the feasibility of predicating the functional dependence of permeability on pressure from independent determination of microstructure, such as from ultrasonic data.

An equation for the diffusion of fluid through cracked rock has been derived by H. Fisher [1], and involves explicitly the permeability $\underline{k}$, and the pressure derivative of a "conduit porosity" θ, with respect to pore pressure P_h.

$$\nabla \cdot (k \cdot \nabla P_h) = \mu \frac{d\theta}{dP_h} \frac{\delta P_h}{\delta t} \tag{1}$$

In equation (1) $\underline{\mu}$ is viscosity and $\underline{t}$ is time. The porosity is defined by its total derivative

$$\frac{d\theta}{dP_h} = \frac{1}{V} \frac{dV_h}{dP_h} \tag{2}$$

where V is the bulk volume and V_h is the pore volume at any pressure and temperature. Difficulty arises in evaluating equation (1). As Fisher notes: "The related quantities k, θ, and $d\theta/dP$ and in particular the dependence on pressure cannot be exactly determined since they also depend on the pore volumes." However, it is possible to approach this problem by seeking to characterize the manner in which the microstructures that form the conduits for fluid flow can be related to elastic properties such as a compliance through their effects on bulk strain and velocities.

In sufficient generality, a normalized elastic compliance ϕ/ϕ_o, (such as compressibility) can be expressed in terms of a "referred porosity" $\underline{\omega}$ and its pressure derivative [2].

$$\omega = 1 - V_o/V \tag{3}$$

where V_o is the reference solid material and

$$-\frac{d\omega}{dR} = (1 - \omega)\phi_o \cdot [\phi/\phi_o - 1] \tag{4}$$

where R is the applied stress state which is used in defining ϕ (for example, R is applied pressure in the case of compressibility). All the porosity affecting the elastic deformation is involved. This is an "elastic porosity" to distinguish it from the conduit porosity.

The expression for $(\phi/\phi_o - 1)$ can be obtained from velocity data or from modeling theory [2,3]. Velocity-pressure data for rocks show systematics which are related to microstructure and which can be inverted into model microstructure by expressing equation (4) in terms of distributions over partial porosities (${}_i\eta$, given by ${}_iV_h/V$) and pore strains (${}_ie_h/e_o$).

$$\phi/\phi_o - 1 + \eta\frac{dP_h}{dP} = \frac{1}{1 - \eta} \sum_i {}_i\eta\left(\frac{{}_ie_h}{e_o}\right) \tag{5}$$

The question arises, can the conduit porosity be related to the elastic porosity? At present this question requires experiments to be answered. One cannot tell yet whether the two porosities are numerically equal, but the pressure derivatives may well be relatable. The quantity $d\theta/dP_h$ can be stated in terms of $d\omega/dR$ through the expressions for the pore strains. In equation (5) the geometries of the pores and their closure laws are not restricted. For cracks, the approximate form of the solution follows, using the distributions in a form involving a pore number density.

If the crack network is characterized by some distributions in length $\underline{a}$ and width $\underline{b}$, then the number density distribution satisfies

$$\int_a \int_b \frac{d^2W}{da\,db}\, da\,db = N \tag{6}$$

where N is the total crack number density and

$$(\phi/\phi_o - 1) \sim \int_a \int_b a^3 \frac{d^2W}{da\,db}\, da\,db \tag{7}$$

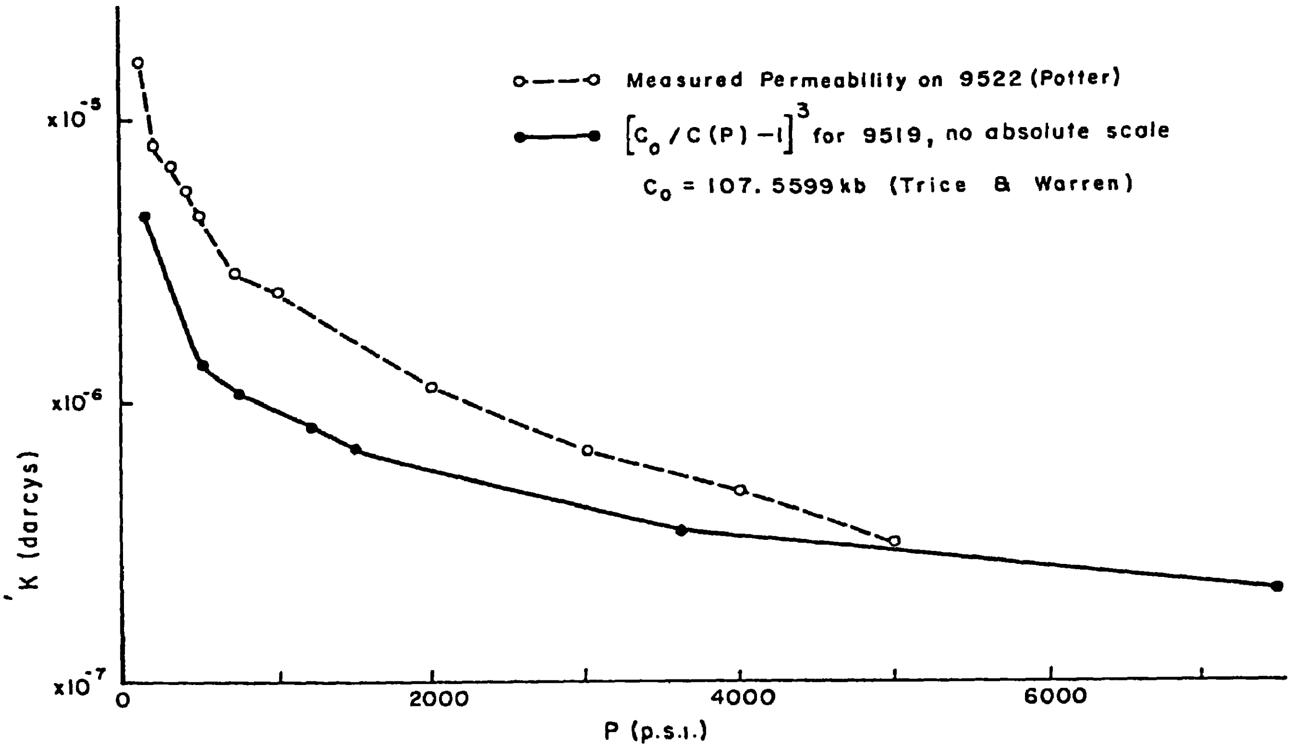

Fig. 1 Permeability and the cubic function of acoustic data versus pressure for GT-2-9522/9519 (permeability data on 9522) from J. Potter, personal communication.

If the Kozenzy relation is assumed for permeability

$$k = c \cdot \theta \bar{b}^2 \tag{8}$$

where $\underline{c}$ is a constant and $\bar{b}$ is the characteristic crack width given by the ratio of pore volume to surface area

$$\bar{b} = \int_a \int_b a^2 b \frac{d^2W}{dadb}\, dadb \Bigg/ \int_a \int_b a^2 \frac{d^2W}{dadb}\, dadb \tag{9}$$

and can be obtained for a model crack spectrum fitting the velocity data. The pressure dependence of $\underline{k}$ is thus expressible as a function of the pressure dependence of the density distribution.

Two conclusions that can be drawn are: 1) that various pressure laws are equally admissible, which may help explain why published power laws fit to permeability have shown such a wide range of exponents (from 2 or 3 to 6), and 2) that comparison of acoustic data or model-crack spectra to permeability-pressure data may elucidate the effects of tortuosity and pinch-off on the conduit porosity. Figure 1 illustrates these points. The figure shows a comparison of permeability data to a cubic function of the compressional wave velocity data for a sample of granodiorite from LASL's Fenton Hills HDR geothermal site, New Mexico [4]. The velocity-pressure data can be modeled assuming cracks with linear closure laws. The parallelness of these curves indicates that the same microcrack network controls both permeability and acoustic properties and that, based on the model, permeability goes as the average crack width (over all pores) cubed.

References

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